

一类非线性耦合方程的孤子解*

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利用双参数假设给出了一类非线性耦合方程的若干孤子解公式,使物理上许多著名的方程作为该方程的特殊情形得到相应的孤子解,指正了一些文献的错误.

关键词:非线性发展方程,双参数假设,孤子解

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1 引 言

考虑非线性耦合方程

$$\sigma'' = \alpha_1 \sigma + \alpha_2 \sigma^3 + \alpha_3 \sigma \rho^2, \quad (1a)$$

$$\rho'' = \beta_1 \rho + \beta_2 \rho^3 + \beta_3 \rho \sigma^2, \quad (1b)$$

其中 $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3$ 为参数; ' 表示对 ξ 求导. 物理学中许多著名的方程都可看作方程(1)的特例. 例如,来源于基本粒子理论中量子化荷电孤子研究的耦合标量场方程^[1].

$$\sigma_{xx} = -\sigma + \sigma^3 + d\sigma\rho^2, \quad (2a)$$

$$\rho_{xx} = (f-d)\rho + \lambda\rho^3 + d\rho\sigma^2, \quad (2b)$$

起源于导电聚合物中弱钉扎电荷密度波方程^[2]

$$\sigma_{tt} - c^2\sigma_{xx} = a\sigma - b\sigma^3 - b\sigma\rho^2, \quad (3a)$$

$$\rho_{tt} - c^2\rho_{xx} = (a-4e)\rho - b\rho^3 - b\rho\sigma^2, \quad (3b)$$

以及 Klein-Gordon 方程, Landou-Ginburg-Higgs 方程等等.

Rajaraman 利用“规道函数”法,对特殊的参数 f, λ 和 d 获得了方程(2)的一种孤子解^[1]. Wang 等人采用函数变换得到了方程(2)的两种新孤子解^[3]. Fan 等人用齐次平衡法得到了方程(2)的另一种新孤子解,同时,利用文献[3]的部分结论得到了方程(2)的其他五种精确解^[4]. 借助于某种变换,文献[4]得到了方程(3)的一组孤子解和一组行波解.

本文首先利用双参数假设,给出了方程(1)当参数满足一定条件时的四种孤子解公式(σ 扭状, ρ 钟状)(σ 钟状, ρ 扭状)(σ 扭状, ρ 扭状)以及(σ

钟状, ρ 钟状).

其次,具体应用到方程(2),方程(3),得到了方程(2)的三种孤子解,方程(3)的四种孤子解.

最后,指正了文献[3][4]的错误.

2 方程(1)的四种孤子解公式

双参数假设,作为构造非线性方程行波解的一种简单方法,我们已用它成功地求得了一些单个因变量非线性方程的精确解^[5]. 对非线性方程

$$u'' = h_1 u + h_3 u^3, \quad (4)$$

其中 h_1, h_3 为非零参数,不难证明如下.

结论 1 方程(4)有孤子解

$$u = \pm \sqrt{-\frac{2h_1}{h_3}} \operatorname{sech}[\sqrt{h_1}(\xi + \xi_0)],$$
$$h_1 > 0, h_3 < 0, \quad (5a)$$

$$u = \pm \sqrt{-\frac{h_1}{h_3}} \tanh\left[\sqrt{-\frac{h_1}{2}}(\xi + \xi_0)\right],$$
$$h_1 < 0, h_3 > 0, \quad (5b)$$

其中 ξ_0 为任意常数. 利用此结论,下面考虑方程(1)的解.

假设

$$\sigma'' = b_1 \sigma + b_3 \sigma^3, \quad (6)$$

其中 b_1, b_3 为非零的待定参数. 将(6)式代入(1a)式的左端并整理得

$$\sigma^2 = \frac{\alpha_3}{b_3 - \alpha_2} \rho^2 + \frac{\alpha_1 - b_1}{b_3 - \alpha_2}, \quad (b_3 - \alpha_2 \neq 0). \quad (7)$$

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将(7)式代入(1b)式的右端得

$$\rho'' = \left[\beta_1 + \frac{\beta_3(\alpha_1 - b_1)}{b_3 - \alpha_2} \right] \rho + \left[\beta_2 + \frac{\alpha_3 \beta_3}{b_3 - \alpha_2} \right] \rho^3 \triangleq a_1 \rho + a_3 \rho^3. \quad (8)$$

(i) 当 $b_1 < 0, b_3 > 0, a_1 > 0, a_3 < 0$ 时, 方程(6)和方程(8)分别有解

$$\begin{aligned} \sigma_1 &= \sqrt{-\frac{b_1}{b_3} \tanh \sqrt{-\frac{b_1}{2}}(\xi + \xi_0)}, \\ \rho_1 &= \sqrt{-\frac{2a_1}{a_3} \operatorname{sech} \sqrt{a_1}(\xi + \xi_0)}. \end{aligned} \quad (9)$$

将(9)式代入(7)式有

$$\begin{aligned} & -\frac{b_1}{b_3} \tanh^2 \sqrt{-\frac{b_1}{2}}(\xi + \xi_0) \\ &= -\frac{2a_1 \alpha_3}{a_3(b_3 - \alpha_2)} \operatorname{sech}^2 \sqrt{a_1}(\xi + \xi_0) \\ &+ \frac{\alpha_1 - b_1}{b_3 - \alpha_2}. \end{aligned} \quad (10)$$

比较(10)式的系数得

$$a_1 = -\frac{b_1}{2}, -\frac{b_1}{b_3} = \frac{2a_1 \alpha_3}{a_3(b_3 - \alpha_2)} = \frac{\alpha_1 - b_1}{b_3 - \alpha_2}. \quad (11)$$

再注意到

$$a_1 = \beta_1 + \frac{\beta_3(\alpha_1 - b_1)}{b_3 - \alpha_2}, a_3 = \beta_2 + \frac{\alpha_3 \beta_3}{b_3 - \alpha_2}, \quad (12)$$

将(11)(12)式联立, 可得如下两组解:

(A₁) $\alpha_3 \neq \beta_2, 2a_2 \alpha_3 \beta_1 + 2\alpha_1 \beta_2 \beta_3 - 2\alpha_2 \beta_1 \beta_2 - \alpha_1 \alpha_3 \beta_3 - \alpha_1 \alpha_2 \beta_2 = 0$ 时,

$$\begin{aligned} b_1 &= \alpha_1(\alpha_3 \beta_3 - \alpha_2 \beta_2) \vee \alpha_2(\alpha_3 - \beta_2), \\ b_3 &= (\alpha_3 \beta_3 - \alpha_2 \beta_2) \vee (\alpha_3 - \beta_3), \\ a_1 &= (\alpha_2 \beta_1 - \alpha_1 \beta_3) \vee \alpha_2, \\ a_3 &= (\alpha_3 \beta_3 - \alpha_2 \beta_2) \vee (\beta_3 - \alpha_2). \end{aligned} \quad (13)$$

(B₁) $\alpha_3 = \beta_2, \alpha_2 = \beta_3$ 时,

$$\begin{aligned} b_1 &= \alpha_1(\alpha_1 - \beta_1), b_3 = \alpha_1(\alpha_1 - \beta_1) \beta_3 / \alpha_1, \\ a_1 &= \beta_1 - \alpha_1, a_3 = \alpha_1(\beta_1 - \alpha_1) \beta_2 / (2\beta_1 - \alpha_1). \end{aligned} \quad (14)$$

将(13)(14)式代入(9)式, 即得方程(1)的一组孤子解(σ_1 扭状, ρ_1 钟状).

(ii) 当 $b_1 > 0, b_3 < 0, a_1 < 0, a_3 > 0$ 时

(iii) 当 $b_1 < 0, b_3 > 0, a_1 < 0, a_3 < 0$ 时

(iv) 当 $b_1 > 0, b_3 < 0, a_1 > 0, a_3 < 0$ 时

仿照(i)的求解过程, 可得方程(1)的另三种孤子

解(σ_2 钟状, ρ_2 扭状)(σ_3 扭状, ρ_3 扭状)(σ_4 钟状, ρ_4 钟状), 因此, 关于方程(1)的孤子解, 我们有

公式 A 当 $\alpha_3 \neq \beta_2$ 时,

$$\begin{aligned} \sigma_1 &= \sqrt{-\frac{\alpha_1}{\alpha_2} \tanh \sqrt{\frac{\alpha_2 \beta_1 - \alpha_1 \beta_3}{\alpha_2}}(\xi + \xi_0)}, \\ \rho_1 &= \sqrt{\frac{\alpha_2 \alpha_2 - \beta_3 \vee \alpha_2 \beta_1 - \alpha_1 \beta_3}{\alpha_2 \vee (\alpha_3 \beta_3 - \alpha_2 \beta_2)}} \\ &\cdot \operatorname{sech} \sqrt{\frac{\alpha_2 \beta_1 - \alpha_1 \beta_3}{\alpha_2}}(\xi + \xi_0), \end{aligned} \quad (15)$$

其中参数有约束关系 $2\alpha_2 \alpha_3 \beta_1 + 2\alpha_1 \beta_2 \beta_3 - 2\alpha_2 \beta_1 \beta_2 - \alpha_1 \alpha_3 \beta_3 - \alpha_1 \alpha_2 \beta_2 = 0$.

$$\begin{aligned} \sigma_2 &= \sqrt{\frac{2\alpha_1(\beta_2 - \alpha_3)}{2\alpha_2 \alpha_3 - \alpha_2 \beta_2 - \alpha_3 \beta_3}} \\ &\cdot \operatorname{sech} \sqrt{\frac{\alpha_1(\alpha_3 \beta_3 - \alpha_2 \beta_2)}{2\alpha_2 \alpha_3 - \alpha_2 \beta_2 - \alpha_3 \beta_3}}(\xi + \xi_0), \\ \rho_2 &= \sqrt{\frac{2\alpha_1(\beta_3 - \alpha_2)}{2\alpha_2 \alpha_3 - \alpha_2 \beta_2 - \alpha_3 \beta_3}} \\ &\cdot \tanh \sqrt{\frac{\alpha_1(\alpha_3 \beta_3 - \alpha_2 \beta_2)}{2\alpha_2 \alpha_3 - \alpha_2 \beta_2 - \alpha_3 \beta_3}}(\xi + \xi_0), \end{aligned} \quad (16)$$

约束关系为 $2\alpha_2 \alpha_3 \beta_1 + 2\alpha_1 \beta_2 \beta_3 - 2\alpha_1 \alpha_2 \beta_2 - \alpha_2 \beta_1 \beta_2 - \alpha_3 \beta_1 \beta_3 = 0$.

$$\begin{aligned} \sigma_3 &= \sqrt{\frac{\alpha_1(\beta_2 - \alpha_3)}{\alpha_3 \beta_3 - \alpha_2 \beta_2} \tanh \sqrt{-\frac{\alpha_1}{2}}(\xi + \xi_0)}, \\ \rho_3 &= \sqrt{\frac{\alpha_1(\alpha_2 - \beta_3)}{\alpha_3 \beta_3 - \alpha_2 \beta_2} \tanh \sqrt{-\frac{\alpha_1}{2}}(\xi + \xi_0)}. \end{aligned} \quad (17)$$

约束关系为 $\alpha_1 = \beta_1 < 0$.

$$\begin{aligned} \sigma_4 &= \sqrt{\frac{2\alpha_1(\beta_2 - \alpha_3)}{\alpha_3 \beta_3 - \alpha_2 \beta_2} \operatorname{sech} \sqrt{\alpha_1}(\xi + \xi_0)}, \\ \rho_4 &= \sqrt{\frac{2\alpha_1(\alpha_2 - \beta_3)}{\alpha_3 \beta_3 - \alpha_2 \beta_2} \operatorname{sech} \sqrt{\alpha_1}(\xi + \xi_0)}. \end{aligned} \quad (18)$$

约束关系为 $\alpha_1 = \beta_1 > 0$.

公式 B 当 $\alpha_3 = \beta_2$ 时,

$$\begin{aligned} \sigma_1 &= \sqrt{-\frac{\alpha_1}{\alpha_2} \tanh \sqrt{\beta_1 - \alpha_1}(\xi + \xi_0)}, \\ \rho_1 &= \sqrt{\frac{\alpha_1 - 2\beta_1}{\beta_2} \operatorname{sech} \sqrt{\beta_1 - \alpha_1}(\xi + \xi_0)}, \end{aligned} \quad (19)$$

其中参数的约束关系为 $\alpha_2 = \beta_3$.

$$\sigma_2 = \sqrt{\frac{\beta_1 - 2\alpha_1}{\beta_3} \operatorname{sech} \sqrt{\alpha_1 - \beta_1}(\xi + \xi_0)},$$

$$\rho_2 = \sqrt{-\frac{\beta_1}{\beta_2}} \tanh \sqrt{\alpha_1 - \beta_1} (\xi + \xi_0), \quad (20)$$

约束关系为 $\alpha_2 = \beta_3$.

$$\begin{aligned} \sigma_3 &= \sqrt{-\frac{\alpha_1}{\tau}} \tanh \sqrt{-\frac{\alpha_1}{2}} (\xi + \xi_0), \\ \rho_3 &= \sqrt{\frac{\alpha_1(\beta_3 - \tau)}{\tau\beta_2}} \tanh \sqrt{-\frac{\alpha_1}{2}} (\xi + \xi_0), \end{aligned} \quad (21)$$

其中 τ 为任意实数 约束关系为 $\alpha_2 = \beta_3, \alpha_1 = \beta_1 < 0$.

$$\begin{aligned} \sigma_4 &= \sqrt{-\frac{2\alpha_1}{\tau}} \operatorname{sech} \sqrt{\alpha_1} (\xi + \xi_0), \\ \rho_4 &= \sqrt{\frac{2\alpha_1(\beta_3 - \tau)}{\tau\beta_2}} \operatorname{sech} \sqrt{\alpha_1} (\xi + \xi_0), \end{aligned} \quad (22)$$

τ 为任意实数 约束关系 $\alpha_2 = \beta_3, \alpha_1 = \beta_1 > 0$.

上述公式 A(15)(16)(17)(18) 式和公式 B(19)(20)(21)(22) 式中, 参数 $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \tau$ 的取值范围是分别使根号内的值大于零, 即使表达式分别有意义的一切实数.

3 方程(2), 方程(3)的有关结果

对非线性耦合标量场方程(2), 注意到

$$\begin{aligned} \alpha_1 &= -1, \alpha_2 = 1, \alpha_3 = d, \\ \beta_1 &= f - d, \beta_2 = \lambda, \beta_3 = d, \end{aligned}$$

由公式 A 我们得到

结论 2 当 $\lambda \neq d$ 时, 方程(2)有三种孤子解:

$$\begin{aligned} \sigma_1 &= \tanh \sqrt{f} (x + x_0), \\ \rho_1 &= \sqrt{\frac{1-d}{d-\lambda}} \operatorname{sech} \sqrt{f} (x + x_0), \end{aligned} \quad (23)$$

其中 $f = \frac{d^2 - \lambda}{2(d - \lambda)} > 0, \frac{1-d}{d-\lambda} > 0$.

$$\begin{aligned} \sigma_2 &= \sqrt{\frac{2(d-\lambda)}{2d-\lambda-d^2}} \operatorname{sech} \sqrt{\frac{\lambda-d^2}{2d-\lambda-d^2}} (x + x_0), \\ \rho_2 &= \sqrt{\frac{2(1-d)}{2d-\lambda-d^2}} \tanh \sqrt{\frac{\lambda-d^2}{2d-\lambda-d^2}} (x + x_0), \end{aligned} \quad (24)$$

其中 $f = \frac{(2-d)(d^2-\lambda)}{2d-\lambda-d^2}, \frac{d-\lambda}{2d-\lambda-d^2} > 0, \frac{\lambda-d^2}{2d-\lambda-d^2} > 0, \frac{1-d}{2d-\lambda-d^2} > 0$.

$$\begin{aligned} \sigma_3 &= \sqrt{\frac{d-\lambda}{d^2-\lambda}} \tanh \frac{1}{\sqrt{2}} (x + x_0), \\ \rho_3 &= \sqrt{\frac{d-1}{d^2-\lambda}} \tanh \frac{1}{\sqrt{2}} (x + x_0), \end{aligned} \quad (25)$$

其中 $f = d - 1 (d - \lambda)(d^2 - \lambda) > 0 (d - 1)(d^2 - \lambda) > 0$.

由公式 B 有

结论 3 当 $\lambda = d = 1$ 时, 方程(2)有下列三种孤子解:

$$\begin{aligned} \sigma_1 &= \tanh \sqrt{f} (x + x_0) \\ \rho_1 &= \sqrt{1-2f} \operatorname{sech} \sqrt{f} (x + x_0) \end{aligned} \quad 0 < f < \frac{1}{2}, \quad (26)$$

$$\begin{aligned} \sigma_2 &= \sqrt{f+1} \operatorname{sech} \sqrt{-f} (x + x_0) \\ \rho_2 &= \sqrt{1-f} \tanh \sqrt{-f} (x + x_0) \end{aligned} \quad -1 < f < 0, \quad (27)$$

$$\sigma_3 = \sqrt{\frac{1}{\tau}} \tanh \frac{1}{\sqrt{2}} (x + x_0)$$

$$\rho_3 = \sqrt{\frac{\tau-1}{\tau}} \tanh \frac{1}{\sqrt{2}} (x + x_0)$$

$f = 0, \tau > 1, \tau$ 为任意实数. (28)

对弱钉扎电荷密度波方程(3), 其行波解 $\sigma = \rho(\xi), \rho = \rho(\xi), \xi = x - kt$ 满足

$$\begin{aligned} (k^2 - c^2)\sigma'' &= a\sigma - b\sigma^3 - b\sigma\rho^2, \\ (k^2 - c^2)\rho'' &= (a - 4e)\rho - b\rho^3 - b\rho\sigma^2. \end{aligned}$$

注意到

$$\alpha_1 = \frac{a}{k^2 - c^2}, \alpha_2 = \frac{-b}{k^2 - c^2}, \alpha_3 = \frac{-b}{k^2 - c^2}.$$

$$\beta_1 = \frac{a-4e}{k^2 - c^2}, \beta_2 = \frac{-b}{k^2 - c^2}, \beta_3 = \frac{-b}{k^2 - c^2}.$$

显然有 $\alpha_3 = \beta_2, \alpha_2 = \beta_3$, 从而由公式 B 有

结论 4 方程(3)有四种孤子解:

当 $e \neq 0$ 时,

$$\begin{aligned} \sigma_1 &= \sqrt{\frac{a}{b}} \tanh \sqrt{\frac{4e}{c^2 - k^2}} (x - kt + \xi_0), \\ \rho_1 &= \sqrt{\frac{a-8e}{b}} \operatorname{sech} \sqrt{\frac{4e}{c^2 - k^2}} (x - kt + \xi_0), \end{aligned} \quad (30)$$

其中 $ab > 0, a(c^2 - k^2) > 0, b(a - 8e) > 0$.

$$\begin{aligned} \sigma_2 &= \sqrt{\frac{a+4e}{b}} \operatorname{sech} \sqrt{\frac{4e}{k^2 - c^2}} (x - kt + \xi_0), \\ \rho_2 &= \sqrt{\frac{4e-a}{b}} \tanh \sqrt{\frac{4e}{k^2 - c^2}} (x - kt + \xi_0), \end{aligned} \quad (31)$$

其中 $b(a+4e) > 0, a(k^2 - c^2) > 0, b(4e - a) > 0$.

当 $e = 0$ 时,

$$\sigma_3 = \sqrt{\frac{a}{a(c^2 - k^2)}} \tanh \sqrt{\frac{a}{2(c^2 - k^2)}} (x - kt + \xi_0),$$

$$\rho_3 = \sqrt{\frac{a}{b} + \frac{a}{(k^2 - c^2)\tau}} \cdot \tanh \sqrt{\frac{a}{2(c^2 - k^2)}}(x - kt + \xi_0), \quad (32)$$

其中 $a\tau(c^2 - k^2) > 0, a(c^2 - k^2) > 0, \frac{a}{b} + \frac{a}{(k^2 - c^2)\tau} > 0, \tau$ 为任意实数.

$$\begin{aligned} \sigma_4 &= \sqrt{\frac{2a}{\tau(c^2 - k^2)}} \operatorname{sech} \sqrt{\frac{a}{(k^2 - c^2)}}(x - kt + \xi_0), \\ \rho_4 &= \sqrt{\frac{2a}{b} + \frac{2a}{(k^2 - c^2)\tau}} \cdot \operatorname{sech} \sqrt{\frac{a}{(k^2 - c^2)}}(x - kt + \xi_0), \end{aligned} \quad (33)$$

其中 $a\tau(c^2 - k^2) > 0, a(k^2 - c^2) > 0, \frac{a}{b} + \frac{a}{(k^2 - c^2)\tau} > 0, \tau$ 为任意实数.

4 关于文献 3 的修正

文献 3 讨论非线性耦合标量场方程(2), 通过作非线性变换, 即文献 3 中(20)式:

$$\begin{aligned} \sigma_x &= \rho \sqrt{A\rho^2 + B\sigma^2 + C}, \\ \rho_x &= D\sigma \sqrt{A\rho^2 + B\sigma^2 + C}, \end{aligned}$$

得到文献 3 中(22)式:

$$\begin{aligned} DC &= -1, BD = 1, 2AD + B = d, \\ DC &= f - d, AD = \lambda, 2BD + AD^2 = d. \end{aligned}$$

从而定出如下的参数值, 即文献 3 中(23)式:
 $A = \lambda^2(d - 2), B = d - 2\lambda, C = (2 - d)\lambda^2,$
 $D = 2 - 3/d, f = d - 1, \lambda = d(d - 2)(2d - 3).$
 这里关于 C 的值算错了, 正确的应为 $C = d(3 - 2d)$. 因此文献[3]中的第(27)式应为 $\sigma_x = \sigma \sqrt{(d - 1)\sigma^2 + (d - 1)(d - 2)}$. 由此得到方程(2)的(σ 钟状, ρ 扭状)的孤子解应为

$$\begin{aligned} \sigma &= \sqrt{\frac{1}{2 - d}} \operatorname{sech} \sqrt{\frac{d - 1}{d - 2}}(x + x_0), \\ \rho &= \sqrt{\frac{2d - 3}{d(d - 2)}} \tanh \sqrt{\frac{d - 1}{d - 2}}(x + x_0). \end{aligned}$$

它是本文(24)式的特殊情形(取 $f = d - 1$). 此外, 文献 3 中的第(29)式 $\sigma_x = \pm \sigma \sqrt{(AD^2 + BD)\sigma^2 + C}$ 也是错误的, 应为 $\sigma_x = \pm \sigma \sqrt{(AD^2 + BD)\sigma^2 + CD}$, 即 $\sigma_x = \pm \sigma \sqrt{(d - 1)\sigma^2 - 1}$. 因此, 不能得到文献 3]

中第(31)式给出的方程(2)的孤子解(σ 钟状, ρ 钟状), 只能得到方程(2)的(σ 扭状, ρ 扭状)的孤子解

$$\begin{aligned} \sigma &= \sqrt{\frac{1}{2(d - 1)}} \tanh \frac{1}{\sqrt{2}}(x + x_0), \\ \rho &= \sqrt{\frac{2d - 3}{2d(d - 1)}} \tanh \frac{1}{\sqrt{2}}(x + x_0). \end{aligned}$$

这在本文中的(25)式中取 $\lambda = d(d - 2)(2d - 3)$ 即可得到.

文献 4]由于借用了文献 3]的结论, 因此文献 [4]中的第(9)式

$$\sigma_x = \sigma \sqrt{(d - 1)\sigma^2 + \frac{(2d - 3)^2}{d^2(2 - d)}}$$

也是错误的, 由此得到的 $3/2 < d < 2$ 时方程(2)的孤子解以及 $d > 2$ 时的三角函数周期解均是错误的.

5 说 明

本文给出了方程(1)四种孤子解的一般公式, 使得求方程(2), 方程(3)的孤子解的过程变得非常简单. 文献 1]得到的结果是我们结论中的(23)式, 文献 3]得到的结果是结论中的(24)式的特殊情形, 文献 4]得到的结果是结论中的(25)式和(30)式. 本文得到的其他孤子解我们尚未见文献给出过, 它将有助于方程得到更广泛的应用.

顺便指出, 由于方程(4)还有如下形式的解:

$$u = \begin{cases} \sqrt{\frac{h_1}{h_3}} \tan \sqrt{\frac{h_1}{2}}(\xi + \xi_0), & h_1 > 0, h_3 > 0, \\ \sqrt{-\frac{2h_1}{h_3}} \operatorname{sec} \sqrt{-h_1}(\xi + \xi_0), & h_1 < 0, h_3 > 0. \end{cases}$$

仿照本文的计算过程, 可得到方程(1)的其他形式的精确解公式, 进而可得到方程(2),(3)的其他精确解, 它包括了文献 4]得到的五种精确解, 在此不再详述.

本文所用的方法适用于其他耦合的非线性方程组, 有关结果已另外成文.

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THE SOLITON SOLUTIONS FOR A CLASS OF NONLINEAR COUPLED EQUATIONS*

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ABSTRACT

Some soliton solutions for a class of nonlinear coupled equations are obtained by double parameter hypothesis. Many well-known equations in physics are special cases of the coupled equations presented in this paper. Some incorrect conclusion in the literature are corrected.

Keywords : Nonlinear evolution equations , Double parameter hypothesis , Soliton solutions

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