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SPL(2,1)超代数的不可分解表示和不可约表示

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利用 SPL(2,1)的非齐次玻色-费米实现,研究了 SPL(2,1)在 Heisenberg-Weyl 超代数的广义包络代数的子空间和商空间上的不可分解表示和不可约表示,并给出了它的全部有限维不可约表示.

PACC:0365;0210;0220

1 引 言

李超代数在超统一理论、核物理、超引力等领域研究中具有极其重要的应用[2—4].建立和发展李超代数的表示理论,是一项很有意义的工作[5—9].李超代数的不可分解表示,在描述不稳定粒子体系中起着重要作用[10].Gruber 等人已经深入研究了一些李代数和李超代数的不可分解表示[11—13].利用李超代数的非齐次玻色-费米实现,研究李超代数的不可分解表示,是一种十分有效的方法.在本文中,我们将涉及李超代数 SPL(2,1)[1].本文的目的,就是利用 SPL(2,1)的非齐次玻色-费米实现,在 Heisenberg 超代数的广义包络代数的子空间和商空间上研究 SPL(2,1)的不可分解表示.以此为基础,在广义 Fock 空间的子空间上,自然地得到 SPL(2,1)的全部有限维不可约表示.

2 SPL(2,1)的不可分解表示

根据文献[14],SPL(2,1)超代数的生成元标记为

$$\{Q_3,Q_+,Q_-,B\in \mathrm{SPL}(2,1)_{\bar 0}\mid V_+,V_-,W_+,W_-\in \mathrm{SPL}(2,1)_{\bar 1}\},\tag{1}$$

并满足如下对易和反对易关系:

$$\begin{aligned}
&[Q_3,Q_\pm]=\pm Q_\pm,[Q_+,Q_-]=2Q_3,\\
&[B,Q_\pm]=[B,Q_3]=0,\\
&[Q_3,V_\pm]=\pm\frac{1}{2}V_\pm,\quad [Q_3,W_\pm]=\pm\frac{1}{2}W_\pm,\\
&[Q_\pm,V_\mp]=V_\pm,\quad [Q_\pm,W_\mp]=W_\pm,\\
&[Q_\pm,V_\pm]=0,\quad [Q_\pm,W_\pm]=0,\\
&[B,V_\pm]=\frac{1}{2}V_\pm,\quad [B,W_\pm]=-\frac{1}{2}W_\pm,\\
&\{V_\pm,V_\pm\}=\{V_\pm,V_\mp\}=\{W_\pm,W_\pm\}=\{W_\pm,W_\mp\}=0,\\
&\{V_\pm,W_\pm\}=\pm Q_\pm,\quad \{V_\pm,W_\mp\}=-Q_3\pm B.
\end{aligned}\tag{2}$$

按照文献[1],SPL(2,1)超代数的非齐次玻色-费米实现,可以由一对玻色子(b^+,b)和两对费米子(a_1^+,a_1;a_2^+,a_2)表示为

$$\begin{aligned}
Q_3&=-\frac{1}{2}n+b^+b+\frac{1}{2}a_1^+a_1+\frac{1}{2}a_2^+a_2,\\
B&=\frac{1}{2}a_2^+a_2-\frac{1}{2}a_1^+a_1,\\
Q_+&=nb^+-b^{+2}b-b^+a_1^+a_1-b^+a_2^+a_2,\\
Q_-&=b,\\
V_+&=\frac{1}{\sqrt2}na_2^++\frac{1}{\sqrt2}b^+a_1-\frac{1}{\sqrt2}a_2^+b^+b-\frac{1}{\sqrt2}a_2^+a_1^+a_1,\\
V_-&=-\frac{1}{\sqrt2}a_2^+b+\frac{1}{\sqrt2}a_1,\\
W_+&=\frac{1}{\sqrt2}na_1^++\frac{1}{\sqrt2}b^+a_2-\frac{1}{\sqrt2}a_1^+b^+b-\frac{1}{\sqrt2}a_1^+a_2^+a_2,\\
W_-&=-\frac{1}{\sqrt2}a_1^+b+\frac{1}{\sqrt2}a_2.
\end{aligned}\tag{3}$$

考虑(1+2)态 Heisenberg-Weyl 超代数 H:$\{b^+,b,a_1^+,a_1,a_2^+,a_2,E\}$,其中 E 为单位算符,根据 Poincare-Birchhoff-Witt 定理,我们将它的广义包络代数 Ω 的基选为

$$\begin{aligned}
\{&\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t)\\
&\equiv b^{+k}b^{l}a_1^{+\alpha_1}a_1^{\beta_1}a_2^{+\alpha_2}a_2^{\beta_2}E^t\mid k,l,t\in Z^+,\\
&\alpha_1,\alpha_2,\beta_1,\beta_2=0,1\}
\end{aligned}\tag{4}$$

其中 Z^+ 表示全体非负整数的集合.空间 Ω 的每一个矢量都是具有复系数的基矢的线性组合.现在,我们将代数空间 Ω 扩充到 $\bar\Omega$,空间 $\bar\Omega$ 中的每一个元素都是以 Grassmann 代数 $\widetilde G$ 的元素作为系数的基矢的线性组合.

H 在 $\bar{\Omega}$ 的表示定义为

$$
\begin{aligned}
& f(b^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& \phi(k+1,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t), \\
& f(b)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& \phi(k,l+1,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
& + k\phi(k-1,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t+1), \\
& f(a_1^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& (1-\alpha_1)\phi(k,l,\alpha_1+1,\beta_1,\alpha_2,\beta_2,t), \\
& f(a_1)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& (-1)^{\alpha_1}\phi(k,l,\alpha_1,\beta_1+1,\alpha_2,\beta_2,t) \\
& + \alpha_1\phi(k,l,\alpha_1-1,\beta_1,\alpha_2,\beta_2,t+1), \\
& f(a_2^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& (-1)^{\alpha_1+\beta_1}(1-\alpha_2)\phi(k,l,\alpha_1,\beta_1,\alpha_2+1,\beta_2,t), \\
& f(a_2)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,t) \\
=& (-1)^{\alpha_1+\beta_1+\alpha_2}\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2+1,t) \\
& + (-1)^{\alpha_1+\beta_1}\alpha_2\phi(k,l,\alpha_1,\beta_1,\alpha_2-1,\beta_2,t+1).
\end{aligned} \tag{5}
$$

设 I 是由元素 E-1 生成的左理想，则商空间 $V=\bar{\Omega}/I$ 的基可以取为

$$
\begin{aligned}
& \{\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& =\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2,0) \mid k,l\in Z^+, \\
& \alpha_1,\alpha_2,\beta_1,\beta_2=0,1\}.
\end{aligned} \tag{6}
$$

显然，H 的表示(5)式在 V 上诱导的表示为

$$
\begin{aligned}
& f(b^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \phi(k+1,l,\alpha_1,\beta_1,\alpha_2,\beta_2), \\
& f(b)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \phi(k,l+1,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& + k\phi(k-1,l,\alpha_1,\beta_1,\alpha_2,\beta_2), \\
& f(a_1^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& (1-\alpha_1)\phi(k,l,\alpha_1+1,\beta_1,\alpha_2,\beta_2), \\
& f(a_1)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& (-1)^{\alpha_1}\phi(k,l,\alpha_1,\beta_1+1,\alpha_2,\beta_2) \\
& + \alpha_1\phi(k,l,\alpha_1-1,\beta_1,\alpha_2,\beta_2), \\
& f(a_2^+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& (-1)^{\alpha_1+\beta_1}(1-\alpha_2)\phi(k,l,\alpha_1,\beta_1,\alpha_2+1,\beta_2), \\
& f(a_2)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& (-1)^{\alpha_1+\beta_1+\alpha_2}\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2+1) \\
& + (-1)^{\alpha_1+\beta_1}\alpha_2\phi(k,l,\alpha_1,\beta_1,\alpha_2-1,\beta_2).
\end{aligned} \tag{7}
$$

利用

$$
\begin{aligned}
& L(F(b^+,b,a_1^+,a_1,a_2^+,a_2)) \\
=& F(f(b^+),f(b),f(a_1^+),f(a_1),f(a_2^+),f(a_2)),
\end{aligned} \tag{8}
$$

此处 F 表示 SPL(2,1)的生成元，以及 SPL(2,1)的玻色-费米实现(3)式，可得 SPL(2,1)在 V 上的表示 L：

$$
\begin{aligned}
& L(Q_3)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \left(-\frac{n}{2}+k+\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& + \phi(k+1,l+1,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& + \frac{1}{2}(-1)^{\alpha_1}(1-\alpha_1)\phi(k,l,\alpha_1+1,\beta_1+1, \\
& \alpha_2,\beta_2) + \frac{1}{2}(-1)^{\alpha_2}(1-\alpha_2) \\
& \cdot\phi(k,l,\alpha_1,\beta_1,\alpha_2+1,\beta_2+1), \\
& L(B)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \left(\frac{\alpha_2}{2}-\frac{\alpha_1}{2}\right)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& + \frac{1}{2}(-1)^{\alpha_2}(1-\alpha_2)\phi(k,l,\alpha_1,\beta_1, \\
& \alpha_2+1,\beta_2+1) - \frac{1}{2}(-1)^{\alpha_1}(1-\alpha_1) \\
& \cdot\phi(k,l,\alpha_1+1,\beta_1+1,\alpha_2,\beta_2), \\
& L(Q_+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& (n-k-\alpha_1-\alpha_2)\phi(k+1,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& - \phi(k+2,l+1,\alpha_1,\beta_1,\alpha_2,\beta_2) - (-1)^{\alpha_1} \\
& \cdot(1-\alpha_1)\phi(k+1,l,\alpha_1+1,\beta_1+1,\alpha_2,\beta_2) \\
& - (-1)^{\alpha_2}(1-\alpha_2)\phi(k+1,l,\alpha_1, \\
& \beta_1,\alpha_2+1,\beta_2+1), \\
& L(Q_-)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \phi(k,l+1,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
& + k\phi(k-1,l,\alpha_1,\beta_1,\alpha_2,\beta_2),
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
& L(V_+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2) \\
=& \frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}(n-k-\alpha_1)(1-\alpha_2) \\
& \cdot\phi(k,l,\alpha_1,\beta_1,\alpha_2+1,\beta_2) \\
& + \frac{1}{\sqrt{2}}(-1)^{\alpha_1}\phi(k+1,l,\alpha_1,\beta_1+1,\alpha_2,\beta_2) \\
& + \frac{1}{\sqrt{2}}\alpha_1\phi(k+1,l,\alpha_1-1,\beta_1, \\
& \alpha_2,\beta_2) - \frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}(1-\alpha_2) \\
& \cdot\phi(k+1,l+1,\alpha_1,\beta_1,\alpha_2+1,\beta_2) \\
& - \frac{1}{\sqrt{2}}(-1)^{\beta_1}(1-\alpha_1)(1-\alpha_2)
\end{aligned}
$$

$$\cdot\phi(k+1,l+1,\alpha_1,\beta_1,\alpha_2+1,\beta_2),$$

$$L(V_-)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2)$$

$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\phi(k,l,\alpha_1,\beta_1+1,\alpha_2,\beta_2)+\frac{1}{\sqrt{2}}\alpha_1\phi(k,l,\alpha_1-1,\beta_1,\alpha_2,\beta_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}(1-\alpha_2)\phi(k,l+1,\alpha_1,\beta_1,\alpha_2+1,\beta_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}k(1-\alpha_2)\cdot\phi(k,l,\alpha_1,\beta_1,\alpha_2+1,\beta_2),$$

$$L(W_+)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2)$$

$$=\frac{1}{\sqrt{2}}(n-k-\alpha_2)(1-\alpha_1)\phi(k,l,\alpha_1+1,\beta_1,\alpha_2,\beta_2)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1+\alpha_2}\cdot\phi(k+1,l,\alpha_1,\beta_1,\alpha_2,\beta_2+1)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}\alpha_2\phi(k+1,l,\alpha_1,\beta_1,\alpha_2-1,\beta_2)-\frac{1}{\sqrt{2}}(1-\alpha_1)\phi(k+1,l+1,\alpha_1+1,\beta_1,\alpha_2,\beta_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_2}(1-\alpha_1)\cdot(1-\alpha_2)\phi(k,l,\alpha_1+1,\beta_1,\alpha_2+1,\beta_2+1),$$

$$L(W_-)\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2)$$

$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1+\alpha_2}\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2+1)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\beta_1}\alpha_2\phi(k,l,\alpha_1,\beta_1,\alpha_2-1,\beta_2)-\frac{1}{\sqrt{2}}(1-\alpha_1)\phi(k,l+1,\alpha_1+1,\beta_1,\alpha_2,\beta_2)-\frac{1}{\sqrt{2}}k(1-\alpha_1)\phi(k-1,l,\alpha_1+1,\beta_1,\alpha_2,\beta_2).$$

注意到(9)式中$(l+\beta_1+\beta_2)$的值并不减少，故由非负整数$m\in Z^+$确定了一个不变子空间$V^{[m]}$：

$$V^{[m]}=\{\phi(k,l,\alpha_1,\beta_1,\alpha_2,\beta_2)\in V\mid l+\beta_1+\beta_2\geqslant m\},\tag{10}$$

但不存在其不变的补空间. 因此，我们获得的(9)式是不可分解表示.

设J是由$\{b-\lambda,a_1-\eta_1,a_2-\eta_2\mid\lambda\in C,\eta_1,\eta_2\in\widetilde{G}\}$生成的$V$的左理想，则商空间$Y=V/J$是广义 Fock 空间，其基可以取为

$$Y:\{\phi(k,\alpha_1,\alpha_2)\equiv\phi(k,0,\alpha_1,0,\alpha_2,0)\times\operatorname{mod}J\mid k\in Z^+,\alpha_1,\alpha_2=0,1\}.\tag{11}$$

(9)式在Y上诱导的表示为

$$L(Q_3)\phi(k,\alpha_1,\alpha_2)$$

$$=\left(-\frac{n}{2}+k+\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)\phi(k,\alpha_1,\alpha_2)+\lambda\phi(k+1,\alpha_1,\alpha_2)+\frac{1}{2}(1-\alpha_1)\eta_1\phi(k,\alpha_1+1,\alpha_2)+\frac{1}{2}(1-\alpha_2)\eta_2\phi(k,\alpha_1,\alpha_2+1),$$

$$L(B)\phi(k,\alpha_1,\alpha_2)$$

$$=\left(\frac{\alpha_2}{2}-\frac{\alpha_1}{2}\right)\phi(k,\alpha_1,\alpha_2)+\frac{1}{2}(1-\alpha_2)\eta_2\phi(k,\alpha_1,\alpha_2+1)-\frac{1}{2}(1-\alpha_1)\eta_1\phi(k,\alpha_1+1,\alpha_2),$$

$$L(Q_+)\phi(k,\alpha_1,\alpha_2)$$

$$=(n-k-\alpha_1-\alpha_2)\phi(k+1,\alpha_1,\alpha_2)-\lambda\phi(k+2,\alpha_1,\alpha_2)-(1-\alpha_1)\eta_1\phi(k+1,\alpha_1+1,\alpha_2)-(1-\alpha_2)\eta_2\phi(k+1,\alpha_1,\alpha_2+1),$$

$$L(Q_-)\phi(k,\alpha_1,\alpha_2)$$

$$=\lambda\phi(k,\alpha_1,\alpha_2)+k\phi(k-1,\alpha_1,\alpha_2),\tag{12}$$

$$L(V_+)\phi(k,\alpha_1,\alpha_2)$$

$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(n-k-\alpha_1)(1-\alpha_2)\cdot\phi(k,\alpha_1,\alpha_2+1)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\eta_1\cdot\phi(k+1,\alpha_1,\alpha_2)+\frac{1}{\sqrt{2}}\alpha_1\phi(k+1,\alpha_1-1,\alpha_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)\lambda\phi(k+1,\alpha_1,\alpha_2+1)-\frac{1}{\sqrt{2}}(1-\alpha_1)(1-\alpha_2)\eta_1\phi(k,\alpha_1+1,\alpha_2+1),$$

$$L(V_-)\phi(k,\alpha_1,\alpha_2)$$

$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\eta_1\phi(k,\alpha_1,\alpha_2)+\frac{1}{\sqrt{2}}\alpha_1\phi(k,\alpha_1-1,\alpha_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)\lambda\phi(k,\alpha_1,\alpha_2+1)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)k\phi(k-1,\alpha_1,\alpha_2+1),$$

$$L(W_+)\phi(k,\alpha_1,\alpha_2)$$

$$=\frac{1}{\sqrt{2}}(n-k-\alpha_2)(1-\alpha_1)\phi(k,\alpha_1+1,\alpha_2)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\alpha_2}\eta_2\phi(k+1,\alpha_1,\alpha_2)+\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2\phi(k+1,\alpha_1,\alpha_2-1)-\frac{1}{\sqrt{2}}(1-\alpha_1)\lambda\phi(k+1,\alpha_1+1,\alpha_2)$$

$$-\frac{1}{\sqrt{2}}(-1)^{\alpha_2}(1-\alpha_1)(1-\alpha_2)\eta_2$$
$$\cdot\phi(k,\alpha_1+1,\alpha_2+1),$$
$$L(W_-)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1+\alpha_2}\eta_2\phi(k,\alpha_1,\alpha_2)$$
$$+\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2\phi(k,\alpha_1,\alpha_2-1)$$
$$-\frac{1}{\sqrt{2}}(1-\alpha_1)\lambda\phi(k,\alpha_1+1,\alpha_2)$$
$$-\frac{1}{\sqrt{2}}(1-\alpha_1)k\phi(k-1,\alpha_1+1,\alpha_2).$$

如果 λ,η_1,η_2 不全为零,(12)式是一个无穷维的不可约表示.当 $\lambda=\eta_1=\eta_2=0$ 时,(12)式变成

$$L(Q_3)\phi(k,\alpha_1,\alpha_2)$$
$$=\left(-\frac{n}{2}+k+\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)\phi(k,\alpha_1,\alpha_2),$$
$$L(B)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{2}(\alpha_2-\alpha_1)\phi(k,\alpha_1,\alpha_2),$$
$$L(Q_+)\phi(k,\alpha_1,\alpha_2)$$
$$=(n-k-\alpha_1-\alpha_2)\phi(k+1,\alpha_1,\alpha_2),$$
$$L(Q_-)\phi(k,\alpha_1,\alpha_2)$$
$$=k\phi(k-1,\alpha_1,\alpha_2), \tag{13}$$
$$L(V_+)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(n-k-\alpha_1)(1-\alpha_2)\phi(k,\alpha_1,$$
$$\alpha_2+1)+\frac{1}{\sqrt{2}}\alpha_1\phi(k+1,\alpha_1-1,\alpha_2),$$
$$L(V_-)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{\sqrt{2}}\alpha_1\phi(k,\alpha_1-1,\alpha_2)-\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)k$$
$$\cdot\phi(k-1,\alpha_1,\alpha_2+1),$$
$$L(W_+)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{\sqrt{2}}(n-k-\alpha_2)(1-\alpha_1)\phi(k,\alpha_1+1,\alpha_2)$$
$$+\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2\phi(k+1,\alpha_1,\alpha_2-1),$$
$$L(W_-)\phi(k,\alpha_1,\alpha_2)$$
$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2\phi(k,\alpha_1,\alpha_2-1)$$
$$-\frac{1}{\sqrt{2}}(1-\alpha_1)k\phi(k-1,\alpha_1+1,\alpha_2).$$

由此可见,如果 $n\notin Z^+$,(13)式是无穷维不可约表示.如果 $n\in Z^+$,显然存在下面的不变子空间:

$$Y(n):\{\phi(k,\alpha_1,\alpha_2)\in Y\,|\,k+\alpha_1+\alpha_2\leqslant n,$$
$$k\in Z^+,\alpha_1,\alpha_2=0,1\},$$
$$\dim Y(n)=4n, \tag{14}$$

但不存在其不变的补空间,故当 $n\in Z^+$ 时,(13)式是不可分解表示.只要我们将(13)式限制到 $Y(n)$ 上,实质上给出了 SPL(2,1)的有限维不可约表示.此问题的详细讨论,将在下一节进行.

3 SPL(2,1)的有限维不可约表示

为了简明起见,我们把 $Y(n)$ 的基重新定义如下:

$$|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{\sqrt{(j+m)!(j-m-\alpha_1)!(j-m-\alpha_2)!}}$$
$$\cdot\phi(j+m,\alpha_1,\alpha_2). \tag{15}$$

其中

$$j=\frac{1}{2}n=0,\frac{1}{2},1,\frac{3}{2},2,\cdots$$
$m=-j,-j+1,\cdots,j$,当 $\alpha_1=0,\alpha_2=0$,
$m=-j,-j+1,\cdots,j-1$,当 $\alpha_1=0,\alpha_2=1$,
$m=-j,-j+1,\cdots,j-1$,当 $\alpha_1=1,\alpha_2=0$,
$m=-j,-j+1,\cdots,j-2$,当 $\alpha_1=1,\alpha_2=1$,

则 SPL(2,1)在 $Y(n)$ 的新基上的表示为

$$L(Q_3)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\left(m+\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)|j,m,\alpha_1,\alpha_2\rangle,$$
$$L(B)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{2}(\alpha_2-\alpha_1)|j,m,\alpha_1,\alpha_2\rangle, \tag{16}$$
$$L(Q_+)|j,m,\alpha_1,\alpha_2\rangle$$
$$=(j-m-\alpha_1-\alpha_2)\sqrt{\frac{j+m+1}{(j-m-\alpha_1)(j-m-\alpha_2)}}$$
$$|j,m+1,\alpha_1,\alpha_2\rangle,$$
$$L(Q_-)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\sqrt{(j+m)(j-m+1-\alpha_1)(j-m+1-\alpha_2)}$$
$$|j,m-1,\alpha_1,\alpha_2\rangle,$$
$$L(V_+)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)(j-m-\alpha_1)$$
$$\cdot\frac{1}{\sqrt{j-m-\alpha_2}}|j,m,\alpha_1,\alpha_2+1\rangle$$
$$+\frac{1}{\sqrt{2}}\alpha_1\sqrt{\frac{j+m+1}{j-m-\alpha_2}}|j,m+1,\alpha_1-1,\alpha_2\rangle,$$

$$L(V_-)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{\sqrt{2}}\alpha_1\sqrt{j-m+1-\alpha_1}\,|j,m,\alpha_1-1,\alpha_2\rangle$$
$$-\frac{1}{\sqrt{2}}(-1)^{\alpha_1}(1-\alpha_2)\sqrt{(j+m)(j-m+1-\alpha_1)}$$
$$\cdot|j,m-1,\alpha_1,\alpha_2+1\rangle,$$
$$L(W_+)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{\sqrt{2}}(1-\alpha_1)(j-m-\alpha_2)\frac{1}{\sqrt{j-m-\alpha_1}}$$
$$\cdot|j,m,\alpha_1+1,\alpha_2\rangle+\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2$$
$$\cdot\sqrt{\frac{j+m+1}{j-m-\alpha_1}}\,|j,m+1,\alpha_1,\alpha_2-1\rangle,$$
$$L(W_-)|j,m,\alpha_1,\alpha_2\rangle$$
$$=\frac{1}{\sqrt{2}}(-1)^{\alpha_1}\alpha_2\sqrt{j-m+1-\alpha_2}\,|j,m,\alpha_1,\alpha_2-1\rangle$$
$$-\frac{1}{\sqrt{2}}(1-\alpha_1)\sqrt{(j+m)(j-m+1-\alpha_2)}$$
$$\cdot|j,m-1,\alpha_1+1,\alpha_2\rangle.$$

其中置$|j,j+1,\alpha_1,\alpha_2\rangle=|j,-j-1,\alpha_1,\alpha_2\rangle=0$. 显然，以 j 为标记的所有$|j,m,\alpha_1,\alpha_2\rangle$架设的表示空间 $Y(2j)$，在 L 的作用下不变. 其次，在 $Y(2j)$中不存在 L 作用下的真子空间. 因此，上述表示是一个 $8j$ 维的不可约表示.

作为一个特例，我们讨论 $j=\frac{1}{2}$时，SPL(2,1)的(2+2)维不可约表示. 考虑到当 $j=\frac{1}{2}$时，(m,α_1,α_2)可取四个值，$(\frac{1}{2},0,0)$，$(-\frac{1}{2},0,0)$，$(-\frac{1}{2},1,0)$，$(-\frac{1}{2},0,1)$，因此，对于 $L(Q_3)$，我们有

(m',α_1',α_2') \ (m,α_1,α_2)	$(\frac{1}{2},0,0)$	$(-\frac{1}{2},0,0)$	$(-\frac{1}{2},1,0)$	$(-\frac{1}{2},0,1)$
$(\frac{1}{2},0,0)$	$\frac{1}{2}$	0	0	0
$(-\frac{1}{2},0,0)$	0	$-\frac{1}{2}$	0	0
$(-\frac{1}{2},1,0)$	0	0	0	0
$(-\frac{1}{2},0,1)$	0	0	0	0

即

$$L(Q_3)=\begin{bmatrix}\frac{1}{2}&0&0&0\\0&-\frac{1}{2}&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{17}$$

同理可得

$$L(B)=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&-\frac{1}{2}&0\\0&0&0&\frac{1}{2}\end{bmatrix},$$

$$L(Q_+)=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$L(Q_-)=\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$L(V_+)=\begin{bmatrix}0&0&\frac{1}{\sqrt{2}}&0\\0&0&0&0\\0&0&0&0\\0&\frac{1}{\sqrt{2}}&0&0\end{bmatrix},$$

$$L(V_-)=\begin{bmatrix}0&0&0&0\\0&0&\frac{1}{\sqrt{2}}&0\\0&0&0&0\\-\frac{1}{\sqrt{2}}&0&0&0\end{bmatrix}, \tag{18}$$

$$L(W_+)=\begin{bmatrix}0&0&0&\frac{1}{\sqrt{2}}\\0&0&0&0\\0&\frac{1}{\sqrt{2}}&0&0\\0&0&0&0\end{bmatrix},$$

$$L(W_-)=\begin{bmatrix}0&0&0&0\\0&0&0&\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}&0&0&0\\0&0&0&0\end{bmatrix}.$$

显然，这种(2+2)维不可约表示 L，实质上就是我们构造 SPL(2,1)非齐次微分实现时所选取的不可约表示 D. 由此说明，在广义 Fock 空间的子空间上，可以自然地得到李超代数的有限维不可约表示.

我们已经获得了 SPL(2,1)超代数的不可分解表示和有限维不可约表示. 我们的方法可以推广到任意李超代数.

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INDECOMPOSABLE AND IRREDUCIBLE REPRESENTATIONS OF THE SPL(2,1) SUPERALGEBRA

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ABSTRACT

Using inhomogeneous boson-ferimon realization the indecomposable and irreducible representations of the SPL(2,1) are studied on subspace and quotient spaces of the universal enveloping algebra of Heisenberg-weyl superalgebra. All the finite dimensional irreducible representations of the SPL(2,1) are given.

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