

高温超导体中的涡旋运动和 Hall 效应

依赖时间的金兹堡-朗道理论近似

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从依赖时间的金兹堡-朗道方程(TDGL 方程)出发研究高温超导体的涡旋运动序参量, 考虑为一个复数. 利用层状模型(Lawrence-Doniach 模型), 由 TDGL 方程与 L-D 模型结合, 从而讨论高温超导体的 Hall 效应, 涡旋运动的数字系数, 用序参量的解计算出来. 建立模型, 从 TDGL 出发, 对涡旋运动方程进行偏微分, 从而计算出纵向传导和 Hall 传导, 讨论 Hall 角的符号改变和 Hall 效应的奇异性. 这些讨论拟合 Hall 效应的符号改变与混合态高温超导体观察到的 Hall 效应的结果.

PACC: 7430; 7460; 7200

1 引 言

在第二类超导体中涡旋运动的研究, 特别是高温超导体中混合态的奇异传输性质的研究, 继续引起理论物理学家和实验物理学家的兴趣, 这些性质中最令人烦恼之一的是奇异 Hall 效应行为, 奇异 Hall 效应行为在超导体混合态中被观察到符号改变. 早期的工作, Hall 效应数据勉强符合超导涡旋运动模型, 这向理论物理学家提出了挑战. 不少理论物理学家和实验物理学家在 Hall 效应方面作了不少工作^[1-15]. 本文试图用高温超导体(HTSC)的 L-D 模型和依赖时间的金兹堡-朗道(TDGL)方程对第 II 类超导体的涡旋性质和 Hall 效应作一探讨.

2 涡旋运动方程

因为高温超导体可以看作为强超导区和弱超导区的交替堆垛体. L-D 模型是处理这类层状体系的理论基础. 出发点是紧束缚近似的修正金兹堡-朗道理论(G-L 理论), 假定层内序参量相同, 层间序参量差为常数, 用对应于每层($z_n = ns$, n 为层数是正整数)的波函数 $\psi_n(r)$ 取代 G-L 理论中连续复序参量 $\psi(r)$, 当 $s \gg \lambda_c$ 时, L-D 自由能函数为 F ^[12, 14-17]

$$\text{TDGL 方程} \quad \left[\partial_t + \frac{i\tilde{\mu}}{\hbar} \right] \psi = - \Gamma \frac{\delta F}{\delta \psi^*}, \quad (1)$$

L-D 自由能

$$F = \int d^2 r s \sum_n \left\{ a |\psi_n(\mathbf{r})|^2 + \frac{1}{2} b |\psi_n|^4 + \frac{\hbar^2}{2m} \left| \left(-i \frac{\partial}{\partial \mathbf{r}} - \frac{2e}{c} \mathbf{A} \right) \psi_n \right|^2 + \frac{1}{2Ms^2} \left| \psi_{n+1} \exp \left[-\frac{2ie}{c} \int_{ns}^{(n+1)s} dz A_z \right] - \psi_n \right|^2 + \frac{H^2}{8\pi} \right\}, \quad (2)$$

其中 s 是层距, m 是准粒子的有效质量, M 是有效金兹堡-朗道质量, z 轴是沿晶体的 c 轴, $\mathbf{r} = \mathbf{r}(x, y)$, $\psi_n(\mathbf{r})$ 在层内忽略各向异性, \mathbf{A} 是矢势, $a = a_0(T/T_c - 1)$, a_0 是待定常量, T 是开氏温度, T_c 是临界温度, $\Gamma = \Gamma_1 + i\Gamma_2$ 是一个复杂的无维关系比率^[11].

现考虑对 TDGL 方程作一个修改并将 L-D 模型用到 Hall 效应中, 那么依赖时间的金兹堡-朗道方程可以写为如下形式:

$$\hbar\gamma \left[\partial_t + i \frac{2e\tilde{\Phi}}{\hbar} \right] \psi = - \frac{\partial F}{\partial \psi^*}, \quad (3a)$$

其中 $\tilde{\Phi} = \Phi + \frac{\mu}{2e}$, $\tilde{\mu} = \mu + 2e\Phi + \frac{\delta F}{\delta n_s}$, Φ 为电势, μ 是化学势, $n_s = |\psi|^2$ 是超流密度, $\frac{\delta F}{\delta n_s}$

是超流动能, $\gamma = \gamma_1 + i\gamma_2 = \frac{\Gamma_1 - i(1 + \Gamma_2)}{\Gamma_1^2 + (1 + \Gamma_2)^2}$, 忽略 $\tilde{\Phi}$ 与 Φ 的不同^[11].

以第 n 层为考虑对象, 考虑序参量为 $\psi(\mathbf{r}, t) = \sum_n \psi_n(\mathbf{r}, t)$ 和 L-D 模型, 我们的超导序参量满足的方程为

$$\hbar\gamma \left[\partial_t + i \frac{2e\tilde{\Phi}}{\hbar} \right] \psi_n(\mathbf{r}) = \left\{ a\psi_n + b |\psi_n|^2 \psi_n + \frac{\hbar^2}{2m} \left(-i \frac{\partial}{\partial \mathbf{r}} - \frac{2e}{c} \mathbf{A} \right)^2 \psi_n - \frac{1}{2Ms^2} \left[\psi_{n+1} \exp \left(-\frac{2ie}{c} \int_{ns}^{(n+1)s} dz A_z \right) - \psi_n \right] \right\}, \quad (3b)$$

$$\hbar\gamma \left[\partial_t + i \frac{2e\tilde{\Phi}}{\hbar} \right] \psi_{n+1}(\mathbf{r}) = \left\{ a\psi_{n+1} + b |\psi_{n+1}|^2 \psi_{n+1} + \frac{\hbar^2}{2m} \left(-i \frac{\partial}{\partial \mathbf{r}} - \frac{2e}{c} \mathbf{A} \right)^2 \psi_{n+1} - \frac{1}{2Ms^2} \left[\psi_n \exp \left(-\frac{2ie}{c} \int_{(n+1)s}^{ns} dz A_z \right) - \psi_{n+1} \right] \right\}. \quad (3c)$$

由 Amperes' 定律

$$\nabla \times \nabla \times \mathbf{A} = 4\pi(\mathbf{J}_n + \mathbf{J}_s), \quad (4)$$

于是

$$\nabla \cdot (\mathbf{J}_n + \mathbf{J}_s) = 0, \quad (5)$$

$$\mathbf{J}_n = \boldsymbol{\sigma}^{(n)} \cdot \mathbf{E} = \boldsymbol{\sigma}^{(n)} \cdot (-\nabla\Phi - \partial_t \mathbf{A}). \quad (6)$$

这里 \mathbf{J}_s 是超流, \mathbf{J}_n 是正常流, $\boldsymbol{\sigma}^{(n)}$ 正常态电导率张量.

$$\mathbf{J}_s = \frac{\hbar e}{mi} (\psi_n^* \nabla \psi_n - \psi_n \nabla \psi_n^*) - \frac{(2e)^2}{m} |\psi_n|^2 \mathbf{A},$$

$$\mathbf{J}'_s = \frac{\hbar e}{mi} (\psi_{n+1}^* \nabla \psi_{n+1} - \psi_{n+1} \nabla \psi_{n+1}^*) - \frac{(2e)^2}{m} |\psi_{n+1}|^2 \mathbf{A}.$$

3 在 $\mathbf{B} \ll \mathbf{H}_{c_2}$ 下的涡旋运动方程

考虑了一系列近似计算和含时复序参量的涡旋运动的纯数字解. 我们发现

$$\begin{aligned} \frac{1}{k} \int \mathbf{d}\mathbf{s} \cdot (\mathbf{J}_{1s} \omega_d - \mathbf{J}_d \omega_1) = & - \int d^2\mathbf{r} \left\{ \gamma_1 f_{n,\nu} f_{n,d} - \gamma_1 \omega_d f_{n,0}^2 \mathbf{P} + \gamma_2 \mathbf{P} f_{n,d} f_{n,0} + \gamma_2 \omega_d f_{n,0} f_{n,\nu} \right. \\ & - \frac{1}{2Ms^2 a} (f_{n,d} f_{n+1,1} - f_{n,1} f_{n+1,d}) \cos \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] \\ & - \frac{\omega_d}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0}) \\ & \left. \times \sin \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] \right\}, \end{aligned} \quad (7a)$$

$$\begin{aligned} \frac{1}{k} \int \mathbf{d}\mathbf{s} \cdot (\mathbf{J}_{1s} \omega_d - \mathbf{J}_d \omega_1) = & - \int d^2\mathbf{r} \left\{ \gamma_1 f_{n+1,\nu} f_{n+1,d} - \gamma_1 \omega_d f_{n+1,0}^2 \mathbf{P} + \gamma_2 \mathbf{P} f_{n+1,d} f_{n+1,0} + \gamma_2 \omega_d f_{n+1,0} f_{n+1,\nu} \right. \\ & - \frac{1}{2Ms^2 a} (f_{n+1,d} f_{n,1} - f_{n+1,1} f_{n,d}) \cos \left[\chi_n - \chi_{n+1} + \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] \\ & - \frac{\omega_d}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0}) \\ & \left. \times \sin \left[\chi_n - \chi_{n+1} + \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] \right\}, \end{aligned} \quad (7b)$$

其中 $\psi_n(\mathbf{r}, t) = f_n(\mathbf{r}, t) \exp[i\chi_n(\mathbf{r}, t)]$, c' 为常数, $\chi_n(\mathbf{r}, t)$ 是一个相角, $\mathbf{P} \equiv \Phi + \partial_t \chi$, $k = l/\xi$ 为金兹堡-朗道参量, $l = [mb/4\pi(2e)^2|a|]^{1/2}$ 为磁场渗入深度, $\xi = \hbar/(2m|a|)^{1/2}$ 为相干长度, $f_n = f_{n,0} + f_{n,1}$ 和 $f_{n+1} = f_{n+1,0} + f_{n+1,1}$, $f_{n,\nu} \equiv \mathbf{V}_L \cdot \nabla f_{n,0}$, $f_{n+1,\nu} \equiv \mathbf{V}_L \cdot \nabla f_{n+1,0}$, \mathbf{V}_L 为涡旋线速度, \mathbf{d} 是一个无限小位移, $f_{n,d} \equiv \mathbf{d} \cdot \nabla f_{n,0}$, $f_{n+1,d} \equiv \mathbf{d} \cdot \nabla f_{n+1,0}$, $\mathbf{Q} \equiv \mathbf{A} - \nabla \chi/k$, $\mathbf{Q} = \mathbf{Q}_0 + \mathbf{Q}_1$, $\mathbf{Q}_d \equiv (\mathbf{d} \cdot \nabla) \mathbf{Q}_0$, $\mathbf{Q}_d \approx -\nabla \omega_d/k$, $\mathbf{Q}_1 \approx -\nabla \omega_1/k$ 磁场 $\mathbf{h} = \nabla \times \mathbf{Q}$, 电场 $\mathbf{E} = -\frac{1}{k} \nabla P - \partial_t \mathbf{Q}^{[11]}$, $\mathbf{J}_{1s} = -f_{n+1,0}^2 \mathbf{Q}_1 - 2f_{n+1,0} f_{n+1,1} \mathbf{Q}_0 = -f_{n,0}^2 \mathbf{Q}_1 - 2f_{n,0} f_{n,1} \mathbf{Q}_0$ 和 $\mathbf{J}_d = (\mathbf{d} \cdot \nabla) \mathbf{J}_0 = -2f_{n,0} f_{n,d} \mathbf{Q}_0 - f_{n,0}^2 \mathbf{Q}_d$.

进一步化简和近似可得如下方程:

$$\begin{aligned} \frac{1}{k^2} \frac{1}{r} \frac{d}{dr} \left[r \frac{df_{n,0}}{dr} \right] - \mathbf{Q}_0^2 f_{n,0} + f_{n,0} - f_{n,0}^3 + \frac{1}{2Ms^2 a} f_{n,0} - \frac{1}{2Ms^2 a} f_{n+1,0} \\ \times \cos \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] = 0, \end{aligned} \quad (8a)$$

$$\begin{aligned} \frac{1}{k^2} \frac{1}{r} \frac{d}{dr} \left[r \frac{df_{n+1,0}}{dr} \right] - \mathbf{Q}_0^2 f_{n+1,0} + f_{n+1,0} - f_{n+1,0}^3 + \frac{1}{2Ms^2 a} f_{n+1,0} - \frac{1}{2Ms^2 a} f_{n,0} \\ \times \cos \left[-\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} d\mathbf{z} \mathbf{A}_z \right] = 0, \end{aligned} \quad (8b)$$

$$\frac{d}{dr} \frac{1}{r} \frac{d(r\mathbf{Q}_0)}{dr} - f_{n,0}^2 \mathbf{Q}_0 = 0, \quad (9a)$$

$$\frac{d}{dr} \frac{1}{r} \frac{d(r\mathbf{Q}_0)}{dr} - f_{n+1,0}^2 \mathbf{Q}_0 = 0, \quad (9b)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(r\mathbf{P}_1)}{dr} - \gamma_1 f_{n,0}^2 \mathbf{P}_1 - \frac{1}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0})$$

$$\times \sin \left[\chi_{n+1} - \chi_n - \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \right] = 0, \quad (10a)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(rP_1)}{dr} - \gamma_1 f_{n+1,0}^2 P_1 - \frac{1}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0})$$

$$\times \sin \left[\chi_n - \chi_{n+1} + \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \right] = 0, \quad (10b)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(rP_2)}{dr} - \gamma_1 f_{n+1,0}^2 P_2 - \frac{1}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0})$$

$$\times \sin \left[\chi_{n+1} - \chi_n - \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \right] = \gamma_2 f_{n,0} \frac{\partial f_{n,0}}{\partial r} - \frac{\sigma_{xy}^{(n)}}{k} \frac{\partial h_0}{\partial r}, \quad (11a)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(rP_2)}{dr} - \gamma_1 f_{n+1,0}^2 P_2 - \frac{1}{2Ms^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0})$$

$$\times \sin \left[\chi_n - \chi_{n+1} + \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \right] = \gamma_2 f_{n+1,0} \frac{\partial f_{n+1,0}}{\partial r} - \frac{\sigma_{xy}^{(n)}}{k} \frac{\partial h_0}{\partial r}, \quad (11b)$$

其中,度量不变量的项 $\mathbf{Q} \equiv \mathbf{A} - \nabla \chi / k$ 和 $\mathbf{P} \equiv \mathbf{\Phi} + \partial_t \chi$, $P(r) = V_L [P_1(r) \cos(\theta - \theta_H) + P_2(r) \sin(\theta - \theta_H)]$, $h_0 = \nabla \times \mathbf{Q}_0^{[11]}$.

$$\mathbf{J}_t \times \mathbf{e}_z = \frac{\alpha_1 k}{2} \mathbf{V}_L + \frac{\alpha_2 k}{2} \mathbf{V}_L \times \mathbf{e}_z. \quad (12a)$$

这里 α_1, α_2 以后说明, \mathbf{e}_z 为 z 轴方向.

第一种情况 $\chi_{n+1} - \chi_n - \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow 0$

$$\alpha_1 = \gamma_1 \int_0^\infty (f'_{n,0})^2 r dr + \frac{2\sigma_{xx}^{(n)}}{k^2} P_1^{(1)} - \frac{\gamma_2}{2} \int_0^\infty (f_{n,0}^2)' P_2 r dr, \quad (13a)$$

$$\alpha_2 = -\frac{2\sigma_{xx}^{(n)}}{k^2} P_2^{(1)} + \frac{1}{k} \sigma_{xy}^{(n)} h_0(0) - \frac{\gamma_2}{2} \int_0^\infty (f_{n,0}^2)' P_1 r dr. \quad (14a)$$

方程(13a)等号右端最后一项,积分一般十分小,是 $o(\gamma_2^2, \gamma_2 \sigma_{xy}^{(n)})$,从现在起将其舍去,最后利用超流速度 $\mathbf{J}_s = \frac{k}{2} f_n^2 \mathbf{V}_s$ 和 $\mathbf{J}'_s = \frac{k}{2} f_{n+1}^2 \mathbf{V}'_s$,在边界($f_{n,0} = 1$), $\mathbf{V}_s = 2\mathbf{J}_t / k$ 可写方程(12a)为

$$\mathbf{V}_{s1} \times \mathbf{e}_z = \alpha_1 \mathbf{V}_L + \alpha_2 \mathbf{V}_L \times \mathbf{e}_z. \quad (12a')$$

计算传导,用 Faraday 定律 $\langle \mathbf{E} \rangle = -\mathbf{V}_L \times \mathbf{B}$ 可得^[11-13,16]

$$\mathbf{J}_t = \frac{\alpha_1 k}{2B} \langle \mathbf{E} \rangle + \frac{\alpha_2 k}{2B} \langle \mathbf{E} \rangle \times \mathbf{e}_z,$$

故而获得纵向传导

$$\sigma_{xx} = \frac{\alpha_1 k}{2B},$$

Hall 传导

$$\sigma_{xy} = \frac{\alpha_2 k}{2B},$$

而

$$\tan \theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{\rho_{xy}}{\rho_{xx}} \Rightarrow \tan \theta_H = \alpha_2 / \alpha_1. \quad (15a)$$

第二种情况 $\chi_{n+1} - \chi_n - \frac{2e'}{c'} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow \frac{\pi}{2}$

$$\alpha_1 = \gamma_1 \int_0^\infty (f_{n,0}')^2 r dr + \gamma_1 \int_0^\infty (f_{n,0}^2) P_1 dr - \frac{\gamma_2}{2} \int_0^\infty (f_{n,0}^2)' P_2 r dr, \quad (13a')$$

$$\alpha_2 = -\frac{\gamma_2}{2} \int_0^\infty (f_{n,0}^2)' P_1 r dr - \gamma_1 \int_0^\infty f_{n,0}^2 P_2 dr + \frac{1}{4 M_s^2 a}. \quad (14a')$$

$$\text{同第一类情况, 可得} \quad \tan \theta_H = \alpha_2 / \alpha_1. \quad (15a')$$

4 结果与讨论

因为求解方程(8a)—(11a)与(8b)—(11b)形式完全一样, 故只求解一组(8a)—(11a)涡旋运动偏微分方程即可, 于是得到下列运动方程:

$$\frac{1}{k^2} \frac{1}{r} \frac{d}{dr} \left[r \frac{df_{n,0}}{dr} \right] - Q_0^2 f_{n,0} + f_{n,0} - f_{n,0}^3 + \frac{1}{2 M_s^2 a} f_{n,0} - \frac{1}{2 M_s^2 a} f_{n+1,0} \\ \times \cos \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} dz A_z \right] = 0, \quad (16)$$

$$\frac{d}{dr} \frac{1}{r} \frac{d(rQ_0)}{dr} - f_{n,0}^2 Q_0 = 0, \quad (17a)$$

$$\frac{d}{dr} \frac{1}{r} \frac{d(rQ_0)}{dr} - f_{n+1,0}^2 Q_0 = 0, \quad (17b)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(rP_1)}{dr} = \gamma_1 f_{n,0}^2 P_1 = \frac{1}{2 M_s^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0}) \\ \times \sin \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} dz A_z \right] = 0, \quad (18)$$

$$\frac{\sigma_{xx}^{(n)}}{k^2} \frac{d}{dr} \frac{1}{r} \frac{d(rP_2)}{dr} = \gamma_1 f_{n,0}^2 P_2 = \frac{1}{2 M_s^2 a} (f_{n,0} f_{n+1,0} + f_{n,0} f_{n+1,1} + f_{n,1} f_{n+1,0}) \\ \times \sin \left[\chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} dz A_z \right] = \gamma_2 f_{n,0} \frac{\partial f_{n,0}}{\partial r} - \frac{\sigma_{xy}^{(n)}}{k} \frac{\partial h_0}{\partial r}, \quad (19)$$

$$h_0(r) = \frac{1}{r} \frac{\partial [rQ_0(r)]}{\partial r}. \quad (20)$$

由(17a)和(17b)式可得

$$f_{n,0} = f_{n+1,0} e^{i\delta} \quad (\text{而 } \delta \neq 0 \text{ 是不考虑层状, 且均匀});$$

$$f_{n,0} = -f_{n+1,0} e^{i\delta} \quad (\text{取 } \delta = 0).$$

要完全求解上述方程(16)—(20)是不可能的, 下面分两种极限情况来讨论.

$$\text{第一种情况} \quad \chi_{n+1} - \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} dz A_z \rightarrow 0.$$

$$\text{选取}^{[17]} \quad f_{n,0}(r) = M_s^2 a \frac{r}{[r^2 + \xi_v^2]^{1/2}}. \quad (21)$$

以上 ξ_v 是一个测量序参量的拟合长度的参量, 数值上接近于 $1^{[11]}$, 取 $\lambda = \frac{1}{M_s^2 a}$,

$R = [r^2 + \xi_v^2]^{1/2}$ 于是有

$$\begin{aligned} \alpha_1 &= \gamma_1 \int_0^\infty \left[\left(\frac{r}{R} \frac{1}{\lambda} \right)' \right]^2 r dr + \gamma_1 \int_0^\infty \left[\frac{r}{\lambda [r^2 + \xi_v^2]^{1/2}} \right]^2 \left(-\frac{RK_1(R/\lambda_1)}{r\xi_v} \frac{1}{K_1(\xi_v/\lambda_1)} \right) dr \\ &\quad - \frac{\gamma_2}{2} \int_0^\infty \left[\left(\frac{r}{\lambda (r^2 + \xi_v^2)^{1/2}} \right)' \right]^2 A \left[\frac{R K_1(R/\xi)}{\xi_v r K_1(\xi_v/\xi)} - \frac{1}{r} \right] r dr \\ &= \frac{\gamma_1}{4\lambda^2} + \frac{\gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v \lambda^2 K_1(\xi_v/\xi)}, \end{aligned} \quad (22)$$

$$\begin{aligned} \alpha_2 &= -\frac{\gamma_2}{2} \int_0^\infty \left[\left(\frac{r}{\lambda (r^2 + \xi_v^2)^{1/2}} \right)' \right]^2 \left(-\frac{RK_1(R/\lambda_1)}{r\xi_v K_1(\xi_v/\lambda_1)} \right) r dr - \frac{\gamma_2}{2} \\ &\quad - \gamma_1 \int_0^\infty \left(\frac{r}{\lambda} \frac{1}{R} \right)^2 A \left[\frac{R K_1(R/\xi)}{\xi_v r K_1(\xi_v/\xi)} - \frac{1}{r} \right] dr \\ &= -\frac{\gamma_2}{2} \int_0^\infty \left[\left(\frac{r}{\lambda} \frac{1}{R} \right)' \right]^2 \left[-\frac{RK_1(R/\lambda_1)}{r\xi_v K_1(\xi_v/\lambda_1)} \right] r dr - \frac{2\sigma_{xx}^{(n)}}{k^2} P_2^{(1)} + \frac{1}{k} \sigma_{xy}^{(n)}(0) h_0(0) \\ &= -\frac{\gamma_2}{2\lambda^2} I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] + \frac{1}{k} \sigma_{xy}^{(n)}(0) h_0(0), \end{aligned} \quad (23)$$

$$\tan \theta_H = \frac{\alpha_2}{\alpha_1} = \frac{-\frac{\gamma_2}{2\lambda^2} I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] + \frac{1}{k} \sigma_{xy}^{(n)}(0) h_0(0)}{\frac{\gamma_1}{4\lambda^2} + \frac{\gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v \lambda^2 K_1(\xi_v/\xi)}}, \quad (24a)$$

其中 $\lambda_1 = (\sigma_{xx}^{(n)} / \gamma_1 k^2)^{1/2} \lambda = \lambda \xi / k$, $\xi = (\sigma_{xx}^{(n)} / \gamma_1)^{1/2}$, $h_0(0)$ 是涡旋中心场, $h_0(0) = \frac{1}{\xi_v \lambda} \frac{K_0(\xi_v/\lambda)}{K_1(\xi_v/\lambda)} = \frac{1}{\lambda^2} (\ln \lambda - 0.231)$, $K_0(z)$, $K_1(z)$ 是变型贝塞耳函数, $I(z) = \frac{2z}{K_1(z)} \times \int_0^\infty \frac{K_1(x)}{x^2} dx$.

Hall 传导包括两部分贡献, 一部分是依赖时间的序参量虚部 γ_2 , 另一部分是正常态 Hall 传导. 如果 $\gamma_2 < 0$ 这两部分贡献有相同的符号, 导致 Hall 效应. 从 (24a) 式可以看出, 如果 $\gamma_2 > 0$, 至少有 $\alpha_2 < 0$ 的可能, 即 $\tan \theta_H < 0$ 的可能.

当 $\gamma_2 > 0$ 时, $\tan \theta_H < 0$ 十分可能, 奇异 Hall 效应才可能发生.

1) 当 $\xi_v/\lambda \rightarrow 0$ 即 $\lambda \rightarrow \infty$ 也即 $T/T_c - 1 \rightarrow 0$,

$$\begin{aligned} \tan \theta_H &= \frac{\alpha_2}{\alpha_1} = \frac{-\frac{\gamma_2}{2} I(\xi_v/\xi) - \frac{\lambda^2 \xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] + \frac{\lambda^2}{k} \sigma_{xy}^{(n)}(0) h_0(0)}{\frac{\gamma_1}{4} + \frac{\gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v K_1(\xi_v/\xi)}} \\ &= \frac{-\frac{\gamma_2}{2} I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\frac{1}{\gamma_2} + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] \left[Ms^2 a_0 \left(\frac{T}{T_c} - 1 \right) \right]^{-2} - \frac{\sigma_{xy}^{(n)}(0)}{k}}{\frac{\gamma_1}{4} + \frac{\gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v K_1(\xi_v/\xi)}}. \end{aligned} \quad (25a)$$

因为 $x \rightarrow 0$, $k_0(x) \sim -\ln(x/2)$, $k_n(x) \sim \frac{(n-1)!}{2} \left(\frac{x}{2} \right)^{-n}$, $n \gg 1$; $x \rightarrow \infty$, $k_n(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x}$. $T \rightarrow T_c$ 时, 可能产生 $\tan \theta_H < 0$, 即奇异 Hall 效应发生.

$$\tan \theta_H = C_1 + \frac{C_2}{(T/T_c - 1)^2} \rightarrow \cot \theta_H = C'_1 + C'_2 T^2. \quad (26)$$

这与前人提及的实验经验公式相符合^[17-19].

2) 当 $\xi_v/\lambda \rightarrow \infty$ 即 $\xi_v \left[Ms^2 a_0 \left(\frac{T}{T_c} - 1 \right) \right] \rightarrow \infty$,

$$\tan \theta_H = \frac{-\frac{\gamma_2}{2} I(\xi_v/\xi) - \frac{\lambda^2 \xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] + \frac{\sigma_{xy}^{(n)}(0) \lambda}{k \xi_v}}{\frac{\gamma_1}{4} + \frac{\gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v K_1(\xi_v/\xi)}} \quad (27a)$$

若 λ^2 的系数小于零, $\tan \theta_H$ 和 Hall 传导出现符号反转是十分可能的.

$$\tan \theta_H = C_1 + C_2 \lambda + C_3 \lambda^3 = C''_1 + C''_2 \frac{1}{T} + C''_3 \frac{1}{T^2} \Rightarrow \cot \theta_H = T^2 / (C'_1 + C'_2 T + C'_3 T^2). \quad (28)$$

第二种情况 $\chi_{n+1} = \chi_n - \frac{2e}{c'} \int_{ns}^{(n+1)s} dz A_z \rightarrow \frac{\pi}{2}$

选取^[17,19] $f_{n,0} = \frac{r}{\lambda_2 (r^2 + \xi_v^2)^{1/2}}, \quad \lambda_2 = \lambda/2,$

于是有

$$\tan \theta_H = \left\{ (-2 \gamma_2 / \lambda^2) I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{\sigma_{xy}^{(n)}(0)}{k^2} \right] + \frac{1}{k} \sigma_{xy}^{(n)}(0) h_0(0) - \frac{\lambda}{4} \left[1 + \frac{\gamma_2}{2 \gamma_1} \pi \xi_v \right] \right\} / \left\{ \frac{\gamma_1}{\lambda^2} + \frac{\lambda}{2} - \frac{\gamma_2}{\gamma_1} \frac{\pi \lambda \xi_v}{8} + \frac{4 \gamma_1 \xi K_0(\xi_v/\xi)}{\lambda^2 \xi_v K_1(\xi_v/\xi)} \right\}. \quad (24a')$$

这里 $h_0(0) = \frac{2 K_0(2 \xi_v/\lambda)}{\xi_v \lambda K_1(2 \xi_v/\lambda)}, \quad \lambda = \frac{1}{Ms^2 a}, \quad a = a_0 (T/T_c - 1).$

如果 $\gamma_2 > 0$, 则 $\alpha_2 < 0$ 十分可能, 则 $\tan \theta_H < 0$ 是十分可能的, 奇异 Hall 效应发生也是十分可能的.

1) $\xi_v/\lambda \rightarrow 0$ 即 $T \rightarrow T_c, h_0(0) \approx \frac{1}{2 \lambda^2},$

$$\tan \theta_H = \left\{ -2 \gamma_2 I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] \lambda^2 + \frac{\sigma_{xy}^{(n)}(0)}{2k} - \frac{\lambda^3}{4} \left[1 + \frac{\gamma_2}{2 \gamma_1} \pi \xi_v \right] \right\} / \left\{ \gamma_1 + \frac{4 \gamma_1 \xi K_0(\xi_v/\xi)}{\xi_v K_1(\xi_v/\xi)} + \frac{\lambda^3}{2} \left[1 - \frac{\gamma_2}{\gamma_1} \frac{\pi}{4} \xi_v \right] \right\} = \frac{C_1 T^3 + C_2 T + C_3}{D_1 T^3 + D_2}, \quad (25a')$$

即: $\tan \theta_H = \left\{ -2 \gamma_2 I(\xi_v/\xi) + \frac{\sigma_{xy}^{(n)}(0)}{2k} - \frac{\lambda^4}{4} \right\} / \left\{ \gamma_1 + \frac{4 \gamma_1 \xi K_0(\xi_v/\lambda)}{\xi_v K_1(\xi_v/\xi)} + \frac{\lambda^4}{2} \right\}.$

$\gamma_2 > 0$ 时, 具有奇异 Hall 效应是十分可能的, 至少有 $\tan \theta_H < 0.$

2) 若 $\xi_v/\lambda \rightarrow \infty$ 即 $\lambda \rightarrow 0, (T/T_c - 1) Ms^2 a_0 \rightarrow \infty, h_0(0) \sim 2/\xi \lambda,$

$$\tan \theta_H = \left\{ -2 \gamma_2 I(\xi_v/\xi) - \frac{\xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} \left[\gamma_2 + \frac{1}{k^2} \sigma_{xy}^{(n)}(0) \right] \lambda^2 + \frac{2 \lambda}{\xi_v k} \sigma_{xy}^{(n)}(0) \right\}$$

$$\left. -\frac{\lambda^3}{4} \left[1 + \frac{\gamma_2}{2\gamma_1} \pi \xi_v \right] \right\} / \left\{ \gamma_1 + \frac{4\gamma_1 \xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)} + \frac{\lambda^3}{2} \left[1 - \frac{\gamma_2}{\gamma_1} \frac{\pi}{4} \xi_v \right] \right\},$$

$$\tan \theta_H = \frac{-2\gamma_2 I(\xi_v/\xi)}{\gamma_1 + \frac{4\gamma_1 \xi}{\xi_v} \frac{K_0(\xi_v/\xi)}{K_1(\xi_v/\xi)}} \quad \gamma_2 > 0, \tan \theta_H < 0$$

$$= \frac{C_1 T^3 + C_2 T^2 + C_3 T + C_4}{D_1 T^3 + D_4} \quad (27a')$$

5 与前人工作的比较

当 $\alpha_1 = 0.436 \gamma_1$ 时, Dorsey^[11] 得到他的(3.54)至(3.56)式.

$$\sigma_{xx} = 2.62 \sigma_{xx}^{(n)} \frac{H_{c_2}}{B}, \quad \sigma_{xy} = 6 \sigma_{xx}^{(n)} \frac{\alpha_2}{\gamma_1} \frac{H_{c_2}}{B},$$

$$\alpha_2 = -0.140 \gamma_2 - 0.186 \left[\gamma_2 + \frac{2\pi \hbar}{m} \sigma_{xy}^{(n)}(0) \right] + \frac{\pi \hbar}{m} (\ln k) \sigma_{xy}^{(n)}(0) \frac{h_0(0)}{H_{c_1}}.$$

Hall 传导包括两个方面的贡献, 序参量的虚部 γ_2 和正常态 Hall 传导, $\gamma_2 > 0$, 可得到 $\tan \theta_H$ 的符号反转, 那么可得 Hall 效应的奇异性. 在 $\alpha_1 = 0.508 \gamma_1$ 可得^[11]

$$\sigma_{xx} = 1.47 \sigma_{xx}^{(n)} \frac{H_{c_2}}{B}, \quad \sigma_{xy} = 2.89 \sigma_{xx}^{(n)} \frac{\alpha_2}{\gamma_1} \frac{H_{c_2}}{B},$$

$$\alpha_2 = -0.178 \gamma_2 - 0.258 \left[\gamma_2 + \frac{2\pi \hbar}{m} \sigma_{xy}^{(n)}(0) \right] + \frac{\pi \hbar}{m} (\ln k) \sigma_{xy}^{(n)}(0) \frac{h_0(0)}{H_{c_1}}.$$

当 $\gamma_2 > 0$, Hall 角的符号可能反转. Dorsey 认为 Hall 传导的符号依赖于 α 的符号, 符号变化的图象将是物质的详细电子结构的后果^[11]. 他没有得出 $\tan \theta_H$ 的 T 显式表达式, 是较定性的解释实验.

当 $\mathbf{B} \rightarrow \mathbf{H}_{c_2}$ 时, Kopnin 等^[13] 得到他的(36)式过渡为其(37)式, $\tan \theta_H = \sigma_n^H / \sigma_n + (\text{sig } n(e) \xi - \sigma_n^H / \sigma_n) \frac{\mathbf{u} \cdot \mathbf{H}_{c_2} - B}{2 \beta_A H_{c_2}}$, 其中 ξ 是 $(1 - T/T_c)$ 的函数, 可以估计的 $\sigma_n^H / \sigma_n \ll \xi$. 因类电子和类孔洞部分, $e\xi$ 和 σ_n^H 的符号改变量的不同平均值结果, $e\xi$ 和 σ_n^H 的符号可以不同. 最后, 他得出, 费密面的形状是依赖于费密水平的位置. 这也许是为什么 Hall 角的实验数据对不同样品是如此不同的理由. 他的工作 $\tan \theta_H$ 中 ξ 是 T 的函数, 但未得出显式.

Otterlo 等^[15] 的工作研究由一个偏微分方程 $[M_c + M_H] \dot{\mathbf{V}} + \eta_c \mathbf{V} = \{k_H \mathbf{V}_T - [\gamma_c + \gamma_H] \mathbf{V}\} \times \mathbf{z}$, 涡旋运动是由动力贡献项和孔洞贡献项相结合提供的. Hall 角的符号的改变是与 $l \sim \xi$ 关联的, 即在超导体中 Hall 效应的奇异性发生, 意味着相干长度 ξ 的有序自由路径 l 是与超导体的详细微观机制相关联. Hall 角符号改变是去打破电子带状结构的部分孔洞对称有关. 他们着重讨论了动力贡献项^[15] γ_H , 如其(2)式中第二项, 他们工作中 ξ , Δ 是 T 的函数, 但讨论 α_{Hall} 时未给出 T 的显式.

Ao 也运用涡旋动力学方程基于在超导体中选一个适当参量集团, 则钉扎涡旋格构的

空穴运动,也许有 Hall 效应里占支配地位,去讨论 Hall 效应的奇异性. 定量的预言被获得,并可观测空穴运动来直接检测其模型^[14]. 在文献[14]中的(10)和(11)式的 ρ_{yx} , ρ_{xx} 中也隐含有 $T(n_v = n_0 e^{-bE_v/k_B T})$, 有效空穴黏滞系数 $\eta_{\text{eff}} = \eta_0 e^{aE_v/k_B T}$ 也是 T 的显式,所以可以得到 Hall 角与温度 T 的显式表达式,但是指数关系的数学式.

本文(12a')式, $\mathbf{V}_{s1} \times \mathbf{e}_z = \alpha_1 \mathbf{V}_L + \alpha_2 \mathbf{V}_L \times \mathbf{e}_z$ 可变形为 $(\mathbf{V}_{s1} - \alpha_2 \mathbf{V}_L) \times \mathbf{e}_z = \alpha_1 \mathbf{V}_L$, 这式等号左边为 Magnus 力,右边是拖动涡旋的黏滞力. 而本文要详细求解方程(8a)——(11a)和(8b)——(11b)是困难的,所以只考虑了两种特殊情况下的几种特殊情形.

第一种情况 当 $\chi_{n+1} - \chi_n - \frac{2e}{c} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow 0$ 时

1) $T \rightarrow T_c$ 时, $\cot \theta_H = C'_1 + C'_2 T^2$, 即(26)式与文献[18, 19]提及的实验公式相符合(如文献[19]中的实验公式完全一致).

2) $\xi_v Ms^2 a_0 (T/T_c - 1) \rightarrow \infty$ 得出(28)式, $\cot \theta_H = \frac{T^2}{C'_1 T^2 + C'_2 T + C'_3}$.

当 $C'_1 = 5.11 \times 10^{-3}$, $C'_2 = 1.14$, $C'_3 = 12$, 可得如图 1 所示的曲线,在 T 较小时,曲线弯曲了一点, T 较大时类似直线.

第二种情况 $\chi_{n+1} - \chi_n - \frac{2e}{c} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow \frac{\pi}{2}$ 时

1) $T \rightarrow T_c$ 时,即得到 25(a')式.

当取 $C_1 = 1$, $C_2 = 1.4$, $C_3 = 12$, $D_1 = 1.2$, $D_2 = 3$ 时,可得图 2 所示的曲线与文献[19]的图 15 相符合.

2) $(T/T_c - 1) \xi_v Ms^2 a_0 \rightarrow \infty$, 即得到(27a')式. 结果与(25a')式的曲线关系类似.

本文与前人的主要区别是得到 Hall 角与 T 的显式表达式且(26)式与实验公式符合,(25a'), (27a')式也与文献[19]得出的结果相符合,若能完全求解(8a)——(11a)式和(8b)——

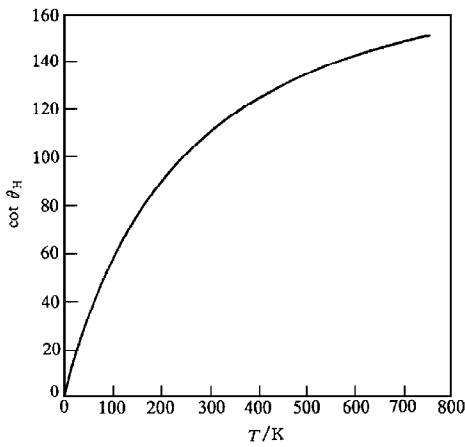


图 1 当 $\chi_{n+1} - \chi_n - \frac{2e}{c} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow 0$ 时, $\xi_v Ms^2 a_0 (T/T_c - 1) \rightarrow \infty$, $\cot \theta_H$ 与 T 关系中的一条曲线(计算机绘图)

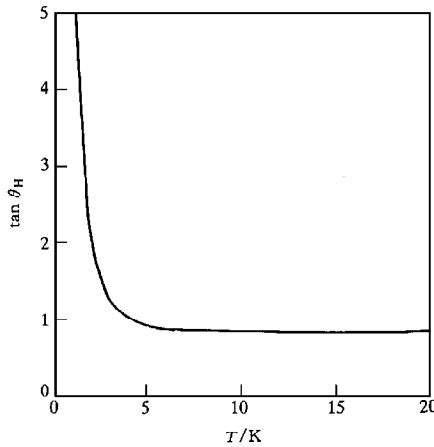


图 2 当 $\chi_{n+1} - \chi_n - \frac{2e}{c} \int_{ns}^{(n+1)s} dz \mathbf{A}_z \rightarrow \pi/2$ 时, 且 $T \rightarrow T_c$ 时, $\tan \theta_H$ 与 T 关系中的一条曲线(计算机绘图)

(11b)式,则相信可以解释更多的实验. Hall 效应符号改变的机制可以认为是由于层状超导体中的超导层和储电层的载流子发生交换而产生的 Hall 效应的符号反转.

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VORTEX MOTION AND THE HALL EFFECT IN HIGH-TEMPERATURE SUPERCONDUCTORS

A TIME-DEPENDANT GINZBURG-LANDAU THEORY APPROACH

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ABSTRACT

Vortex motion in high-temperature superconductors is studied starting from a variant of the time-dependent Ginzberg-Landau(TDGL) equations, in which the order-parameter relaxation time is taken to be complex. Using Lawrence-Doniach model and considering a TDGL model modified to take into account the Hall effect, we have evaluated and discussed the sign change of the Hall angle and anomalous Hall effect. The results are discussed in light of rather puzzling sign change of the Hall effect which has been observed in the mixed state of the high-temperature Superconductors.

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