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關於氫二核之光致蛻變*

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在這裏，我們扼要地說明了如何導出關於氫二核之光致蛻變的截面及角分佈的公式。在求得這些公式的過程中，我們把中子和質子看成同樣的質點，並應用忽魯登變分法所求得之波函數去計算各種躍遷之矩陣素。

平常大家都覺得從氫二核之光致蛻變中可以大致瞭解一些關於核子勢之情況。在這篇文章中，我們扼要地說明氫二核之光致蛻變的截面及角分佈的公式是怎樣求來的。在求得這些公式的過程中，我們把中子和質子看成同樣質點的不同狀態，這種質點叫做核子，核子必須滿足鮑里不相容原理。我們把核子勢取作任意形狀底短力程之中心勢。

氫二核之光致蛻變可以看成一個帶有電荷 e 之二核子系，由於吸收一個動量爲 $\hbar k_\gamma$（能量爲 $E_\gamma=\hbar ck_\gamma$）之光子，從氫二核之 ^3S 基態 Ψ_i 變到氫二核之蛻變態 Ψ_f。在每秒中的躍遷或然率可以由下列公式求出：

$$\Gamma=\frac{2\pi}{\hbar}\left|<f\mid W\mid i>\right|^2\rho_f, \tag{1}$$

此中之 W 是核子與光子之間相互作用，ρ_f 是在每單位能程中蛻變態的密度。用 $P(j)$ 或 $N(j)$ 來表示第 j 個核子的電荷函數按核子 j 是質子或中子，用 $\alpha(j)$ 或 $\beta(j)$ 來表示其自旋函數的兩個自旋排列方向。令

$$\boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2),\qquad \boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2.$$

在初態中，波函數可以寫成

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$$\Psi_i=\psi(\boldsymbol{r},\boldsymbol{R})\frac{1}{\sqrt{2}}\Big[P(1)\,N(2)-P(2)\,N(1)\Big]\begin{cases}\alpha(1)\,\alpha(2)\\ \frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2)+\alpha(2)\,\beta(1)\Big],\\ \beta(1)\,\beta(2)\end{cases} \tag{2}$$

式中
$$\psi(\boldsymbol{r},\boldsymbol{R})=\frac{1}{\sqrt{V}}e^{\frac{i}{\hbar}\boldsymbol{p}\cdot\boldsymbol{R}}\frac{1}{(4\pi)^{1/2}}\frac{u(r)}{r}. \tag{3}$$

質量中心的運動用平面波函數來表示。因爲在初態時，氫二核是靜止的，所以其質量中心之動量 $\boldsymbol{p}$ 爲零。V 爲平面波函數之正化體積。$u=u(r)$ 是氫二核在 ^3S 基態中的徑波函數，並滿足下列正化條件

$$\int_o^\infty u^2\,dr=1,$$

u 同時也滿足方程式

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(-\epsilon-V(r))\,u=0, \tag{4}$$

式中 M 表示核子之質量，$\epsilon\,(>0)$ 表示氫二核之結合能，同時 $v(r)$ 表示適於 ^3S 狀態之核子勢。三重線終態的波函數可以寫成

$$\Psi_f=\frac{1}{\sqrt{V}}e^{\frac{i}{\hbar}\boldsymbol{p}'\cdot\boldsymbol{R}}\frac{1}{k}\sum_{l=o}^{\infty}i^l(2l+1)\frac{1}{\sqrt{2}}\Big[P(1)\,N(2)-(-1)^l P(2)\,N(1)\Big]e^{-i\delta_l}\frac{u_l(r)}{r}P_l(\cos\angle\,\boldsymbol{k},\boldsymbol{r})\begin{cases}\alpha(1)\,\alpha(2)\\ \frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2)+\alpha(2)\,\beta(1)\Big],\\ \beta(1)\,\beta(2)\end{cases} \tag{5}$$

單線終態的波函數可以寫成

$$\Psi_f=\frac{1}{\sqrt{V}}e^{\frac{i}{\hbar}\boldsymbol{p}'\cdot\boldsymbol{R}}\frac{1}{k}\sum_{l=o}^{\infty}i^l(2l+1)\frac{1}{\sqrt{2}}\Big[P(1)\,N(2)+(-1)^l P(2)\,N(1)\Big]e^{-i\delta_l}\frac{u_l(r)}{r}P_l(\cos\angle\,\boldsymbol{k},\boldsymbol{r})\frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2)-\alpha(2)\,\beta(1)\Big], \tag{6}$$

$u_l=u_l(r)$ 滿足

$$\frac{d^2 u_l}{dr^2}+\left(k^2-\frac{M}{\hbar^2}V_l(r)-\frac{l(l+1)}{r^2}\right)u_l=0 \tag{7}$$

式中 V_l 表示適於自旋之核子勢。 u_l 並滿足下列邊界條件

$$u_l=0 \text{ 當 } r=0, \quad u_l\sim\sin\left(kr-\frac{l}{2}\pi+\delta\right) \text{ 當 } r\longrightarrow\infty. \tag{8}$$

$\boldsymbol{p}'$ 表示兩個核子在蛻變之後的總動量，其相對動量用 $\hbar\boldsymbol{k}$ 來表示，其方向是由質子指向中子。 由於不滅原理我們有

$$\boldsymbol{p}'=\boldsymbol{p}+\hbar\boldsymbol{k}_\gamma=\hbar\boldsymbol{k}_\gamma \quad \text{及} \quad E=\frac{\hbar^2 k^2}{M}=E_\gamma-\epsilon-\frac{p'^2}{4M}=E_\gamma-\epsilon-\frac{E_\gamma^2}{4Mc^2}$$

應用忽魯登[1] 的變分法可以得到很近似的波函數 u, u_l 以及週相 δ_l。 u 及 u_l 可以寫成

$$u=(1-e^{-\lambda r})\sum_n B_n e^{-n\lambda r}e^{-\kappa r}, \qquad \kappa=\sqrt{\frac{M\epsilon}{\hbar^2}}, \tag{9}$$

$$u_l=\cos\delta_l\left[\sum_n C_n e^{-n\lambda r}\right]F_l(r)+\sin\delta_l\left[\sum_n C_{-n} e^{-n\lambda r}\right](1-e^{-\lambda r})^{2l+1}G_l(r), \tag{10}$$

式中 B_n, C_n, C_{-n} ($n=0, 1, \cdots$) 是變分參數，$1/\lambda$ 是核子勢的力程（所以 V 及 V_l 趨近於零當 $\lambda r>1$），

$$F_l(r)=\left(\frac{\pi kr}{2}\right)^{1/2}J_{l+\frac{1}{2}}(kr)\sim\sin\left(kr-\frac{l}{2}\pi\right) \text{ 當 } r\longrightarrow\infty, \tag{11}$$

$$G_l(r)=(-1)^l\left(\frac{\pi kr}{2}\right)^{1/2}J_{-l-\frac{1}{2}}(kr)\sim\cos\left(kr-\frac{l}{2}\pi\right) \text{ 當 } r\longrightarrow\infty. \tag{12}$$

應用泰勒定理，我們可以把核子與光子間之相互作用在氫二核之質量中心附近展開。當入射光子的能量不過高時，W 可以寫成[2]

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1. L. Hulthén, *Kgl. Fysiograf. Sällsk. Lund Förhandl.* **14** (1944), No. 8, No. 21. 並參看彭桓武，黃祖洽在本期內發表之論文。

2. 如果有人用介子理論的觀點去討論核子的電磁性，當光子的能量相當低時，此處所寫的 W 之展開式仍然正確。 見 L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1944), Vol. II. p. 449.

$$W=-e\sum_{j=1}^{2}\frac{1+\tau_3^{(j)}}{2}(\boldsymbol{r}_j-\boldsymbol{R})\cdot\boldsymbol{E}_o-e\sum_{j=1}^{2}\frac{1+\tau_3^{(j)}}{2}\frac{1}{2}\left((\boldsymbol{r}_j-\boldsymbol{R})(\boldsymbol{r}_j-\boldsymbol{R})\cdot\nabla_o\right)\cdot\boldsymbol{E}_o$$

$$-\sum_{j=1}^{2}\frac{e\hbar}{2Mc}\left(\frac{1+\tau_3^{(j)}}{2}\mu_p+\frac{1-\tau_3^{(j)}}{2}\mu_n\right)\boldsymbol{\sigma}_j\cdot\boldsymbol{H}_o, \qquad (13)$$

式中 μ_p 和 μ_n 是質子和中子的磁矩，它們的單位用波爾的原子核磁子 $\frac{e\hbar}{2Mc}$ 表示出來。在氫二核的質量中心上，入射光子的電場和磁場的數值是 $\boldsymbol{E}_o$ 和 $\boldsymbol{H}_o$。$\tau_3^{(j)}$ 運算在電荷函數 $P(j)$ 上得到本徵值 $+1$，運算在 $N(j)$ 上得到 -1。

當計算躍遷矩陣素的時候，由 W 的前兩項我們得到所謂光電躍遷，由第三項我們得到所謂光磁躍遷。關於自旋變更的選擇定則對於這兩種躍遷是不同的。光電躍遷永遠把三重線基態變到三重線終態，而同時並不變更其自旋排列方向。光磁躍遷既可以把三重線基態變到三重線終態，又可將其變成單線終態，而同時又永遠變更其自旋排列方向，因此我們能分開來計算由於這兩種躍遷的截面，於是氫二核之光致蛻變的截面就是這兩種躍遷截面的總和。因為 Ψ_i 是 ^3S 狀態，當我們計算 $<f\,|\,w\,|\,i>$ 內角積分的時候，由於電偶極躍遷，我們僅得到 ^3P 終態；由於電四極躍遷，我們僅得到 ^3D 終態。其次我們再談磁偶極躍遷。因為當自旋不同時，核子勢是不相同的，所以 ^1S 終態的波函數並不正交於 ^3S 基態的波函數。於是我們可以知道由於磁偶極磁遷，我們僅能得 ^1S 終態，而不能得到 ^3S 終態。

我們把核子在終態中的自旋加起來，同時我們把在初態中氫二核的自旋排列方向以及入射光子的極化方向平均一下，我們就可以由 (1) 式得到氫二核之光致蛻變的微分截面

$$d\sigma=f(\theta)\,d\Omega=\frac{1}{8\pi}\Big\{2\,\sigma_M+\sin^2\theta\,[3\,\sigma_d+6\,(5\,\sigma_d\,\sigma_q)^{1/2}\cos(\delta_1-\delta_2)\cos\theta$$

$$+15\,\sigma_q\cos^2\theta]\Big\}\,d\Omega \quad \theta=\angle\,\boldsymbol{k},\boldsymbol{k}_r, \quad d\Omega=2\pi\sin\theta\,d\theta \qquad (14)$$

其中電偶極截面 σ_d，電四極截面 σ_q，以及磁偶極截面 σ_m 可以由下列公式求出：

$$\sigma_d=\frac{\pi}{3}\frac{e^2}{\hbar c}\left(\frac{M}{E}\right)^{1/2}\frac{E_\gamma}{\hbar}\,|\,I_1\,|^2, \qquad I_1=\int_o^\infty u_1\,r\,u\,dr; \qquad (15)$$

$$\sigma_q=\frac{\pi}{240}\frac{e^2}{\hbar c}\left(\frac{M}{E}\right)^{1/2}\frac{E_\gamma^3}{\hbar^3c^2}\,|\,I_2\,|^2, \quad I_2=\int_o^\infty u_2\,r^2\,u\,dr; \qquad (16)$$

以及 $$\sigma_M=\frac{\pi}{6}\frac{e^2}{\hbar c}\frac{E_\gamma}{Mc^2}\frac{\hbar}{\sqrt{ME}}(\mu_p-\mu_n)^2\,|I_0|^2,\quad I_0=\int_0^\infty u_0\,u\,dr\,; \tag{17}$$

式中 u_1, u_2 以及 u_0 是 ^3P, ^3D 以及 ^1S 終態的徑波函數。把 (14) 式積分一下，我們得到總截面為

$$\sigma=\sigma_M+\sigma_d+\sigma_q=\sigma_M+\sigma_E\,. \tag{18}$$

如果我們用波函數 (9) 及 (10)，由於下列公式[3]

$$\int_0^\infty e^{-\alpha r}\,r^{l+\frac{1}{2}}\,J_{l+\frac{1}{2}}(kr)\,dr=\frac{(2k)^{l+\frac{1}{2}}\,\Gamma(l+1)}{(\alpha^2+k^2)^{l+1}\sqrt{\pi}}\,;$$

$$\int_0^\infty e^{-\alpha r}\,r^{l+\frac{1}{2}}\,J_{-l-\frac{1}{2}}(kr)\,dr=\frac{(k/2)^{-(l+\frac{1}{2})}}{(\alpha^2+k^2)^{l+1}\,\Gamma(-l+\frac{1}{2})}\,\alpha^{2l+1}\,F\left(-l,-l-\tfrac{1}{2},-l+\tfrac{1}{2};-\frac{k^2}{\alpha^2}\right),$$

我們可以得到積分

$$I_l=\int_0^\infty u_l\,r^l\,u\,dr \qquad (l=1,\ 2,\ 0)\,.$$

當 l=1, 2 或 0 時，超幾何級數變成

$$F\left(-l,-l-\tfrac{1}{2},-l+\tfrac{1}{2};-\frac{k^2}{\alpha^2}\right)=(2l+1)\sum_{i=0}^{l}\binom{l}{i}\frac{1}{2l+1-2i}\left(\frac{k^2}{\alpha^2}\right)^i.$$

令

$$R(n,m;l)=\frac{l\,!}{2}(2k)^{l+1}\left[\frac{1}{[(n\lambda+m\lambda+\kappa)^2+k^2]^{l+1}}-\frac{1}{[(n\lambda+m\lambda+\lambda+\kappa)^2+k^2]^{l+1}}\right],$$

$$S(n,m;l)=\sum_{j=0}^{2l+2}(-)^j\binom{2l+2}{j}\frac{(2l+1)\,!}{2^l\,l\,!}\,\frac{(n\lambda+m\lambda+j\lambda+\kappa)^{2l+1}}{k^l\,[(n\lambda+m\lambda+j\lambda+\kappa)^2+k^2]^{l+1}}\,T(n,m;j;l),$$

式中 $$T(n,m;j;l)=\sum_{i=0}^{l}\binom{l}{i}\frac{1}{2l+1-2i}\left(\frac{k^2}{(n\lambda+m\lambda+j\lambda+\kappa)^2}\right)^i.$$

最後 I_l 可寫為

$$I_l=\cos\delta_l\sum_n\sum_m C_n\,B_m\,R(n,m;l)+\sin\delta_l\sum_n\sum_m C_{-n}\,B_m\,S(n,m;l)\,.$$

$$(l=1,\ 2,\ 0) \tag{19}$$

3. 參看 G. N. Watson, *Theory of Bessel Functions*, 2nd Edition, pp. 385, 386.

如果我們忽略 (14) 式中的 $\sigma_a \cos^2\theta$ 項，我們就得到蛻變質子的角分佈公式如下：

$$f(\theta) \propto a + \sin^2\theta\,(1 + \alpha\cos\theta)\,,\quad a = \frac{2}{3}\,\frac{\sigma_M}{\sigma_d}\,,\quad \alpha = 2\,(5\,\sigma_q/\sigma_d)^{1/2}\cos(\delta_1 - \delta_2) \qquad (20)$$

在實驗室系中，上公式變成

$$f(\theta_{\text{lab.}}) \propto a + \sin^2\theta_{\text{lab.}}\left[1 + \left(\alpha + \frac{2\beta\,E_\gamma}{E_\gamma - \epsilon}\right)\cos\theta_{\text{lab.}}\right],\quad \beta = \left(\frac{E_\gamma - \epsilon}{M c^2}\right)^{1/2}, \qquad (21)$$

上兩式中的 $\alpha\cos\theta$ 項，或 $\alpha\cos\theta_{\text{lab.}}$ 項之所以存在，是由於電偶極躍遷跟電四極躍遷互相干涉而來的。有些作者說電偶極躍遷並不能跟電四極躍遷發生干涉，因爲終態是屬于不同的電荷態，所以當我們把不同的電荷態加起來的時侯，干涉項就不會產生了。但又有作者[4]說如果一種狀態有確定的角動量和自旋，那麼其電荷態就被鮑里的不相容原理所固定，於是我們不可能把電荷態加起來，因之我們仍然可以得到干涉項。在我們的計算中，我們很明顯地包括了電荷函數，我們確實得到了這干涉項。

關於中子被質子的光性俘獲，我們由 (14) 式可以得有其微分俘獲截面爲[5]

$$d\sigma_{\text{cap.}} = \frac{3}{2}\,\frac{1}{ME}\left(\frac{E_\gamma}{e}\right)^2 f(\theta)\,d\Omega_\gamma \qquad (22)$$

此工作多蒙彭桓武先生之指導，作者在此表示謝意。

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4. J. F. Marshall and E. Guth, *Phys. Rev.* **76** (1949), 1879. 並參看 J. F. Marshall and E. Guth, *Phys. Rev.* **78** (1950), 738.
5. 看參 H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York) p. 61. J. Schwinger, 原子核物理講義, 哈佛, 1947 (hectographed notes on nuclear physics, Harvard, 1947).

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NOTE ON THE PHOTO-DISINTEGRATION OF THE DEUTRON*

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ABSTRACT

A brief derivation of the formulae for the cross-sections and angular distribution for the photo-disintegration of the deuteron is given here by treating protons and neutrons as identical particles. Wave functions obtained by Hulthén's variational method are used to calculate the matrix elements for various transitions.

It is generally expected that some information about the nuclear potential can be obtained by studying the photo-disintegration of the deuteron. In this paper, we shall give a brief derivation of the formulae for the cross-sections and the angular distribution of the disintegrated particles, treating protons and neutrons as identical particles, called nucleons, which satisfy Pauli's exclusion principle. The nuclear potential is taken entirely a central potential of short range but arbitrary shape.

The photo-disintegration of the deuteron can be described as a transition of two-nucleon system of total charge e, under the absorption of a photon of momentum $\hbar \boldsymbol{k}_\gamma$ (energy $E_\gamma = \hbar c k_\gamma$), from the ^3S ground state Ψ_i of the deuteron to a state Ψ_f in the continuous energy spectrum which corresponds to the disintegrated deuteron. The transition probability per second is given by the well-known formula

$$\Gamma = \frac{2\pi}{\hbar} \left| < f \mid W \mid i > \right|^2 \rho_f , \qquad (1)$$

where W is the interaction between the nucleous and the photon, and ρ_f is the density of the final states per unit energy. Let the charge wave function for the jth nucleon be denoted by $P(j)$ or $N(j)$ according as the nucleon is a proton or a neutron. Similarly, let the spin wave function be denoted by $\alpha\ (j)$ and $\beta\ (j)$ for the two spin orientations. Let

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$$R=\frac{1}{2}(r_1+r_2), \qquad r=r_1-r_2 .$$

For the initial state, the wave function Ψ_i can be written as

$$\Psi_i=\psi(r,R)\frac{1}{\sqrt{2}}\Big[P(1)\,N(2)-P(2)\,N(1)\Big]\begin{cases}\alpha(1)\,\alpha(2)\\ \frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2)+\alpha(2)\,\beta(1)\Big],\\ \beta(1)\,\beta(2)\end{cases} \tag{2}$$

with

$$\psi(r,R)=\frac{1}{\sqrt{V}}e^{\frac{i}{\hbar}p'\cdot R}\frac{1}{(4\pi)^{1/2}}\frac{u(r)}{r}. \tag{3}$$

Here the motion of the center of mass is described by the plane wave function, normalized in the volume V, with momentum $p=0$, while $u=u(r)$ is the radial wave function for the ^3S ground state of the deuteron, normalized by

$$\int_0^\infty u^2\,dr=1 .$$

The u satisfies

$$\frac{d^2u}{dr^2}+\frac{M}{\hbar^2}(-\epsilon-V(r))\,u=0, \tag{4}$$

where M denotes the mass of the nucleon, ϵ (>0) denotes the binding energy of the deuteron and $V(r)$ denotes the nuclear potential appropriate for the ^3S state. For the final state, the wave function Ψ_f is of the form

$$\Psi_f=\frac{1}{\sqrt{V}}e^{\frac{1}{\hbar}p'\cdot R}\frac{1}{k}\sum_{l=0}^{\infty}i^l(2l+1)\frac{1}{\sqrt{2}}\Big[P(1)\,N(2) - (-1)^l P(2)\,N(1)\Big]e^{-i\delta_l}\frac{u_l(r)}{r}P_l(\cos\angle k,r)\begin{cases}\alpha(1)\,\alpha(2)\\ \frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2) +\alpha(2)\,\beta(1)\Big]\\ \beta(1)\,\beta(2)\end{cases} \tag{5}$$

for a triplet final state, or

$$\Psi_f=\frac{1}{\sqrt{V}}e^{\frac{1}{\hbar}p'\cdot R}\frac{1}{k}\sum_{l=0}^{\infty}i^l(2l+1)\frac{1}{\sqrt{2}}\Big[P(1)\,N(2)+(-1)^l P(2)\,N(1)\Big] e^{-i\delta_l}\frac{u_l(r)}{r}P_l(\cos\angle k,r)\frac{1}{\sqrt{2}}\Big[\alpha(1)\,\beta(2)-\alpha(2)\,\beta(2)\Big] \tag{6}$$

for a singlet final state. The $u_l = u_l\,(r)$ satisfies

$$\frac{d^2 u_l}{dr^2} + \left(k^2 - \frac{M}{\hbar^2} V_l(r) - \frac{l\,(l+1)}{r^2}\right) u_l = 0 \tag{7}$$

with a nuclear potential V_l appropriate for the spin, together with the boundary conditions

$$u_l = 0 \text{ at } r = 0, \quad u_l \sim \sin\,(kr - \frac{l}{2}\pi + \delta_l) \text{ as } r \longrightarrow \infty\,. \tag{8}$$

Here $\boldsymbol{p}'$ denotes he total momentum of the two nucleons after disintegration, the relative momentum being denoted by $\hbar\boldsymbol{k}$ directed from the proton to the neutron. From the conservation laws we have

$$\boldsymbol{p}' = \boldsymbol{p} + \hbar\boldsymbol{k}_r = \hbar\boldsymbol{k}_r \text{ and } E = \frac{\hbar^2 k^2}{M} = E_\gamma - \epsilon - \frac{p'^2}{4M} = E_\gamma - \epsilon - \frac{E_\gamma^2}{4Mc^2}$$

These wave functions u, u_l and the plase δ_l can be approximated quite accurately by using the variational method of Hulthén[1], and are of the forms

$$u = (1 - e^{-\lambda r}) \sum_n B_n\, e^{-n\lambda r} e^{-\kappa r}\,, \quad \kappa = \sqrt{\frac{M\gamma}{\hbar^2}}\,, \tag{9}$$

$$u_l = \cos\delta_l \left[\sum_n C_n\, e^{-n\lambda r}\right] F_l(r) + \sin\delta_l \left[\sum_n C_{-n}\, e^{-n\lambda r}\right] (1 - e^{-\lambda r})^{2l+1}\, G_l(r)\,, \tag{10}$$

involving a number of variational parameters B_n, C_n, C_{-n}, ($n = 0$, 1, etc.) Here $1/\lambda$ stands for the range of the nuclear potential (so V and V_l falls off rapidly for $\lambda r > 1$) and

$$F_l(r) = \left(\frac{\pi kr}{2}\right)^{1/2} J_{l+\frac{1}{2}}(kr) \sim \sin\,(kr - \frac{l}{2}) \quad \text{as } r \longrightarrow \infty\,, \tag{11}$$

$$G_l(r = (-1)^l \left(\frac{\pi kr}{2}\right)^{1/2} J_{-l-\frac{1}{2}}(k) \sim \cos\,(kr - \frac{l}{2}) \quad \text{as } r \longrightarrow \infty\,. \tag{12}$$

For the interaction W between the two nucleous and the electro-magnetic field of the photon, we use Taylor's expansion for the field in the neighbor-

1. L. Hulthén, *Kgl. Fysiograf. Sällsk. Lund Förhandl.* **14** (1944), No. 8, No. 21. See also the preceding paper by H. W. Peng and Huang Tzu-Chia.

hood of the center of mass of the deuteron. Except for very high energy of the photon, the multipoles higher than the magnetic dipole and the electric quadrupole can be neglected. Then we have[2] W

$$W = -e\sum_{j=1}^{2}\frac{1+\tau_3^{(j)}}{2}(\boldsymbol{r}_j-\boldsymbol{R})\cdot\boldsymbol{E}_o - e\sum_{j=1}^{2}\frac{1+\tau_3^{(j)}}{2}\frac{1}{2}((\boldsymbol{r}_j-\boldsymbol{R})(\boldsymbol{r}_j-\boldsymbol{R})\cdot\overset{\nearrow}{\nabla}_o)\cdot\boldsymbol{E}_o$$

$$-\sum_{j=1}^{2}\frac{e\hbar}{2Mc}\left(\frac{1+\tau_3^{(j)}}{2}\mu_p+\frac{1-\tau_3^{(j)}}{2}\mu_n\right)\boldsymbol{\sigma}_j\cdot\boldsymbol{H}_o. \qquad (13)$$

Here μ_p and μ_n are the magnetic moments of proton and neutron, respectively, in the units of Bohr's nuclear magneton $e\hbar/2Mc$. The electric and magnetic fields $\boldsymbol{E}_o$ and $\boldsymbol{H}_o$ of the incident photon are evaluated at the center of mass of the deuteron. The operator $\tau_3^{(j)}$ operates on the charge wave function, with the eigenvalue +1 or -1 for $P(j)$ or $N(j)$.

In evaluating the transition matrix element the first two terms of W give rise to the so-called photo-electric transitions, while the third term gives rise to the so-called photo-magnetic transitions. The selection rules regarding the change of spin are different for these two transitions. The photo-electric transitions always lead from the triplet ground state to a triplet final state and do not change the spin orientation. The photo-magnetic transitions lead to both singlet and triplet final states and always involve a change in the spin orientation. The cross-section for the photo-disintegration of the deuteron is therefore simply the sum of the cross-sections for the photo-electric and photo-magnetic transitions, each of which may be calculated separately. Furthermore, since Ψ_i is an ^3S-state, it follows from the integration over angles in $\langle f|W|i\rangle$ that the electric dipole transition only leads to a final ^3P-state and the electric quadrupole transition only leads to a final ^3D-state. Similarly the magnetic dipole transition only leads to a final ^1S-state (not a final ^3S-state), the wave function for the final ^1S-state is not orthogonal to that of the ^3S ground state owing to the spin dependence of the nuclear potential.

By summing over the spin of the nucleons in the final states and averaging over the spin orientation of the deuteron and over the polarization of the incident photon in the initial state, we derive from (1) the differential cross-section for photo-disintegration

$$d\sigma = f(\theta)\,d\Omega = \frac{1}{8\pi}\Big\{2\,\sigma_M+\sin^2\theta\Big[3\,\sigma_d+6\,(5\,\sigma_d\,\sigma_q)^{1/2}\cos(\delta_1-\delta_2)\cos\theta$$

$$+15\,\sigma_q\cos^2\theta\Big]\,d\Omega,\ \theta = \angle\,\boldsymbol{k},\boldsymbol{k}_r,\ d\Omega = 2\pi\sin\theta\,d\theta\,. \qquad (14)$$

2. If one adopts the meson theory for the electromagnetic properties of nucleons, the expansion for W used here is still correct for photons of low energy, see L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1949), Vol. II, p. 449.

Here the electric dipole cross-section σ_d, the electric quadrupole cross-section σ_q and the magnetic dipole cross-section σ_M are given by

$$\sigma_d = \frac{\pi}{3}\,\frac{e^2}{\hbar c}\left(\frac{M}{E}\right)^{1/2}\frac{E_\gamma}{\hbar}\,|\,I_1\,|^2\,, \qquad I_1 = \int_0^\infty u_1\, r\, u\, dr\,; \tag{15}$$

$$\sigma_q = \frac{\pi}{240}\,\frac{e^2}{\hbar c}\left(\frac{M}{E}\right)^{1/2}\frac{E_\gamma^3}{\hbar^3 c^2}\,|\,I_2^2\,|^2\,, \qquad I_2 = \int_0^\infty u_2\, r^2\, u\, dr\,; \tag{16}$$

and

$$\sigma_M = \frac{\pi}{6}\,\frac{e^2}{\hbar c}\,\frac{E_\gamma}{Mc^2}\,\frac{\hbar}{\sqrt{ME}}\,(\mu_p-\mu_n)^2\,|\,I_o|^2,\ I_o = \int_0^\infty u_o\, u\, dr\,; \tag{17}$$

where u_1, u_2, and u_0 are the radial wave functions of the final ^3P, ^3D and ^1S states. The total cross-section is, by integrating (14),

$$\sigma = \sigma_M + \sigma_d + \sigma_q = \sigma_M + \sigma_E\,. \tag{18}$$

If we use the appropriate wave functions (9) and (10) the integrals

$$I_l = \int_0^\infty u_l\, r^l\, u\, dr \qquad (l = 1,\ 2,\ 0)$$

can be evaluated with the aid of the following formula[3]

$$\int_0^\infty e^{-\alpha r}\, r^{l+\frac{1}{2}}\, J_{l+\frac{1}{2}}(kr)\, dr = \frac{(2k)^{l+\frac{1}{2}}\,\Gamma\,(l+1)}{(\alpha^2+k^2)^{l+1}\sqrt{\pi}}$$

and

$$\int_0^\infty e^{-\alpha r}\, r^{l+\frac{1}{2}}\, J_{-l-\frac{1}{2}}(kr)\, dr$$

$$= \frac{(k/2)^{-(l+\frac{1}{2})}}{(\alpha^2+k^2)^{l+1}\,\Gamma(-l+\frac{1}{2})}\,\alpha^{2l+1}\, F\left(-l,\, -l-\tfrac{1}{2};\, -l+\tfrac{1}{2};\, -\frac{k^2}{\alpha^2}\right).$$

For $l = 1$, 2, or 0, the hypergeometric series terminates and becomes

$$F\left(-l,\, -l-\tfrac{1}{2};\, -l+\tfrac{1}{2};\, -\frac{k^2}{\alpha^2}\right) = (2l+1)\sum_{i=0}^{l}\binom{l}{i}\frac{1}{2l+1-2i}\left(\frac{k^2}{\alpha^2}\right)^i.$$

3. Cf. G. N. Watson, *Theory of Bessel Functions*, 2nd Edition, pp. 385, 386.

Let

$$R(n, m; l) = \frac{l!}{2}(2k)^{l+1}\left[\frac{1}{[(n\lambda+m\lambda+\kappa)^2+k^2]^{l+1}} - \frac{1}{[(n\lambda+m\lambda+\lambda+\kappa)^2+k^2]^{l+1}}\right],$$

$$S(n, m; l) = \sum_{j=o}^{2l+2}(-)^j\binom{2l+2}{j}\frac{(2l+1)!}{2^l\, l!}\frac{(n\lambda+m\lambda+j\lambda+\kappa)^{2l+1}}{k^l\,[(n\lambda+m\lambda+j\lambda+\kappa)^2+k^2]^{l+1}}\,T(n, m; j; l),$$

where

$$T(n, m; j; l) = \sum_{i=o}^{l}\binom{l}{i}\frac{1}{2l+1-2i}\left(\frac{k^2}{(n\lambda+m\lambda+j\lambda+\kappa)^2}\right)^i.$$

Then, we get for I_l

$$I_l = \cos\delta_l \sum_n \sum_m C_n B_m R(n, m; l) + \sin\delta_l \sum_n \sum_m C_{-n} B_m S(n, m; l)$$
$$(l = 1, 2, 0)\ . \tag{19}$$

As regards the angular distribution of disintegrated protons, we have from (14), neglecting the $\sigma_q \cos^2\theta$ term,

$$f(\theta) \propto a + \sin^2\theta\,(1 + \alpha\cos\theta),\quad a = \frac{2}{3}\,\frac{\sigma_M}{\sigma_d},\quad \alpha = 2\,(5\sigma_q/\sigma_d)^{\frac{1}{2}}\cos(\delta_1 - \delta_2). \tag{20}$$

or, in the laboratory system,

$$f(\theta_{\rm lab}) \propto a + \sin^2\theta_{\rm lab}\left[1 + \left(\alpha + \frac{2\beta\, E_\gamma}{E_\gamma - \epsilon}\right)\cos\theta_{\rm lab}\right],\ \beta = \left(\frac{E_\gamma - \epsilon}{Mc^2}\right)^{1/2}.$$

The term $\alpha\cos\theta$ arises from the interference between the electric dipole and electric quadrupole transitions and leads to a forward asymmetry. Several authors have stated that there is no interference between electric dipole and electric quadrupole transitions by arguing that the final states belong to different charge states and the interference term would vanish on summation over the different charge states. It has been pointed out[4], however, that for a state of definite angular momentum and spin, the possibility of the charge state is determined by the Pauli's exclusion principle, so there is no summation over different charge states and the interference term should remain. Our

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4. J. F. Marshall and E. Guth, *Phys. Rev.* **76** (1949), 1879; see also J. F. Marshall and E. Guth, *Phys. Rev.* **78** (1950), 738.

calculation including explicitly the charge wave functions, as given by (5) and (6), does give rise to this interference term.

For the radiative capture of neutrons by protons, which is the inverse process to the photo-disintegration of the deuteron, the capture differential cross-section can be obtained from that of the photo-disintegration of the deuteron, namely (14), by the formula[5]

$$d\sigma_{\text{caprature}} = \frac{3}{2}\,\frac{1}{ME}\left(\frac{E_\gamma}{c}\right)^2 f(\theta)\, d\Omega_\gamma\,. \tag{21}$$

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5. Cf. H. A. Bethe, *Elementary Nuclear Theory* (John Wiley and Sons, Inc., New York) p. 61. J. Schwinger, hectographed notes on nuclear physics, Harvard, 1947.