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电磁复合聚焦-偏转球面阴极透镜的相对论象差理论

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提　　要

本文应用变分原理研究了电磁复合聚焦-偏转球面阴极透镜的相对论象差理论. 在考虑阴极面逸出电子具有一定的初能量和初角度分布，物场和象场弯曲以及阴极面上磁场和横向电场不为零的情况下，导出了任意理想象面上的一级近轴横向象差和包括色球差在内的全部三级几何横向象差,以及各种特殊类型象差系数的明显表达式. 本文导出的象差公式不仅普遍适用于宽束和细束阴极透镜,而且普遍适用于相对论或非相对论,阴极和屏为球面或平面时的各种情况. 本文以复数描写轨迹,用矩阵表示象差,形式简洁,适合于计算机计算.

一、引　　言

随着各种新型电子光学器件的出现,电磁复合聚焦-偏转系统的相对论象差理论和带有球面阴极和球面屏系统的电子光学器件已日益受到重视. 目前,在红外变象管、微光象增强器、高速摄影、微光电视、医用X射线象增强器、天文望远镜等许多电子光学器件中都已广泛应用了球面阴极透镜系统. 近来在扫描电子显微镜、电子束布线机、电子束曝光机等扫描偏转电子光学装置和仪器中也要应用电磁复合聚焦-偏转理论.

文献[1—6]讨论了电或磁聚焦与磁偏转复合系统的象差,以及讨论了电磁复合聚焦-偏转系统的相对论象差. 本文在文献[7—10]的基础上,进一步研究了电磁复合聚焦-偏转球面阴极透镜的相对论象差理论.

二、电磁场及变分函数的展开

在以 z 轴为旋转对称轴的无源无旋的真空区域内，与电子光学折射率对应的相对论变分函数可以表示为

$$F=\sqrt{\varphi_r(1+\varepsilon_p\varphi_r)(1+r'^2+r^2\theta'^2)}-\sqrt{\eta/2}(r'A_r+r\theta'A_\theta+A_z). \tag{1}$$

式中 $\varepsilon_p=e/(2m_0c^2)$, $\eta=e/m_0$ 为电子的荷质比, c 为真空中的光速. 撇号一律表示对 z 求导. 在考虑阴极面上逸出电子具有一定的初能量分布以及阴极面上磁场和横向电场不为零的情况下[11]

$$\varphi_r=\varphi-\varphi_0+\varepsilon_0, \tag{2}$$

式中 $\varphi=\varphi(r,\theta,z)$ 为空间电位分布，$\varphi_0=\varphi_0(r_0,\theta_0,z_0)$ 为空间电位分布在阴极面上的值，ε_0 为阴极面上电子逸出初电位．在圆柱坐标系 (r,θ,z) 中，电位分布 φ 和磁标位分布 ψ 以及磁矢位 A 的三个分量 A_r，A_θ，A_z 可分别展开为傅里叶级数[6]：

$$\varphi(r,\theta,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n m!}{4^n n!(m+n)!}r^{m+2n}(\Phi_m^{(2n)},e^{im\theta}),$$

$$\psi(r,\theta,z)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n m!}{4^n n!(m+n)!}r^{m+2n}(\Psi_m^{(2n)},e^{im\theta}),$$

$$rA_\theta=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{n+1} m!}{4^n n!(m+n)!(m+2n+2)}r^{m+2n+2}(\Psi_m^{(2n+1)},e^{im\theta}),$$

$$A_z=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{n+1} m!m}{4^n n!(m+n)!(m+2n)}r^{m+2n}(i\Psi_m^{(2n)},e^{im\theta})$$

$$A_r=0. \tag{3}$$

上式中，$\Phi_m=\Phi_m(z)$，$\Psi_m=\Psi_m(z)$ 均为 z 的复函数，分别称为 φ 和 ψ 的 m 次谐波势，并可以表示为如下的复数形式：

$$\Phi_m=\Phi_{mr}+i\Phi_{mi},\qquad \Psi_m=\Psi_{mr}+i\Psi_{mi}. \tag{4}$$

式中 $i=\sqrt{-1}$，下标 r 和 i 分别表示复数的实部和虚部．同时，假定 Φ_0 和 Ψ_0 为实函数，即

$$\Phi=\Phi_0,\quad B_0=-\Psi_0',\quad h=\sqrt{\eta/8}B_0. \tag{5}$$

在 (3) 式中包含有复数表达式形式为 (a,b)，它被定义为复数 a，b 的内积[6]

$$(a,b)=a_rb_r+a_ib_i. \tag{6}$$

本文所讨论的聚焦-偏转系统是由一个(或 n 个)旋转对称圆形电极或磁极与一对(或 n 对)偏转器所组成，因而有

$$\Phi_{2n}=\Psi_{2n}=0,\quad n=1,2,3,\cdots \tag{7}$$

对于空间同一点在圆柱(固定)坐标系中的坐标 $\{r,\theta,z\}$ 可换用 $\{\rho,z\}$ 来表示，其中

$$\rho=re^{i\theta},\quad \rho_0=r_0e^{i\theta_0}. \tag{8}$$

考虑到阴极和屏为球面时必然会引起物场和象场的变化，因而引入如下坐标变换：

$$z=\xi+\Delta z_0,\quad \zeta=\xi+\Delta z_i,$$

$$\Delta z_0=\frac{r_0^4}{2R_c},\quad \Delta z_i=\frac{M^2r_0^4}{2R_s}. \tag{9}$$

式中 ξ，ζ 是与 z 同方向的新的坐标变量，R_c 和 R_s 分别为阴极和屏的曲率半径，M 为系统的放大率．因为 $(d/d\zeta)=(d/dz)$，可将 ζ 换回来用 z 表示．于是可将电位和磁标位的 m 次谐波势展开如下：

$$\Phi_m(z)=\Phi_m(\zeta+\Delta z_0+\Delta z_i)=\Phi_m(z)+(\Delta z_0+\Delta z_i)\Phi_m'(z)$$
$$+\frac{1}{2}(\Delta z_0+\Delta z_i)^2\Phi_m''(z),$$

$$\Psi_m(z) = \Psi_m(\zeta + \Delta z_0 + \Delta z_i) = \Psi_m(z) + (\Delta z_0 + \Delta z_i)\Psi'_m(z) + \frac{1}{2}(\Delta z_0 + \Delta z_i)^2\Psi''_m(z). \tag{10}$$

Φ_m 和 Ψ_m 的高阶导数以此类推.

引入旋转坐标系 $\{u, z\}$, 它与固定坐标系 $\{\rho, z\}$ 间的关系为

$$u = \rho e^{-i\chi}, \quad \rho = u e^{i\chi}. \tag{11}$$

其中 $\chi(z)$ 是磁场引起轨迹旋转而引入的旋转坐标系 $\{u, z\}$ 相对于固定坐标系 $\{\rho, z\}$ 转过的角度. 不失普遍性可以假设在阴极面上 $\chi_0 = 0$, 则有

$$\chi(z) = \int_{z_0}^{z} \frac{h}{\sqrt{V_p}} dz, \quad \chi'(z) = \frac{h}{\sqrt{V_p}}. \tag{12}$$

式中

$$V_p = V(1 + \varepsilon_p V), \quad V = V(z) = \phi - \phi_0 + \varepsilon_n. \tag{13}$$

$V(z)$ 称为准规范化电位; $\varepsilon_0 = \varepsilon_n + \varepsilon_t$, $\varepsilon_n = \varepsilon_0 \cos^2\alpha_0$, $\varepsilon_t = \varepsilon_0 \sin^2\alpha_0$, ε_n 和 ε_t 分别为逸出电子初速度的法向分量和切向分量所对应的初电位, α_0 为电子逸出角; $\phi = \phi(z)$ 是静电电位沿轴分布, $\phi_0 = \phi(z_0)$ 是 $\phi(z)$ 在阴极面上的值.

现将 u, u', $\Phi_1^{(n)}$, $\Psi_1^{(n)}$ $(n = 0, 1, 2)$, Φ_3, Ψ_3 都看作一级小量,由 (2), (3), (5), (10)—(12) 诸式代入 (1) 式,并考虑到 (7) 式,就可以将变分函数展开为

$$F = F_0 + F_2 + F_4, \tag{14}$$

$$F_0 = \sqrt{V_p}, \tag{15}$$

$$F_2 = -[M(u, u) - M_0] + N(u', u') + [(\alpha_1, u) - (\alpha_{10}, u_0)]. \tag{16}$$

$$F_4 = F_{4C} + F_{4D}, \tag{17}$$

$$\begin{aligned} -F_{4C} = {} & \frac{1}{4}[L(u, u) - L_0](u, u) + \frac{1}{2}[M(u, u) - M_0](u', u') \\ & + \frac{N}{4}(u', u')^2 + \sqrt{V_p}[P(u, u) + Q(u', u') \\ & + K\sqrt{V_p}(iu, u') - P_0](iu, u') - \delta_0. \end{aligned} \tag{18}$$

$$\begin{aligned} -F_{4D} = {} & [(\alpha_3, u^3) - (\alpha_{30}, u_0^3)] + [(\mu, u) - (\mu_0, u_0)](u, u) \\ & + [(\lambda, u) - (\lambda_0, u_0)](u', u') + 2\sqrt{V_p}[(\kappa, u) - (\kappa_0, u_0)](iu, u') \\ & + [(\tau, u) - (\tau_0, u_0)]^2 + [(\nu, u) - (\nu_0, u_0)](u_0, u_0) \\ & + 2(2\eta)^{-1/2}(iu_0, \dot{u}_0)[(\gamma, u) - (\gamma_0, u_0)] \\ & + (2\eta)^{-1}(\dot{u}_0, \dot{u}_0)[(\pi, u) - (\pi_0, u_0)]. \end{aligned} \tag{19}$$

式中 F_{4C} 和 F_{4D} 分别是复合聚焦-偏转系统中聚焦和偏转部分的四级变分函数. 现定义以下函数:

$$T = \frac{\phi''}{4}\sigma + h^2, \quad \sigma = 1 + 2\varepsilon_p V, \quad T_0 = \frac{\phi_0''}{4} + h_0^2,$$

$$W = \frac{\phi''''}{16}\sigma + \frac{\varepsilon_p}{4}\phi''^2 + hh'',$$

$$K=\frac{h^2}{2V_p^{3/2}},\quad L=\frac{1}{2V_p^{1/2}}\left(\frac{T^2}{V_p}-W\right)$$

$$M=\frac{T}{2V_p^{1/2}},\quad N=\frac{1}{2}V_p^{1/2},$$

$$P=\frac{1}{2V_p^{1/2}}\left(\frac{h}{V_p}T-\frac{h''}{4}\right),\quad Q=\frac{h}{2V_p^{1/2}}. \tag{20}$$

$$L_0=\left[(\sigma T-\varepsilon_p V_p\phi'')\frac{S_0}{V_p}-(\Delta z_0+\Delta z_i)\left(\frac{\sigma}{2}\phi'''+4hh'\right)\right]\frac{1}{\sqrt{V_p}},$$

$$M_0=\frac{\sigma S_0}{2V_p^{1/2}},\quad P_0=\frac{\sigma hS_0}{2V_p^{3/2}}-(\Delta z_0+\Delta z_i)\frac{h'}{\sqrt{V_p}},$$

$$S_0=T_0(u_0,u_0)+2(2\eta)^{-1/2}h_0(iu_0,\dot{u}_0)+(2\eta)^{-1}(\dot{u}_0,\dot{u}_0)$$
$$+(\Delta z_0+\Delta z_i)(\phi'-\phi_0'),$$

$$\delta_0=\frac{1}{2\sqrt{V_p}}\left\{\frac{\sigma}{2}(\Delta z_0+\Delta z_i)^2(\phi''-\phi_0'')-\frac{S_0^2}{4V_p}\right.$$
$$\left.+\frac{\sigma}{4}\left[(\Delta z_0+\Delta z_i)\phi_0'''-\frac{\phi_0''''}{16}(u_0,u_0)\right](u_0,u_0)\right\}. \tag{21}$$

$$\alpha_m=-\left[\frac{\sigma\Phi_m}{2\sqrt{V_p}}+i\sqrt{\frac{\eta}{2}}\Psi_m\right]e^{-im\chi},\quad (m=1,3)$$

$$\alpha_{m0}=-\frac{\sigma\Phi_{m0}}{2\sqrt{V_p}},\quad (m=1,3)$$

$$\mu=-\left[\frac{\Phi_1}{4V_p^{3/2}}(\sigma T-\varepsilon_p V_p\phi'')-\frac{\sigma\Psi_1''}{16V_p^{1/2}}+\sqrt{\frac{\eta}{2}}\frac{h\Psi_1'}{3V_p^{1/2}}-i\sqrt{\frac{\eta}{2}}\frac{\Psi_1''}{24}\right]e^{-i\chi},$$

$$\mu_0=-\left[\frac{\Phi_{10}}{4V_p^{3/2}}(\sigma T-\varepsilon_p V_p\phi'')\right]$$

$$\lambda=-\frac{\sigma\Phi_1}{4V_p^{1/2}}e^{-i\chi},\quad \lambda_0=-\frac{\sigma\Phi_{10}}{4V_p^{1/2}},$$

$$\kappa=-\left[\frac{\sigma h\Phi_1}{4V_p^{3/2}}+\sqrt{\frac{\eta}{2}}\frac{\Psi_1'}{6V_p^{1/2}}\right]e^{-i\chi},\quad \kappa_0=-\frac{\sigma h\Phi_{10}}{4V_p^{3/2}},$$

$$\tau=-\frac{\Phi_1}{(4V_p)^{3/4}}e^{-i\chi},\quad \tau_0=-\frac{\Phi_{10}}{(4V_p)^{3/4}},$$

$$\nu=-\left\{\frac{1}{2}\left(\frac{1}{R_c}+\frac{M^2}{R_s}\right)\left[\frac{\sigma\Phi_1'}{2V_p^{1/2}}+i\sqrt{\frac{\eta}{2}}\Psi_1'-\frac{\Phi_1}{4V_p^{3/2}}(\phi'-\phi_0')\right]\right.$$
$$\left.-\frac{\Phi_1T_0}{4V_p^{3/2}}\right\}e^{-i\chi},$$

$$\nu_0=-\left\{\frac{1}{2}\left(\frac{1}{R_c}+\frac{M^2}{R_s}\right)\left[\frac{\sigma\Phi_{10}'}{2V_p^{1/2}}-\frac{\Phi_{10}}{4V_p^{3/2}}(\phi'-\phi_0')\right]-\frac{\Phi_{10}T_0}{4V_p^{3/2}}-\frac{\sigma\Phi_{10}''}{16V_p^{1/2}}\right\},$$

$$\gamma=\frac{h_0\Phi_1}{4V_p^{3/2}}e^{-i\chi},\quad \gamma_0=\frac{h_0\Phi_{10}}{4V_p^{3/2}},$$

$$\pi=\frac{\Phi_1}{4V_p^{3/2}}e^{-i\chi},\quad \pi_0=\frac{\Phi_{10}}{4V_p^{3/2}}. \tag{22}$$

注意到(5)和(20)式,可以看出(20)式中的K, L, M, $\cdots$ 等函数正好与 Glaser[10,11] 给出的四级变分函数形式相类似. 上述各式中 L_0, M_0, $\cdots$, π_0 等下标为零的函数项是因为考虑到逸出电子的初能量和初角度分布, 物场和象场弯曲以及阴极面上磁场和横向电场不为零情况下引入的函数项. 在不同的特殊情况下选择其中某些函数项为零, 即可得出相对论或非相对论,阴极和屏为球面或平面,以及电或磁聚焦和电或磁偏转等特殊形式的变分函数.

三、近轴轨迹方程和三级横向象差方程

由费马原理旋转坐标系下的欧拉方程为

$$\frac{d}{dz}\left(\frac{\partial F}{\partial u'}\right)-\left(\frac{\partial F}{\partial u}\right)=0. \tag{23}$$

将二级变分函数 F_2 的表示式(16)代入,可得相对论情况下电磁复合聚焦-偏转球面阴极透镜带有自由项的近轴轨迹方程为

$$V_p u''+\frac{V_p'}{2}u'+Tu=\sqrt{V_p}\,\alpha_1. \tag{24}$$

这个方程是在考虑阴极面逸出电子具有一定的初能量和初角度分布, 并采用准规范化电位而导出的. 文献[6]中的旁轴轨迹方程仅是上式当 ε_n 等于零或常数时的特例.

令实函数 $v=v(z)$, $w=w(z)$ 为(24)式齐次方程的两个线性无关特解, 它们在阴极面 $z=z_0$ 处满足如下初条件:

$$\begin{aligned} &v(z_0)=0, \quad v'(z_0)\sqrt{\varepsilon_n}=1,\\ &w(z_0)=1, \quad w'(z_0)=0. \end{aligned} \tag{25}$$

对于球面阴极透镜 $w(z)$ 特解应换成

$$w(z)=\tilde{w}(z)+\frac{\sqrt{\varepsilon_n}}{R_c}v(z).$$

在球面阴极上电子逸出点处特解 $\tilde{w}(z)$ 满足以下初条件:

$$\tilde{w}(z_0)=1, \quad \tilde{w}'(z_0)=-\frac{1}{R_c}. \tag{26}$$

v, w 两特解满足的朗斯基行列式为

$$\sqrt{V_p}\,(v'w-vw')=1. \tag{27}$$

由于假定在阴极面 $z=z_0$ 处, 磁场引起的旋转角 $\chi_0=0$, 于是电子从阴极面逸出的初条件为

$$\begin{aligned} &u_0=\rho_0=r_0e^{i\theta_0},\\ &\dot{u}_0=\sqrt{2\eta}(\sqrt{\varepsilon_t}e^{i\beta_0}-ih_0r_0e^{i\theta_0}). \end{aligned} \tag{28}$$

式中 θ_0 为逸出电子的初始位置 r_0 与 X 轴的夹角, β_0 为逸出电子的初始方位角, 即电子逸出方向与 X 轴的夹角.

近轴轨迹方程(24)满足初条件(28)式的通解为

$$u = \mathscr{V} w + \mathscr{W} v. \tag{29}$$

式中

$$\mathscr{V} = u_0 + \mathscr{D}, \quad \mathscr{W} = (2\eta)^{-1/2}\dot{u}_0 + \mathscr{E}. \tag{30}$$

$$\mathscr{D} = -\int_{z_0}^{z} \alpha_1 v dz, \quad \mathscr{E} = \int_{z_0}^{z} \alpha_1 w dz. \tag{31}$$

由 (29)—(31) 式可得出近轴轨迹的斜率为

$$u' = \mathscr{V} w' + \mathscr{W} v'. \tag{32}$$

文献 [9] 指出，系统的理想成象性质应在与某一法向初电位 ε_{n1} 相对应的象面上来讨论．特解 $v(z, \varepsilon_{n1})$ 将决定该象面的位置 z_i

$$v(z_i, \varepsilon_{n1}) = 0. \tag{33}$$

在象面 z_i 上另一特解 $w(z_i, \varepsilon_{n1})$ 将决定系统的放大率 M ($\chi_i = m\pi$ 时)：

$$w(z_i, \varepsilon_{n1}) = (-1)^m M. \tag{34}$$

在象面 z_i 处约化的偏转量为

$$\mathscr{D}_{bn}(z_i, \varepsilon_{n1}) = -\int_{z_0}^{z_i} \alpha_1 v dz. \tag{35}$$

由欧拉方程 (23) 式，利用变动任意常数法和朗斯基行列式 (27)，并进行部分积分，便可求得任意理想象面 z_i 上包括色球差在内的全部三级横向象差表达式 (假定阴极面上 $\Delta u_3(z_0) = \Delta u_3'(z_0) = 0$)：

$$\Delta u_{3i} = v_i \left(\frac{\partial F_4}{\partial u'} w\right)_{z_0} + v_i \int_{z_0}^{z_i} \left[\frac{\partial F_4}{\partial u} w + \frac{\partial F_4}{\partial u'} w'\right] dz$$
$$- w_i \int_{z_0}^{z_i} \left[\frac{\partial F_4}{\partial u} v + \frac{\partial F_4}{\partial u'} v'\right] dz. \tag{36}$$

式中 $\Delta u_{3i} = \Delta u_3(z_i, \varepsilon_n)$，$v_i = v(z_i, \varepsilon_n)$，$w_i = w(z_i, \varepsilon_n)$．文献 [7] 指出在 z_i 象面上，只有与之相应的法向初电位 $\varepsilon_n = \varepsilon_{n1}$ 的电子才能理想成象，即特解 $v(z_i, \varepsilon_{n1}) = 0$；而以法向初电位 $\varepsilon_n \neq \varepsilon_{n1}$ 从阴极面逸出的电子在此象面上不能理想聚焦而形成象差，即特解 $v(z_i, \varepsilon_n) \neq 0$．因此必须计算 (36) 式的所有各项才能导出系统的全部三级横向象差．

四、三级横向象差和象差系数表达式

将 (29) 和 (32) 式代入 (18) 和 (19) 式，就可以求得

$$\begin{aligned}
-F_{4C} = {} & \frac{A}{4}(\mathscr{V}, \mathscr{V})^2 + \frac{B}{4}(\mathscr{W}, \mathscr{W})^2 + C(\mathscr{V}, \mathscr{W})^2 + \frac{D}{2}(\mathscr{V}, \mathscr{V})(\mathscr{W}, \mathscr{W}) \\
& + E(\mathscr{V}, \mathscr{V})(\mathscr{V}, \mathscr{W}) + F(\mathscr{W}, \mathscr{W})(\mathscr{V}, \mathscr{W}) \\
& + 2c(\mathscr{V}, \mathscr{W})(i\mathscr{V}, \mathscr{W}) + e(\mathscr{V}, \mathscr{V})(i\mathscr{V}, \mathscr{W}) \\
& + f(\mathscr{W}, \mathscr{W})(i\mathscr{V}, \mathscr{W}) - \frac{1}{2}(\mathscr{V}, \mathscr{V})[(u_0, u_0)E_{2R} + \varepsilon_t C_e] \\
& - (\mathscr{V}, \mathscr{W})[(u_0, u_0)E_{1R} + \varepsilon_t C_m] - \frac{1}{2}(\mathscr{W}, \mathscr{W})[(u_0, u_0)D_R
\end{aligned}$$

$$+ \varepsilon_t C_d] - (i\mathscr{V}, \mathscr{W})[(u_0, u_0)e_R + \varepsilon_t C_r]. \tag{37}$$

$$\begin{aligned}
- F_{4D} = {} & (\mathscr{W}, \mathscr{W})[(a_0, \mathscr{V}w + \mathscr{W}v) - (a_{00}, u_0)] + 2(\mathscr{V}, \mathscr{W}) \\
& \cdot [(a_1, \mathscr{V}w + \mathscr{W}v) - (a_{10}, u_0)] + (\mathscr{V}, \mathscr{V})[(a_2, \mathscr{V}w + \mathscr{W}v) \\
& - (a_{20}, u_0)] + 2(i\mathscr{V}, \mathscr{W})[(\kappa, \mathscr{V}w + \mathscr{W}v) - (\kappa_0, u_0)] \\
& + [(\tau, \mathscr{V}w + \mathscr{W}v) - (\tau_0, u_0)]^2 + [(\nu, \mathscr{V}w + \mathscr{W}v) \\
& - (\nu_0, u_0)](u_0, u_0) + [(\gamma, \mathscr{V}w + \mathscr{W}v) - (\gamma_0, u_0)] \\
& \cdot [2(2\eta)^{-1/2}(iu_0, \dot{u}_0)] + [(\pi, \mathscr{V}w + \mathscr{W}v) - (\pi_0, u_0)](2\eta)^{-1}(\dot{u}_0, \dot{u}_0) \\
& + \{[\alpha_3, (\mathscr{V}w + \mathscr{W}v)^3] - (\alpha_{30}, u_0^3)\}.
\end{aligned} \tag{38}$$

式中

$$A = -\frac{1}{2\sqrt{V_p}}\left[Ww^4 - \frac{K_{ww}^2}{V_p}\right], \quad B = -\frac{1}{2\sqrt{V_p}}\left[Wv^4 - \frac{K_{vv}^2}{V_p}\right],$$

$$C = -\frac{1}{2\sqrt{V_p}}\left[Wv^2w^2 - \frac{K_{vw}^2}{V_p} + \frac{h^2}{V_p}\right],$$

$$D = -\frac{1}{2\sqrt{V_p}}\left[Wv^2w^2 - \frac{K_{vv}K_{ww}}{V_p} - \frac{2h^2}{V_p}\right],$$

$$E = -\frac{1}{2\sqrt{V_p}}\left[Wvw^3 - \frac{K_{vw}K_{ww}}{V_p}\right],$$

$$F = -\frac{1}{2\sqrt{V_p}}\left[Wv^3w - \frac{K_{vv}K_{vw}}{V_p}\right],$$

$$c = -\frac{1}{2\sqrt{V_p}}\left[\frac{h''}{4}vw - \frac{hK_{vw}}{V_p}\right],$$

$$e = -\frac{1}{2\sqrt{V_p}}\left[\frac{h'}{4}w^2 - \frac{hK_{ww}}{V_p}\right],$$

$$f = -\frac{1}{2\sqrt{V_p}}\left[\frac{h''}{4}v^2 - \frac{hK_{vv}}{V_p}\right],$$

$$C_d = \frac{1}{2\sqrt{V_p}}\left[\frac{\sigma K_{vv}}{V_p} - \varepsilon_p\phi''v^2\right], \quad C_m = \frac{1}{2\sqrt{V_p}}\left[\frac{\sigma K_{vw}}{V_p} - \varepsilon_p\phi''vw\right],$$

$$C_a = \frac{1}{2\sqrt{V_p}}\left[\frac{\sigma K_{ww}}{V_p} - \varepsilon_p\phi''w^2\right], \quad C_r = \frac{1}{2\sqrt{V_p}}\left[\frac{\sigma h}{V_p}\right].$$

$$D_R = \frac{1}{2}\left(\frac{1}{R_C} + \frac{M^2}{R_S}\right)\left[C_d(\phi' - \phi_0') - \frac{v^2}{2\sqrt{V_p}}\left(\frac{\sigma\phi'''}{2} + 4hh'\right)\right],$$

$$E_{1R} = \frac{1}{2}\left(\frac{1}{R_C} + \frac{M^2}{R_S}\right)\left[C_m(\phi' - \phi_0') - \frac{vw}{2\sqrt{V_p}}\left(\frac{\sigma\phi'''}{2} + 4hh'\right)\right],$$

$$E_{2R} = \frac{1}{2}\left(\frac{1}{R_C} + \frac{M^2}{R_S}\right)\left[C_a(\phi' - \phi_0') - \frac{w^2}{2\sqrt{V_p}}\left(\frac{\sigma\phi'''}{2} + 4hh'\right)\right],$$

$$e_R = \frac{1}{2}\left(\frac{1}{R_C} + \frac{M^2}{R_S}\right)\left[C_r(\phi' - \phi_0') - \frac{h'}{\sqrt{V_p}}\right],$$

$$\varepsilon_t = T_0(u_0, u_0) + 2(2\eta)^{-1/2}h_0(iu_0, \dot{u}_0) + (2\eta)^{-1}(\dot{u}_0, \dot{u}_0). \tag{39}$$

式中

$$K_{vv}=Tv^2+Vv'^2,\quad K_{vw}=Vvw+Vv'w',\quad K_{ww}=Tw^2+Vw'^2.$$

a_0, a_1, a_2 等函数将在后面 (43) 式给出．同时将三级横向象差表达式变化如下：

$$\Delta u_{3i}=v_i\left(\frac{\partial F_4}{\partial v'}w\right)_{z_0}+v_i\int_{z_0}^{z_i}\frac{\partial F_4}{\partial \mathscr{V}}dz-w_i\int_{z_0}^{z_i}\frac{\partial F_4}{\partial \mathscr{W}}dz. \tag{40}$$

将 (17), (37), (38) 式代入上式，便可以将相对论下电磁复合聚焦-偏转球面阴极透镜在任意理想象平面 z_i 上的全部三级横向象差表示为以下矩阵形式：

$$\Delta u_{3i}=\begin{bmatrix}(2\eta)^{-1}\dot{u}_0\bar{\dot{u}}_0\\(2\eta)^{-1/2}\dot{u}_0\bar{u}_0\\(2\eta)^{-1/2}\dot{u}_0\bar{\mathscr{D}}_b\end{bmatrix}^T\begin{bmatrix}k_S & k_L & k_L\\k_R & k_F & \mathscr{K}_{F2}\\K_R & \mathscr{K}_{F1} & K_F\end{bmatrix}\begin{bmatrix}(2\eta)^{-1/2}\dot{u}_0\\u_0\\\mathscr{D}_b\end{bmatrix}$$

$$+\left\{\begin{bmatrix}u_0 & u_0\\u_0 & \mathscr{D}_b\\\mathscr{D}_b & \mathscr{D}_b\end{bmatrix}^T\begin{bmatrix}k_A & k_D & \mathscr{K}_{D1}\\\mathscr{K}_A & \mathscr{K}_{D2} & \mathscr{K}_{D3}\\K_A & \mathscr{K}_{D4} & K_D\end{bmatrix}\right.$$

$$\left.+\begin{bmatrix}(2\eta)^{-1/2}\bar{u}_0\bar{\mathscr{D}}_b\\\bar{u}_0\bar{\mathscr{D}}_b\\\bar{\mathscr{D}}_b\bar{\mathscr{D}}_b\end{bmatrix}^T\begin{bmatrix}\tilde{K}_R & \frac{1}{2}\mathscr{K}_A & \frac{1}{2}\mathscr{K}_A\\\frac{1}{2}\mathscr{K}_A & \mathscr{K}_{D2} & \frac{1}{2}\mathscr{K}_{D4}\\\frac{1}{2}\tilde{K}_A & \frac{1}{2}\mathscr{K}_{D4} & \tilde{K}_D\end{bmatrix}\right\}\begin{bmatrix}(2\eta)^{-1/2}\bar{\dot{u}}_0\\\bar{u}_0\\\bar{\mathscr{D}}_b\end{bmatrix}. \tag{41}$$

式中“T”号表示转置矩阵，上加“—”号表示共轭复数．由上式可见，若按 $\dot{u}_0$ 或 $\bar{\dot{u}}_0$ 的幂次来划分，则包括色球差在内的三级几何横向象差可按下标分为色球差 (S)，彗差长度 (L) 和彗差半径 (R)，场曲 (F)，象散 (A) 和畸变 (D) 六种类型．若按场的性质不同可分成三类，即由系数 k 表示的旋转对称圆形透镜的聚焦象差；系数 K 表示的轴上点物的纯偏转象差；系数 $\mathscr{K}$ 表示的有限大的物的混合偏转象差，共 18 种象差．由于偏转场中通常存在着三次谐波势 $\{\Phi_3, \Psi_3\}$，因此出现六种附加象差，用上加“～”号的系数表示．现在将 (41) 式中各象差系数的明显表达式给出如下：

$$k_S=w_i\int_{z_0}^{z_i}\{B-C_d\}dz-v_i\int_{z_0}^{z_i}\{(F-if)-(C_m-iC_r)\}dz,$$

$$k_L=w_i\int_{z_0}^{z_i}\{2(F+if)-(C_m+iC_r)-ih_0C_d\}dz$$
$$-v_i\int_{z_0}^{z_i}\{(C+D)-C_a-ih_0(C_m-iC_r)\}dz,$$

$$k_R=w_i\int_{z_0}^{z_i}\{(F-if)+ih_0C_d\}dz$$
$$-v_i\int_{z_0}^{z_i}\{(C-2ic)+ih_0(C_m-iC_r)\}dz,$$

$$k_F=w_i\int_{z_0}^{z_i}\{(C+D)-(D_R+T_0C_d)+ih_0(C_m+iC_r)\}dz$$

$$- v_i \int_{z_0}^{z_i} \{2(E - ie) - (E_{1R} - ie_R) - T_0(C_m - iC_r) + ih_0 C_a\} dz,$$

$$k_A = w_i \int_{z_0}^{z_i} \{(C + 2ic) - ih_0(C_m + iC_r)\} dz$$

$$- v_i \int_{z_0}^{z_i} \{(E + ie) - ih_0 C_a\} dz,$$

$$k_D = w_i \int_{z_0}^{z_i} \{(E + ie) - T_0(C_m + iC_r) - (E_{1R} + ie_R)\} dz$$

$$- v_i \int_{z_0}^{z_i} \{A - (E_{2R} + T_0 C_a)\} dz + i \left[\frac{h_0''}{8} - \frac{h_0'}{2}\left(\frac{1}{R_C} + \frac{M^2}{R_S}\right)\right],$$

$$K_L = w_i \int_{z_0}^{z_i} \{2l_1 + \pi v - \mathscr{E} C_d - \mathscr{D}(C_m + iC_r)\} dz$$

$$- v_i \int_{z_0}^{z_i} \{l_2 + \pi w - \mathscr{E}(C_m - iC_r) - \mathscr{D} C_a\} dz,$$

$$K_R = w_i \int_{z_0}^{z_i} \bar{l}_1 dz / \bar{\mathscr{D}}_b - v_i \int_{z_0}^{z_i} \bar{l}_3 dz / \bar{\mathscr{D}}_b,$$

$$K_F = w_i \int_{z_0}^{z_i} \{2(\mathscr{E}\bar{l}_1 + \bar{\mathscr{E}} p_0) + \mathscr{D}\bar{l}_2 + \bar{\mathscr{D}} p_1 + \tau\bar{\tau} v^2\} dz / \mathscr{D}_b \bar{\mathscr{D}}_b$$

$$- v_i \int_{z_0}^{z_i} \{\bar{\mathscr{E}} l_2 + 2\mathscr{D} l_4 + \mathscr{D}\bar{p}_2 + 2\mathscr{E}\bar{p}_3 + \tau\bar{\tau} v w\} dz / \mathscr{D}_b \bar{\mathscr{D}}_b,$$

$$K_A = w_i \int_{z_0}^{z_i} \{\mathscr{E}(l_1 + p_0) + \mathscr{D}(l_3 + p_3) + \tau^2 v^2\} dz / \mathscr{D}_b^2$$

$$- v_i \int_{z_0}^{z_i} \left\{\frac{\mathscr{E}}{2}(l_2 + p_1) + \frac{\mathscr{D}}{2}(l_5 + p_2) + \tau^2 v w\right\} dz / \mathscr{D}_b^2$$

$$K_D = w_i \int_{z_0}^{z_i} \{\mathscr{E}^2 \bar{l}_1 + \mathscr{E}\mathscr{D}\bar{l}_2 + \mathscr{D}^2 \bar{l}_4 + \bar{\mathscr{D}}(\mathscr{E} p_1 + \mathscr{D} p_2)$$

$$+ 2\bar{\mathscr{E}}(\mathscr{E} p_0 + \mathscr{D} p_3) + \tau(\bar{\tau} l_0 + \tau \bar{l}_0) v\} dz / \mathscr{D}_b^2 \bar{\mathscr{D}}_b$$

$$- v_i \int_{z_0}^{z_i} \{\mathscr{E}\mathscr{D}\bar{l}_5 + \mathscr{D}^2 \bar{l}_6 + \mathscr{E}^2 \bar{l}_3 + 2\bar{\mathscr{D}}(\mathscr{D} p_5 + \mathscr{E} p_4)$$

$$+ \bar{\mathscr{E}}(\mathscr{D} p_2 + \mathscr{E} p_1) + \tau(\bar{\tau} l_0 + \tau \bar{l}_0) w\} dz / \mathscr{D}_b^2 \bar{\mathscr{D}}_b,$$

$$\mathscr{K}_{F1} = w_i \int_{z_0}^{z_i} \{\bar{l}_2 - \bar{a}_{00}\} dz / \bar{\mathscr{D}}_b - v_i \int_{z_0}^{z_i} \{\bar{l}_5 - \bar{a}_{30}\} dz / \bar{\mathscr{D}}_b$$

$$\mathscr{K}_{F2} = w_i \int_{z_0}^{z_i} \{l_2 - a_{00} - i\gamma v + ih_0 \mathscr{E} C_d + ih_0 \mathscr{D}(C_m + iC_r)\} dz / \mathscr{D}_b$$

$$- v_i \int_{z_0}^{z_i} \{2l_4 - a_{40} - i\gamma w + ih_0 \mathscr{D} C_a + ih_0 \mathscr{E}(C_m - iC_r)\} dz / \mathscr{D}_b,$$

$$\mathscr{K}_A = w_i \int_{z_0}^{z_i} \{2l_3 + i\gamma v - ih_0 \mathscr{E} C_d - ih_0 \mathscr{D}(C_m + iC_r)\} dz / \mathscr{D}_b$$

$$- v_i \int_{z_0}^{z_i} \{l_5 + i\gamma w - ih_0 \mathscr{D} C_a - ih_0 \mathscr{E}(C_m - iC_r)\} dz / \mathscr{D}_b,$$

$$\mathscr{K}_{D1} = w_i \int_{z_0}^{z_i} \{\bar{l}_4 - \bar{a}_{40}\} dz / \bar{\mathscr{D}}_b - v_i \int_{z_0}^{z_i} \{\bar{l}_6 - \bar{a}_{20}\} dz / \bar{\mathscr{D}}_b,$$

$$\mathscr{K}_{D2} = w_i \int_{z_0}^{z_i} \{l_5 + \nu v - a_{30} - \mathscr{E}(T_0 C_d + D_R) - \mathscr{D} T_0(C_m + iC_r)$$

$$
\begin{aligned}
&- \mathscr{D}(E_{1R} + ie_R)\}dz/\mathscr{D}_b \\
&- v_i\int_{z_0}^{z_i}\{2l_6 + vw - a_{20} - \mathscr{D}(T_0C_a + E_{2R}) - \mathscr{E}T_0(C_m - iC_r) \\
&- \mathscr{E}(E_{1R} - ie_R)\}dz/\mathscr{D}_b,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{K}_{D3} = w_i\int_{z_0}^{z_i}&\{\mathscr{E}\bar{l}_2 + 2\mathscr{D}\bar{l}_4 + 2\bar{\mathscr{E}}p_5 + \bar{\mathscr{D}}p_2 + \tau\bar{\tau}vw - \tau\bar{\tau}_0v \\
&- (\mathscr{D}\bar{a}_{40} + \mathscr{E}\bar{a}_{00})\}dz/\mathscr{D}_b\bar{\mathscr{D}}_b \\
- v_i\int_{z_0}^{z_i}&\{2\bar{\mathscr{D}}l_6 + \bar{\mathscr{E}}l_5 + 2\mathscr{D}\bar{p}_5 + \mathscr{E}\bar{p}_2 - \tau\bar{\tau}w^2 - \tau\bar{\tau}_0w \\
&- (\mathscr{D}\bar{a}_{20} + \mathscr{E}\bar{a}_{30})\}dz/\mathscr{D}_b\bar{\mathscr{D}}_b,
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{K}_{D4} = w_i\int_{z_0}^{z_i}&\left\{\frac{\mathscr{E}}{2}(l_2 + p_1) + \frac{\mathscr{D}}{2}(l_5 + p_2) + \tau^2vw - \tau\tau_0v\right. \\
&\left.- (\mathscr{D}a_{30} + \mathscr{E}a_{00})\right\}dz/\mathscr{D}_b^2 \\
- v_i\int_{z_0}^{z_i}&\{\mathscr{E}(l_4 + p_4) + \mathscr{D}(l_6 + p_5) + \tau^2w^2 - \tau\tau_0w \\
&- (\mathscr{D}a_{20} - \mathscr{E}a_{40})\}dz/\mathscr{D}_b^2,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{K}_R &= w_i\int_{z_0}^{z_i}\{3\alpha_3v^3\}dz/\bar{\mathscr{D}}_b - v_i\int_{z_0}^{z_i}\{3\alpha_3v^2w\}dz/\bar{\mathscr{D}}_b, \\
\tilde{K}_A &= w_i\int_{z_0}^{z_i}\{6\alpha_3v^2\bar{l}_0\}dz/\bar{\mathscr{D}}_b^2 - v_i\int_{z_0}^{z_i}\{6\alpha_3vw\bar{l}_0\}dz/\bar{\mathscr{D}}_b^2, \\
\tilde{\mathscr{K}}_A &= w_i\int_{z_0}^{z_i}\{6\alpha_3v^2w\}dz/\bar{\mathscr{D}}_b - v_i\int_{z_0}^{z_i}\{6\alpha_3vw^2\}dz/\bar{\mathscr{D}}_b, \\
\tilde{K}_D &= w_i\int_{z_0}^{z_i}\{3\alpha_3v\bar{l}_0^2\}dz/\bar{\mathscr{D}}_b^3 - v_i\int_{z_0}^{z_i}\{3\alpha_3w\bar{l}_0^2\}dz/\bar{\mathscr{D}}_b^3, \\
\tilde{\mathscr{K}}_{D2} &= w_i\int_{z_0}^{z_i}\{3\alpha_3vw^2\}dz/\bar{\mathscr{D}}_b - v_i\int_{z_0}^{z_i}\{3\alpha_3w^3\}dz/\bar{\mathscr{D}}_b, \\
\tilde{\mathscr{K}}_{D4} &= w_i\int_{z_0}^{z_i}\{6\alpha_3vw\bar{l}_0\}dz/\bar{\mathscr{D}}_b^2 - v_i\int_{z_0}^{z_i}\{6\alpha_3w^2\bar{l}_0\}dz/\bar{\mathscr{D}}_b^2.
\end{aligned}
\tag{42}
$$

式中

$$
\begin{aligned}
&a_0 = \mu v^2 + \lambda v'^2, && a_{00} = \mu_0v^2 + \lambda_0v'^2, \\
&a_1 = \mu vw + \lambda v'w', && a_{10} = \mu_0vw + \lambda_0v'w', \\
&a_2 = \mu w^2 + \lambda w'^2, && a_{20} = \mu_0w^2 + \lambda_0w'^2, \\
&a_3 = a_1 + i\kappa, && a_{30} = a_{10} + i\kappa_0, \\
&a_4 = a_1 - i\kappa, && a_{40} = a_{10} - i\kappa_0, \\
&p_0 = a_0v, && p_1 = a_0w + a_4v, \\
&p_2 = a_2v + a_3w, && p_3 = a_3v, \\
&p_4 = a_4w, && p_5 = a_2w, \\
&l_0 = \mathscr{D}w + \mathscr{E}v, && l_1 = \mathscr{E}B + \mathscr{D}(F + if) + p_0
\end{aligned}
$$

$$
\begin{aligned}
l_2 &= \mathscr{D}(C + D) + 2\mathscr{E}(F - if) + p_1, \\
l_3 &= \mathscr{D}(C + 2ic) + \mathscr{E}(F + if) + p_3, \\
l_4 &= \mathscr{E}(C - 2ic) + \mathscr{D}(E - ie) + p_4,
\end{aligned}
$$

$$l_5 = \mathscr{E}(C + D) + 2\mathscr{D}(E + ie) + p_2,$$
$$l_6 = \mathscr{D}A + \mathscr{E}(E - ie) + p_5. \tag{43}$$

由于(41)式是用旋转坐标系的表观参量来表示象差的,因此还需要变换成用固定坐标系的实在参量来表示三级横向象差[7,8]。对于(41)式进行高斯变换[10]并注意到(28)式,于是得到

$$\Delta u_{3i} = \begin{bmatrix} \varepsilon_t \\ r_0\sqrt{\varepsilon_t}\, e^{i(\beta_0-\theta_0)} \\ \bar{\mathscr{D}}_b\sqrt{\varepsilon_t}\, e^{i\beta_0} \end{bmatrix}^T \begin{bmatrix} k_S & k_L - 2ih_0k_S & K_L \\ k_R + ih_0k_S & k_F + ih_0(k_L - 2k_R) + 2h_0^2k_S & \mathscr{K}_{F2} + ih_0K_L \\ K_R & \mathscr{K}_{F1} - 2ih_0K_R & K_F \end{bmatrix}$$
$$\cdot \begin{bmatrix} \sqrt{\varepsilon_t}\, e^{i\beta_0} \\ r_0e^{i\theta_0} \\ \mathscr{D}_b \end{bmatrix}$$
$$+ \begin{bmatrix} r_0^2e^{2i\theta_0} \\ r_0\mathscr{D}_be^{i\theta_0} \\ \mathscr{D}_b\mathscr{D}_b \end{bmatrix}^T \begin{bmatrix} k_A - ih_0k_L - h_0^2k_S & k_D + ih_0(k_A - k_F) + h_0^2(k_L - k_R - ih_0k_S) & \mathscr{K}_{D1} - ih_0\mathscr{K}_{F1} - h_0^2K_R \\ \mathscr{K}_A - ih_0K_L & \mathscr{K}_{D2} + ih_0(\mathscr{K}_A - \mathscr{K}_{F2}) + h_0^2K_L & \mathscr{K}_{D3} - ih_0K_F \\ K_A & \mathscr{K}_{D4} + ih_0K_A & K_D \end{bmatrix} \begin{bmatrix} \sqrt{\varepsilon_t}\, e^{-i\beta_0} \\ r_0e^{-i\theta_0} \\ \bar{\mathscr{D}}_b \end{bmatrix}$$
$$+ \begin{bmatrix} \sqrt{\varepsilon_t}\, \bar{\mathscr{D}}_be^{-i\beta_0} \\ r_0\bar{\mathscr{D}}_be^{-i\theta_0} \\ \bar{\mathscr{D}}_b\bar{\mathscr{D}}_b \end{bmatrix}^T \begin{bmatrix} \tilde{K}_R & \frac{1}{2}\tilde{\mathscr{K}}_A + ih_0\tilde{\mathscr{K}}_R & \frac{1}{2}\tilde{\mathscr{K}}_A \\ \frac{1}{2}\tilde{\mathscr{K}}_A + ih_0\tilde{K}_R & \tilde{\mathscr{K}}_{D2} + ih_0\tilde{\mathscr{K}}_A - h_0^2\tilde{\mathscr{K}}_R & \frac{1}{2}(\tilde{\mathscr{K}}_{D4} + ih_0\tilde{K}_A) \\ \frac{1}{2}\tilde{K}_A & \frac{1}{2}(\tilde{\mathscr{K}}_{D4} + ih_0\tilde{\mathscr{K}}_A) & \tilde{K}_D \end{bmatrix} \begin{bmatrix} \sqrt{\varepsilon_t}\, e^{-i\beta_0} \\ r_0e^{-i\theta_0} \\ \bar{\mathscr{D}}_b \end{bmatrix}. \tag{44}$$

将以上矩阵进行乘法运算展开可得

$$\begin{aligned}
\Delta u_{3i} = {} & (\varepsilon_t)^{3/2}e^{i\beta_0}k_S && \text{——色球差} \\
& + \varepsilon_tr_0e^{i\theta_0}(k_L - 2ih_0k_S) + \varepsilon_t\mathscr{D}_bK_L && \text{——彗差长度} \\
& + \varepsilon_tr_0e^{i(2\beta_0-\theta_0)}(k_R + ih_0k_S) + \varepsilon_t\bar{\mathscr{D}}_be^{2i\beta_0}K_R \\
& + \varepsilon_t\bar{\mathscr{D}}_be^{-2i\beta_0}\tilde{K}_R && \text{——彗差半径} \\
& + \sqrt{\varepsilon_t}\, r_0^2e^{i\beta_0}[k_F + ih_0(k_L - 2k_R) + 2h_0^2k_S] + \sqrt{\varepsilon_t}\,\mathscr{D}_b\bar{\mathscr{D}}_be^{i\beta_0}K_F \\
& + \sqrt{\varepsilon_t}\, r_0\bar{\mathscr{D}}_be^{i(\beta_0+\theta_0)}(\mathscr{K}_{F1} - 2ih_0k_R) \\
& + \sqrt{\varepsilon_t}\, r_0\mathscr{D}_be^{i(\beta_0-\theta_0)}(\mathscr{K}_{F2} + ih_0K_L) && \text{——场曲}
\end{aligned}$$

$$
\begin{aligned}
&+\sqrt{\varepsilon_t}\, r_0 \mathscr{D}_b e^{-i(\beta_0-\theta_0)}(\mathscr{K}_A - ih_0 K_L) + \sqrt{\varepsilon_t}\, \bar{\mathscr{D}}_b^2 e^{-i\beta_0}\tilde{K}_A \\
&+\sqrt{\varepsilon_t}\, r_0 \bar{\mathscr{D}}_b e^{-i(\beta_0+\theta_0)}(\tilde{\mathscr{K}}_A + 2ih_0\tilde{K}_R) \qquad \text{——象散} \\
&+ r_0^3 e^{i\theta_0}[k_D + ih_0(k_A - k_F) + h_0^3(k_L - k_R - ih_0 k_S)] + \mathscr{D}_b^2\bar{\mathscr{D}}_b K_D \\
&+ r_0^2\bar{\mathscr{D}}_b e^{2i\theta_0}(\mathscr{K}_{D1} - ih_0\mathscr{K}_{F1} - h_0^2 K_R) + \bar{\mathscr{D}}_b^3\tilde{K}_D \\
&+ r_0\mathscr{D}_b\bar{\mathscr{D}}_b e^{i\theta_0}(\mathscr{K}_{D3} - ih_0 K_F) \\
&+ r_0^2\mathscr{D}_b[\mathscr{K}_{D2} + ih_0(\mathscr{K}_A - \mathscr{K}_{F2}) + h_0^2 K_L] \\
&+ r_0\mathscr{D}_b^2 e^{-i\theta_0}(\mathscr{K}_{D4} + ih_0 K_A) \\
&+ r_0^2\bar{\mathscr{D}}_b e^{-2i\theta_0}(\tilde{\mathscr{K}}_{D2} + ih_0\tilde{\mathscr{K}}_A - h_0^2\tilde{K}_R) \\
&+ r_0\bar{\mathscr{D}}_b^2 e^{-i\theta_0}(\tilde{\mathscr{K}}_{D4} + ih_0\tilde{K}_A). \qquad \text{——畸变(45)}
\end{aligned}
$$

在固定坐标系内一级近轴横向象差为

$$
\begin{aligned}
\Delta r_1(z_i, \varepsilon_n) = &\left\{\left[1 - \frac{ih_i}{2\sqrt{V_p(z_i)}}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right) r_0^2\right][(r_0 e^{i\theta_0} + \mathscr{D}_b) w_i \right. \\
&+ (\sqrt{\varepsilon_t}\, e^{i\beta_0} - ih_0 r_0 e^{i\theta_0} + \mathscr{E}_b) v_i] \\
&- \frac{r_0^2}{2}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right)[(r_0 e^{i\theta_0} + \mathscr{D}_b) w_i' \\
&\left. + (\sqrt{\varepsilon_t}\, e^{i\beta_0} - ih_0 r_0 e^{i\theta_0} + \mathscr{E}_b) v_i']\right\} e^{i\chi(z_i, \varepsilon_n)} \\
&- \left\{(-1)^m r_0 e^{i\theta_0} M \left[1 - \frac{ih_i}{2\sqrt{V_p(z_i)}}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right) r_0^2\right]\right. \\
&- \frac{r_0^2}{2}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right)[(r_0 e^{i\theta_0} + \mathscr{D}_{bn}) w'(z_i, \varepsilon_{n1}) \\
&\left. + (\sqrt{\varepsilon_t}\, e^{i\beta_0} - ih_0 r_0 e^{i\theta_0} + \mathscr{E}_{bn}) v'(z_i, \varepsilon_{n1})] e^{i\chi(z_i, \varepsilon_{n1})}\right\}. \qquad (46)
\end{aligned}
$$

相对论情况下实际轨迹可以表示为

$$
\begin{aligned}
r_{\text{real}}(z_i, \varepsilon_n) = &\left\{\left[1 - \frac{ih_i}{2\sqrt{V_p(z_i)}}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right) r_0^2\right]\right. \\
&\cdot [(r_0 e^{i\theta_0} + \mathscr{D}_b) w_i + (\sqrt{\varepsilon_t}\, e^{i\beta_0} - ih_0 r_0 e^{i\theta_0} + \mathscr{E}_b) v_i] \\
&- \frac{r_0^2}{2}\left(\frac{1}{R_c} + \frac{M^2}{R_s}\right)[(r_0 e^{i\theta_0} + \mathscr{D}_b) w_i' \\
&\left. + (\sqrt{\varepsilon_t}\, e^{i\beta_0} - ih_0 r_0 e^{i\theta_0} + \mathscr{E}_b) v_i'] + \Delta u_{3i}\right\} e^{i\chi(z_i, \varepsilon_n)}. \qquad (47)
\end{aligned}
$$

式中 $\mathscr{D}_{bn} = \mathscr{D}(z_i, \varepsilon_{n1})$，$\mathscr{E}_{bn} = \mathscr{E}(z_i, \varepsilon_{n1})$ 分别为 z_i 象面上的约化偏转量与偏转灵敏度，$\mathscr{E}_b = \mathscr{E}(z_i, \varepsilon_n)$，$\mathscr{D}_b = \mathscr{D}(z_i, \varepsilon_n)$。

如果选择本文象差公式中某些函数为零，则可以讨论电或磁聚焦以及电或磁偏转等复合系统的象差；还可以讨论扫描束电子光学器件中使用的可动物镜系统和存在预偏转的电磁复合系统的象差[10]。

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RELATIVISTIC ABERRATION THEORY FOR A COMBINED ELECTROMAGNETIC FOCUSING-DEFLECTION SYSTEM POSSESSING A SPHERICAL CATHODE LENS

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ABSTRACT

In the present paper, by means of the variational principle, the relativistic aberration theory for a combined electromagnetic focusing-deflection system possessing a spherical cathode lens is studied. We have discussed the following general cases: electrons emitted from the cathode possess definite energy and angular initial distributions; the objective and image fields have finite curvatures; the cathode is situated in both magnetic and transversal electric fields. For these general cases the general formulae of aberration coefficients for first and third order transversal geometrical and chromatic aberrations on any ideal image plane are given in explicit form. The above-mentioned aberration coefficient formulae can be applied to the following cases: either the wide electron beam system or the narrow electron beam system; either the relativistic or the non-relativistic electron optical system; either the cathode or the screen having the spherical or the planar shapes. In the present work, the electron trajectories are described in the complex form. The aberrations are expressed in the matrix form, and are thus suitable for computer computations.