

# 一类动态黑洞的非热辐射\*

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研究了一类动态黑洞视界附近的粒子能级交错,给出了交错能级最大值的表达式,所得结果覆盖了所有已知的动态黑洞的结果.对于动态非球对称黑洞,其非热辐射的频率范围不仅依赖于时间,而且依赖于角度,这是新的结果.

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## 1 引 言

Damour 和 Deruelle<sup>[1]</sup>研究了弯曲时空中的 Hamilton-Jacobi 方程,根据此方程,人们已对稳态黑洞的非热辐射进行了研究<sup>[2]</sup>.近年来,对动态黑洞物理特征的研究引起了人们的广泛兴趣<sup>[3-9]</sup>.动态黑洞的非热辐射也是一个值得研究的课题.我们采用推广的 Tortoise 坐标变换,找到了导出粒子能级表达式的新方法,有效地研究了一类动态黑洞的非热辐射.

## 2 计算结果

一般而言,动态时空线元可以表示为

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \quad (1)$$

采用球坐标 ( $x^0 = \nu, x^1 = r, x^2 = \theta, x^3 = \phi$ ), 并利用零曲面条件

$$g^{\mu\nu} \frac{\partial F}{\partial x^\mu} \frac{\partial F}{\partial x^\nu} = 0, \quad (2)$$

得出黑洞视界面方程为

$$g^{00} \left( \frac{\partial r}{\partial \nu} \right)^2 - 2g^{01} \left( \frac{\partial r}{\partial \nu} \right) + 2g^{02} \left( \frac{\partial r}{\partial \nu} \right) \left( \frac{\partial r}{\partial \theta} \right) + 2g^{03} \left( \frac{\partial r}{\partial \nu} \right) \left( \frac{\partial r}{\partial \phi} \right) + g^{11}$$

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$$-2g^{12}\left(\frac{\partial r}{\partial \theta}\right) - 2g^{13}\left(\frac{\partial r}{\partial \phi}\right) + g^{22}\left(\frac{\partial r}{\partial \theta}\right)^2 + 2g^{23}\left(\frac{\partial r}{\partial \theta}\right)\left(\frac{\partial r}{\partial \phi}\right) + g^{33}\left(\frac{\partial r}{\partial \phi}\right)^2 = 0. \quad (3)$$

满足此方程的  $r_H$  就是局部事件视界<sup>[10]</sup>.

在(1)式表述的动态时空中,粒子的运动方程为

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu} - \beta A_\mu\right)\left(\frac{\partial S}{\partial x^\nu} - \beta A_\nu\right) - \mu^2 = 0, \quad (4)$$

即

$$\begin{aligned} & g^{\nu\nu}\left(\frac{\partial S}{\partial \nu} - \beta A_\nu\right)^2 + 2g^{01}\left(\frac{\partial S}{\partial \nu} - \beta A_\nu\right)\left(\frac{\partial S}{\partial r} - \beta A_1\right) + 2g^{02}\left(\frac{\partial S}{\partial \nu} - \beta A_\nu\right) \\ & \quad \times \left(\frac{\partial S}{\partial \theta} - \beta A_2\right) + 2g^{03}\left(\frac{\partial S}{\partial \nu} - \beta A_\nu\right)\left(\frac{\partial S}{\partial \phi} - \beta A_3\right) + g^{11}\left(\frac{\partial S}{\partial r} - \beta A_1\right)^2 \\ & \quad + 2g^{12}\left(\frac{\partial S}{\partial r} - \beta A_1\right)\left(\frac{\partial S}{\partial \theta} - \beta A_2\right) + 2g^{13}\left(\frac{\partial S}{\partial r} - \beta A_1\right) \\ & \quad \times \left(\frac{\partial S}{\partial \phi} - \beta A_3\right) + g^{22}\left(\frac{\partial S}{\partial \theta} - \beta A_2\right)^2 + 2g^{23}\left(\frac{\partial S}{\partial \theta} - \beta A_2\right) \\ & \quad \times \left(\frac{\partial S}{\partial \phi} - \beta A_3\right) + g^{33}\left(\frac{\partial S}{\partial \phi} - \beta A_3\right)^2 - \mu^2 = 0, \end{aligned} \quad (5)$$

式中  $A_\mu$  是物质场的广义规范势,  $\beta$  是相应于  $A_\mu$  的“荷”. 当  $A_\mu$  是电磁四矢时,  $\beta$  是粒子电荷.

为了研究黑洞视界附近的非热辐射,对(5)式作推广的 Tortoise 坐标变换<sup>[9,10]</sup>

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa} \ln [r - r_H(\nu, \theta, \phi)], \quad \theta_* = \theta - \theta_0, \\ \nu_* &= \nu - \nu_0, \quad \phi_* = \phi - \phi_0. \end{aligned} \quad (6)$$

由此可得部分关系式

$$\begin{aligned} \frac{\partial}{\partial r} &= \frac{2\kappa(r - r_H) + 1}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}, \quad \frac{\partial}{\partial \nu} = \frac{\partial}{\partial \nu_*} - \frac{\left(\frac{\partial r_H}{\partial \nu}\right)}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial \theta} &= \frac{\partial}{\partial \theta_*} - \frac{\left(\frac{\partial r_H}{\partial \theta}\right)}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}, \quad \frac{\partial}{\partial \phi} = \frac{\partial}{\partial \phi_*} - \frac{\left(\frac{\partial r_H}{\partial \phi}\right)}{2\kappa(r - r_H)} \frac{\partial}{\partial r_*}. \end{aligned} \quad (7)$$

把(7)式代入方程(5)中,得到方程(5)在推广的 Tortoise 坐标下的形式,经过一定的计算可得

$$\frac{\partial S}{\partial r_*} = \frac{2\kappa(r - r_H) [-\tilde{B} \pm (\tilde{B}^2 - \tilde{A}\tilde{C})^{1/2}]}{\tilde{A}}, \quad (8)$$

式中

$$\tilde{A} = g^{\nu\nu}\left(\frac{\partial r_H}{\partial \nu}\right)^2 - 2g^{01}\left(\frac{\partial r_H}{\partial \nu}\right)[2\kappa(r - r_H) + 1] + 2g^{02}\left(\frac{\partial r_H}{\partial \nu}\right)\left(\frac{\partial r_H}{\partial \theta}\right)$$

$$\begin{aligned}
& + 2g^{03} \left( \frac{\partial r_{\text{H}}}{\partial v} \right) \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) + g^{11} [2\kappa(r - r_{\text{H}}) + 1]^2 - 2g^{12} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \\
& \times [2\kappa(r - r_{\text{H}}) + 1] - 2g^{13} \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) [2\kappa(r - r_{\text{H}}) + 1] \\
& + g^{33} \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right)^2 + g^{22} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right)^2 + 2g^{23} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right), \tag{9}
\end{aligned}$$

$$\begin{aligned}
\tilde{B} = & -g^{00} \left( \frac{\partial S}{\partial v_{*}} \right) \left( \frac{\partial r_{\text{H}}}{\partial v} \right) + g^{00} \beta A_0 \left( \frac{\partial r_{\text{H}}}{\partial v} \right) + g^{01} \left( \frac{\partial S}{\partial v_{*}} \right) [2\kappa(r - r_{\text{H}}) + 1] \\
& + g^{01} \beta A_1 \left( \frac{\partial r_{\text{H}}}{\partial v} \right) - g^{01} \beta A_0 [2\kappa(r - r_{\text{H}}) + 1] \\
& - g^{02} \left( \frac{\partial r_{\text{H}}}{\partial v} \right) \left( \frac{\partial S}{\partial \theta_{*}} \right) - g^{02} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \left( \frac{\partial S}{\partial v_{*}} \right) + g^{02} \beta A_2 \left( \frac{\partial r_{\text{H}}}{\partial v} \right) \\
& + g^{02} \beta A_0 \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) - g^{03} \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) \left( \frac{\partial S}{\partial v_{*}} \right) - g^{03} \left( \frac{\partial r_{\text{H}}}{\partial v} \right) \left( \frac{\partial S}{\partial \phi_{*}} \right) \\
& + g^{03} \beta A_3 \left( \frac{\partial r_{\text{H}}}{\partial v} \right) + g^{03} \beta A_0 \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) - g^{11} \beta A_1 [2\kappa(r - r_{\text{H}}) + 1] \\
& + g^{12} \left( \frac{\partial S}{\partial \theta_{*}} \right) [2\kappa(r - r_{\text{H}}) + 1] - g^{12} \beta A_2 [2\kappa(r - r_{\text{H}}) + 1] \\
& + g^{13} [2\kappa(r - r_{\text{H}}) + 1] \left( \frac{\partial S}{\partial \phi_{*}} \right) + g^{12} \beta A_1 \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \\
& - g^{13} \beta A_3 [2\kappa(r - r_{\text{H}}) + 1] - g^{13} \beta A_1 \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) - g^{22} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \left( \frac{\partial S}{\partial \theta_{*}} \right) \\
& + g^{22} \beta A_2 \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) - g^{23} \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) \left( \frac{\partial S}{\partial \theta_{*}} \right) - g^{23} \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) \left( \frac{\partial S}{\partial \phi_{*}} \right) \\
& + g^{23} \beta A_3 \left( \frac{\partial r_{\text{H}}}{\partial \theta} \right) + g^{23} \beta A_2 \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) + g^{33} \beta A_3 \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) \\
& - g^{33} \left( \frac{\partial r_{\text{H}}}{\partial \phi} \right) \left( \frac{\partial S}{\partial \phi_{*}} \right), \tag{10}
\end{aligned}$$

$$\begin{aligned}
\tilde{C} = & g^{00} \left( \frac{\partial S}{\partial v_{*}} \right)^2 - 2g^{00} \beta A_0 \left( \frac{\partial S}{\partial v_{*}} \right) + g^{00} \beta^2 A_0^2 - 2g^{01} \beta A_1 \left( \frac{\partial S}{\partial v_{*}} \right) \\
& + 2g^{01} \beta^2 A_0 A_1 + 2g^{02} \left( \frac{\partial S}{\partial v_{*}} \right) \left( \frac{\partial S}{\partial \theta_{*}} \right) - 2g^{02} \beta A_2 \left( \frac{\partial S}{\partial v_{*}} \right) \\
& - 2g^{02} \beta A_0 \left( \frac{\partial S}{\partial \theta_{*}} \right) + 2g^{03} \left( \frac{\partial S}{\partial v_{*}} \right) \left( \frac{\partial S}{\partial \phi_{*}} \right) - 2g^{03} \beta A_3 \left( \frac{\partial S}{\partial v_{*}} \right) \\
& - 2g^{03} \beta A_0 \left( \frac{\partial S}{\partial \phi_{*}} \right) + g^{11} \beta^2 A_1^2 - 2g^{12} \beta A_1 \left( \frac{\partial S}{\partial \theta_{*}} \right) + 2g^{12} \beta^2 A_1 A_2 \\
& - 2g^{13} \beta A_1 \left( \frac{\partial S}{\partial \phi_{*}} \right) + g^{13} \beta^2 A_1 A_3 + g^{22} \left( \frac{\partial S}{\partial \theta_{*}} \right)^2 - 2g^{22} \beta A_2 \left( \frac{\partial S}{\partial \theta_{*}} \right)
\end{aligned}$$

$$\begin{aligned}
& + g^{22}\beta A_1^2 + 2g^{23}\beta^2 A_2 A_3 + 2g^{23}\left(\frac{\partial S}{\partial \theta_*}\right)\left(\frac{\partial S}{\partial \phi_*}\right) - 2g^{23}\beta A_3\left(\frac{\partial S}{\partial \theta_*}\right) \\
& + g^{33}\left(\frac{\partial S}{\partial \phi_*}\right) - 2g^{23}\beta A_2\left(\frac{\partial S}{\partial \phi_*}\right) - 2g^{33}\beta A_3\left(\frac{\partial S}{\partial \phi_*}\right) \\
& + g^{33}\beta^2 A_3^2 + 2g^{02}\beta^2 A_0 A_2 + 2g^{03}\beta^2 A_0 A_3 - \mu^2, \tag{11}
\end{aligned}$$

式中  $S$  是 Hamilton 主函数,  $\frac{\partial S}{\partial r_*}$  应为实数. 因此, 由(8)式得到

$$\tilde{B}^2 - \tilde{A}\tilde{C} \geq 0, \tag{12}$$

式中  $\tilde{B}$  和  $\tilde{C}$  均与  $\partial S / \partial v_*$  有关, 而  $\partial S / \partial v_*$  又与粒子能量有关, 所以, (12)式就是粒子能量所满足的关系式, 只有满足此式的粒子能级才存在. 定义

$$\frac{\partial S}{\partial v_*} = -\omega; \quad \frac{\partial S}{\partial \theta_*} = p_2; \quad \frac{\partial S}{\partial \phi_*} = p_3. \tag{13}$$

把(13), (11), (10)和(9)式代入(12)式, 得

$$\begin{aligned}
& \omega^2 \left\{ -g^{00}\left(\frac{\partial r_H}{\partial v}\right) + g^{01}[2\kappa(r - r_H) + 1] - g^{02}\left(\frac{\partial r_H}{\partial \theta}\right) - g^{03}\left(\frac{\partial r_H}{\partial \phi}\right) \right\}^2 \\
& - 2\omega\tilde{b} \left\{ -g^{00}\left(\frac{\partial r_H}{\partial v}\right) + g^{01}[2\kappa(r - r_H) + 1] - g^{02}\left(\frac{\partial r_H}{\partial \theta}\right) \right. \\
& \left. - g^{03}\left(\frac{\partial r_H}{\partial \phi}\right) \right\} + \tilde{b}^2 - \omega^2\tilde{A}g^{00} + \tilde{A}\omega(-2g^{00}\beta A_0 - 2g^{01}\beta A_1 + 2g^{02}p_2 \\
& - 2g^{03}\beta A_2 + 2g^{03}p_3 - 2g^{03}\beta A_3) - \tilde{A}\tilde{C}' \geq 0. \tag{14}
\end{aligned}$$

取上式等号解之得

$$\omega_{\pm} = \frac{H\tilde{b} - \tilde{A}(-\beta g^{00}A_0 + g^{02}p_2 + g^{03}p_3)}{H^2 - \tilde{A}g^{00}} \pm D, \tag{15}$$

式中

$$\begin{aligned}
D &= \frac{1}{H^2 - \tilde{A}g^{00}} \left\{ \tilde{A}^2 [(-\beta g^{00}A_0 + g^{02}p_2 + g^{03}p_3)^2 - g^{00}\tilde{C}'] - \tilde{A}H \right. \\
& \left. \times [2\tilde{b}(-\beta g^{00}A_0 + g^{02}p_2 + g^{03}p_3)H\tilde{C}'] - \tilde{A}g^{00}\tilde{b}^2 \right\}^{1/2}, \tag{16}
\end{aligned}$$

$$H = -g^{00}\left(\frac{\partial r_H}{\partial v}\right) + g^{01}[2\kappa(r - r_H) + 1] - g^{02}\left(\frac{\partial r_H}{\partial \theta}\right) - g^{03}\left(\frac{\partial r_H}{\partial \phi}\right), \tag{17}$$

$$\begin{aligned}
\tilde{b} &= g^{00}\beta A_0\left(\frac{\partial r_H}{\partial v}\right) + g^{01}\beta A_1\left(\frac{\partial r_H}{\partial v}\right) - g^{01}\beta A_0[2\kappa(r - r_H) + 1] \\
& - g^{02}p_2\left(\frac{\partial r_H}{\partial v}\right) + g^{02}\beta A_2\left(\frac{\partial r_H}{\partial v}\right) + g^{02}\beta A_0\left(\frac{\partial r_H}{\partial \theta}\right) - g^{03}p_3\left(\frac{\partial r_H}{\partial v}\right) \\
& + g^{03}\beta A_3\left(\frac{\partial r_H}{\partial v}\right) + g^{03}\beta A_0\left(\frac{\partial r_H}{\partial \phi}\right) - g^{11}\beta A_1[2\kappa(r - r_H) + 1] \\
& + g^{12}p_2[2\kappa(r - r_H) + 1] - g^{12}\beta A_2[2\kappa(r - r_H) + 1] \\
& + g^{13}\beta A_1\left(\frac{\partial r_H}{\partial \theta}\right) - g^{13}p_3[2\kappa(r - r_H) + 1] - g^{13}\beta A_3[2\kappa(r - r_H) + 1]
\end{aligned}$$

$$\begin{aligned}
& + g^{13}\beta A_1 \left( \frac{\partial r_H}{\partial \phi} \right) - g^{22}p_2 \left( \frac{\partial r_H}{\partial \theta} \right) - g^{23}p_2 \left( \frac{\partial r_H}{\partial \phi} \right) + g^{22}\beta A_2 \left( \frac{\partial r_H}{\partial \theta} \right) \\
& - g^{23}p_3 \left( \frac{\partial r_H}{\partial \theta} \right) + g^{23}\beta A_3 \left( \frac{\partial r_H}{\partial \theta} \right) + g^{23}\beta A_2 \left( \frac{\partial r_H}{\partial \phi} \right) \\
& - g^{33}p_3 \left( \frac{\partial r_H}{\partial \phi} \right) + g^{33}\beta A_3 \left( \frac{\partial r_H}{\partial \phi} \right), \tag{18}
\end{aligned}$$

$$\begin{aligned}
\tilde{C}' = & g^{00}\beta^2 A_0^2 + 2g^{01}\beta^2 A_0 A_1 - 2g^{02}\beta A_0 p_2 - 2g^{03}\beta A_0 p_3 + g^{11}\beta^2 A_1^2 - 2g^{12}\beta A_1 p_2 \\
& + 2g^{12}\beta^2 A_1 A_2 - 2g^{13}\beta A_1 p_3 + 2g^{13}\beta^2 A_1 A_3 + g^{22}p_2^2 - 2g^{22}\beta A_2 p_2 \\
& + g^{22}\beta^2 A_2^2 + 2g^{23}\beta^2 A_2 A_3 + 2g^{23}p_2 p_3 - 2g^{23}\beta A_3 p_2 - 2g^{23}\beta A_2 p_3 \\
& + g^{33}p_3^2 - 2g^{33}\beta A_3 p_3 + g^{33}\beta^2 A_3^2 + 2g^{02}\beta^2 A_0 A_2 + 2g^{03}\beta^2 A_0 A_3 - \mu^2. \tag{19}
\end{aligned}$$

由(14)和(15)式可知,粒子能级分布为

$$\omega \geq \omega^+; \omega \leq \omega^-. \tag{20}$$

粒子能量处于  $\omega^- < \omega < \omega^+$  的状态是禁戒的,禁区的宽度为

$$\Delta\omega = 2D. \tag{21}$$

在以上导出粒子能级分布的过程中,未取  $r$  的特殊值,因此,计算所得的结论在黑洞视界附近及远处都适合.下面研究黑洞视界附近的能级分布情形.

当  $r \rightarrow r_H$  时,得到

$$\begin{aligned}
\tilde{A}|_{r \rightarrow r_H} = & g^{00} \left( \frac{\partial r_H}{\partial v} \right)^2 - 2g^{01} \left( \frac{\partial r_H}{\partial v} \right) + 2g^{02} \left( \frac{\partial r_H}{\partial v} \right) \left( \frac{\partial r_H}{\partial \theta} \right) \\
& + 2g^{03} \left( \frac{\partial r_H}{\partial v} \right) \left( \frac{\partial r_H}{\partial \phi} \right) + g^{11} - 2g^{12} \left( \frac{\partial r_H}{\partial \theta} \right) \\
& - 2g^{13} \left( \frac{\partial r_H}{\partial \phi} \right) + g^{33} \left( \frac{\partial r_H}{\partial \phi} \right)^2 + 2g^{23} \left( \frac{\partial r_H}{\partial \theta} \right) \left( \frac{\partial r_H}{\partial \phi} \right) \\
& + g^{22} \left( \frac{\partial r_H}{\partial \theta} \right)^2 = 0. \tag{22}
\end{aligned}$$

此式与方程(3)一致,是局部事件视界  $r_H$  所满足的方程.

由(22),(21)和(15)–(19)式可得

$$\Delta\omega|_{r \rightarrow r_H} = 0, \tag{23}$$

$$\omega^+|_{r \rightarrow r_H} - \omega^-|_{r \rightarrow r_H} = \omega_0 = \frac{\tilde{\omega}}{H}|_{r \rightarrow r_H}$$

$$\begin{aligned}
= & \frac{\beta A_\mu \left[ g^{\mu j} \left( \frac{\partial r_H}{\partial x^j} \right) - g^{\mu 1} \right]}{g^{01} - g^{00} \left( \frac{\partial r_H}{\partial v} \right) - g^{02} \left( \frac{\partial r_H}{\partial \theta} \right) - g^{03} \left( \frac{\partial r_H}{\partial \phi} \right)} \Bigg|_{r \rightarrow r_H} \\
& + \frac{p_2 \left[ g^{12} - g^{j2} \left( \frac{\partial r_H}{\partial x^j} \right) \right] - p_3 \left[ g^{13} + g^{j3} \left( \frac{\partial r_H}{\partial x^j} \right) \right]}{g^{01} - g^{00} \left( \frac{\partial r_H}{\partial v} \right) - g^{02} \left( \frac{\partial r_H}{\partial \theta} \right) - g^{03} \left( \frac{\partial r_H}{\partial \phi} \right)} \Bigg|_{r \rightarrow r_H}, \tag{24}
\end{aligned}$$

式中  $j = 0, 2, 3; \mu = 0, 1, 2, 3$ .

(23)和(24)式表明,在一类动态黑洞视界附近,出现了粒子正负能级交错现象,在能级的交错区,禁区的宽度为零,处于负能态而能量高于最低正能态的粒子,将通过量子隧道效应穿过禁区成为出射正能粒子,这是与黑洞温度无关的自发辐射过程。 $\omega_0$ 是交错能级的最大值,由(24)式表述。显然, $\omega_0$ 由两项组成,第一项取决于物质场的广义规范势,第二项主要由广义动量  $p_2$  和  $p_3$  决定。只要广义规范势  $A_\mu$  不为零或黑洞有加速度,就会在黑洞视界附近出现正负能级交错,导致自发辐射的产生,这是对费密子而言。对于玻色子,除了自发辐射过程外,还会发生受激辐射。自发辐射和受激辐射都属于与黑洞温度无关的非热辐射,其辐射频率的最大值由(24)式决定。

### 3 结果与讨论

根据上面的计算结果,对特殊情况进行讨论。

#### 3.1 Vaidya-Bonner 黑洞

Vaidya-Bonner 黑洞是球对称带电蒸发黑洞,时空线元为

$$ds^2 = -\left[1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2}\right] dv^2 + 2dvdr + r^2 dQ^2. \quad (25)$$

由(24)和(25)式可得

$$\omega_0 = eQ/r_H, \quad (26)$$

根据(25)和(15)式得出

$$\omega^\pm|_{r \rightarrow \infty} \rightarrow \pm \mu, \quad (27)$$

(26),(27)式与已知结论完全一致。对于 Vaidya-Bonner 黑洞,辐射粒子的频率范围是

$$\mu < \omega \leq eQ/r_H. \quad (28)$$

辐射频率的最大值与角度无关,这是特殊情况。

#### 3.2 动态 Kinnersley 黑洞

变加速直线运动的 Kinnersley 黑洞,时空线元为

$$ds^2 = (1 - 2ar \cos \theta - r^2 f - 2Mr^{-1})dv^2 - 2dvdr + 2r^2 f dv d\theta - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2. \quad (29)$$

由(29)和(24)式得到

$$\omega_0 = p_2 \left[ f - \frac{1}{r_H^2} \left( \frac{\partial r_H}{\partial \theta} \right) \right] - \beta \left\{ A_0 + A_1 \left[ \left( \frac{\partial r_H}{\partial v} \right) + \left( \frac{\partial r_H}{\partial \theta} \right) \right] \cdot \left[ f - \frac{1}{r_H^2} \left( \frac{\partial r_H}{\partial \theta} \right) \right] \right\} + A_2 \left[ f - \frac{1}{r_H^2} \left( \frac{\partial r_H}{\partial \theta} \right) \right]. \quad (30)$$

这是变加速直线运动黑洞非热辐射频率的最大值,与文献 [11] 的结论一致。对于动态 Kinnersley 黑洞而言,非热辐射的频率范围不仅与时间有关,而且还与角度有关。

### 4 结 论

(24)式具有一定的综合性,包括了一类动态黑洞的情形,对于不同的特殊情况,非热

辐射频率的最大值不同。应该强调的是,对于动态非球对称黑洞,交错能级最大值  $\omega_0$  不仅随时间变化,而且与角度有关。因此,非热辐射的频率范围不仅依赖于时间,而且与角度有关,这是具有重要意义的结论。

- [1] T. Damour, N. Deruelle, in *Proceedings of the First Marcel Grossmann Meeting on General Relativity*, by R. Ruffini (North-Holland Publishing Company, Amsterdam, 1975), p.476.
- [2] 赵峥, *物理学报*, **32**(1983), 1233.
- [3] S. W. Kim, *Phys. Lett.* **A41**(1989), 238.
- [4] R. Balbinot, *Phys. Rev.*, **D33**(1986), 1611.
- [5] W.A. Hiscock, *Phys. Rev.*, **D23** (1981), 2813.
- [6] J.W. York, Jr., *Phys. Rev.*, **D33**(1986), 2092.
- [7] R. Balbinot, A. Barletta, *Class. Quantum Grav.*, **6**(1989), 195.
- [8] Zhao Zheng, Dai Xian-xin, *Modern Phys. Lett.*, **A7** (1992), 1771.
- [9] 赵峥, *中国科学*, **23**(1993), 178.
- [10] 罗志强、赵峥, *物理学报*, **42**(1993), 506.
- [11] 杨树政、肖兴国、赵峥, *科学通报*, **38**(1993), 1371.

## NON-THERMAL RADIATIONS OF A TYPE OF NON-STATIONARY BLACK HOLE

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### ABSTRACT

The crossing of the particle energy levels near the event horizons of a type of non-stationary black hole is studied. The representative of the maximum value of the crossing energy levels is shown. The conclusion given covers all known results for non-stationary black holes. A new interesting result of this paper is that the frequency distribution of the non-thermal radiation, from a non-spherically symmetric and non-stationary black hole, depends on not only the time, but also the angle.

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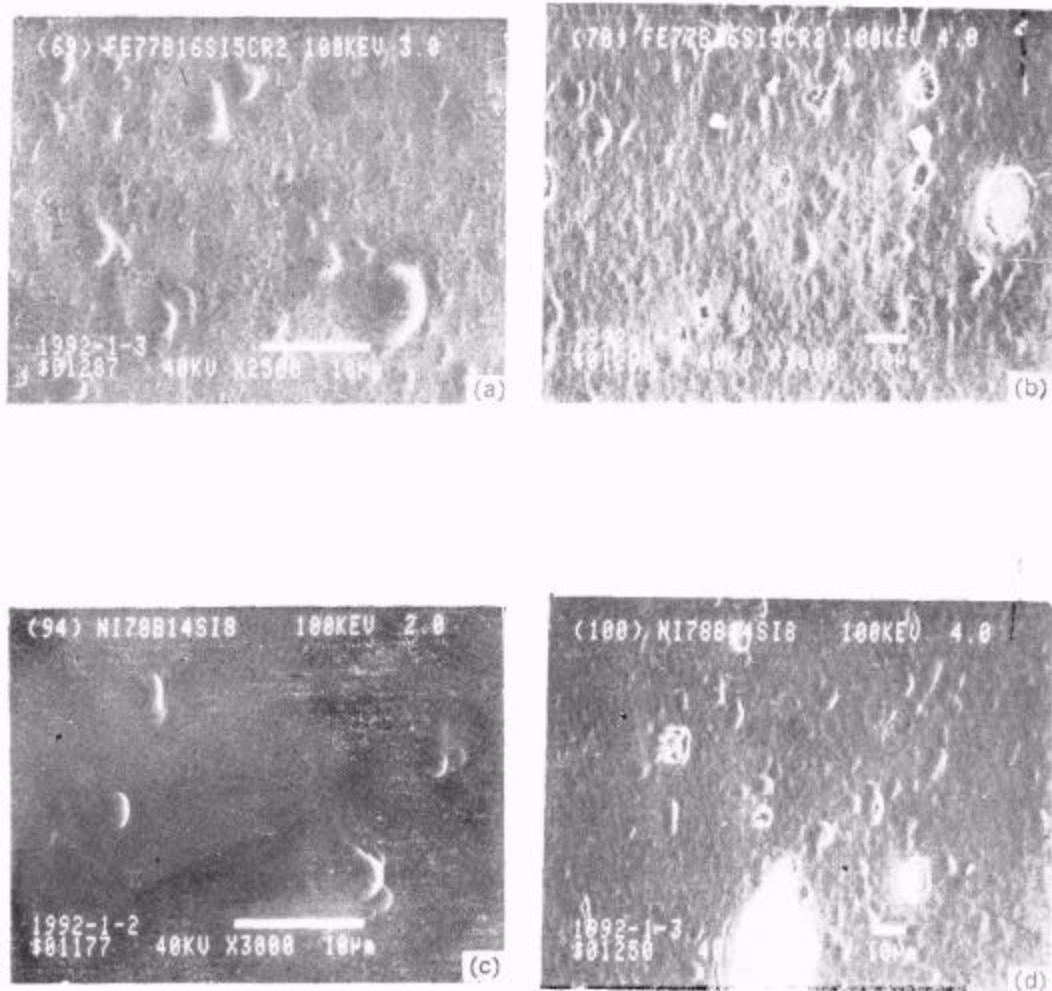


图1 100keV  $\text{He}^{+}$  辐照非晶态合金  $\text{Fe}_{77}\text{B}_{16}\text{Si}_{15}\text{Cr}_2$  和  $\text{Ni}_{78}\text{B}_{14}\text{Si}_8$  的 SEM 图  
图上方分别给出样品号、材料、辐照能量和剂量 ( $\times 10^{18}\text{He}/\text{cm}^2$ )



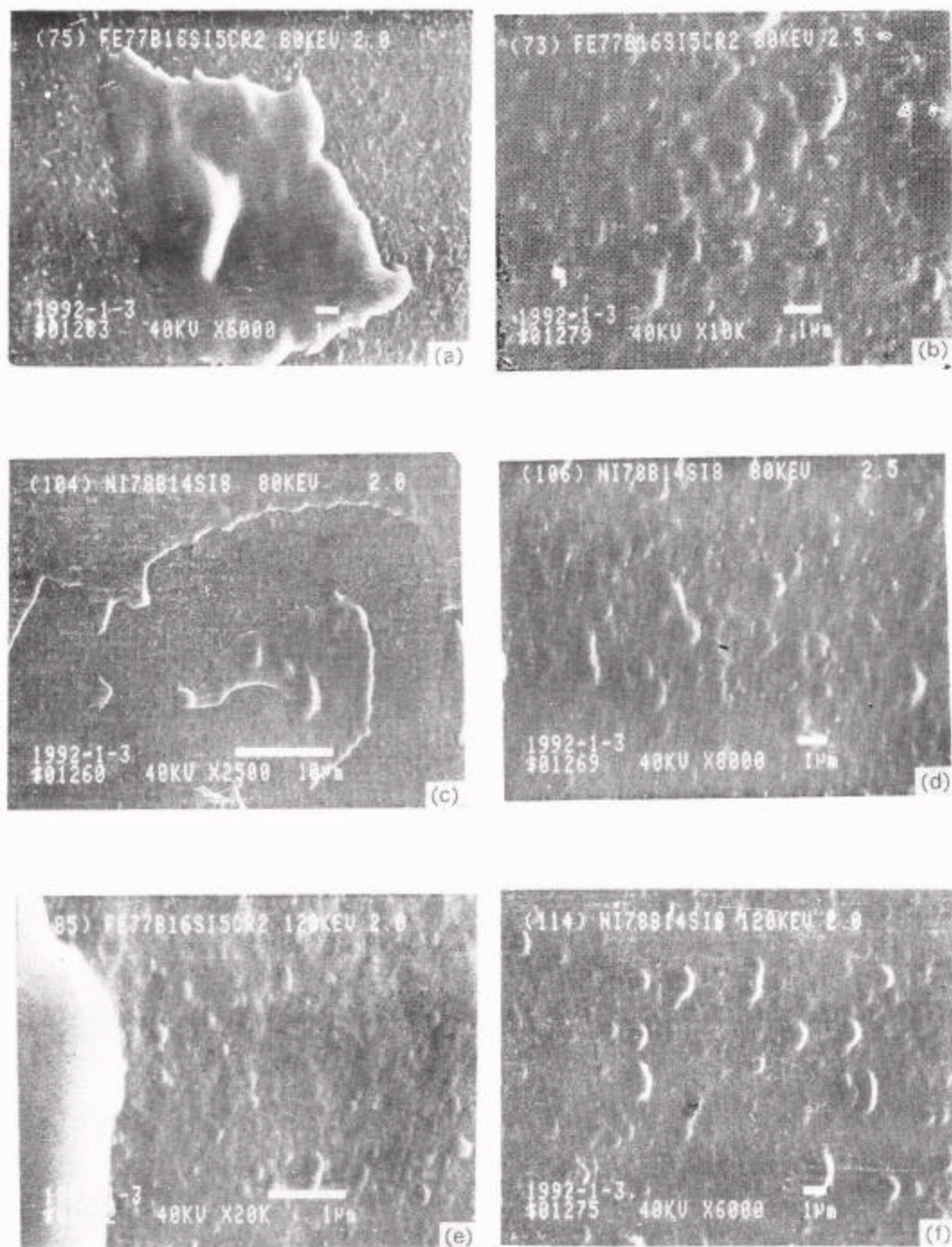


图3 80keV 和 120keV  $\text{He}^+$  辐照  $\text{Fe}_{77}\text{B}_{16}\text{Si}_{15}\text{Cr}_2$  和  $\text{Ni}_{78}\text{B}_{14}\text{Si}_8$  的 SEM 图  
图上方分别给出样品号、材料、辐照能量和剂量 ( $\times 10^{16}\text{He}/\text{cm}^2$ )