有旋电子在电磁场及二维谐振子场中运动的双波描述

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研究有旋非相对论性电子在相互垂直的均匀电磁场及二维谐振子场中运动的量子单波及量子双波描述.

关键词:双波描述,有旋电子,电磁场,谐振子场
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1 引 言

作为一种决定论性的量子力学理论,双波函数描述已经被广泛应用到束缚态、非束缚态、原子辐射和原子双能级理论等量子问题,并取得了成功[1—10].本文作者曾经讨论过不同条件下带电谐振子的双波描述,相应地也得到了令人满意的结果[11—13].但是,除了相对论性粒子的双波描述涉及到粒子的自旋以外[14,15],至今发表的关于有旋非相对论性粒子的双波描述仍然少见.本文讨论有旋电子在相互垂直的均匀电场和均匀磁场及二维谐振子场中运动的非相对论双波描述,得到了一系列有别于普通量子力学的结果.文中讨论了有旋电子的退化问题,并将其双波描述的经典极限与经典力学结果相比较,得到了相一致的结论.

2 量子单波描述

考虑有旋电子在相互垂直的均匀电场 E(沿 x 方向)和均匀磁场 B(沿 z 方向)及二维各向同性谐振子场 $U(x,z)=\frac{1}{2}\mu\omega^2(x^2+z^2)$ 中运动(各向异性谐振子场见讨论部分).如果忽略电子自旋与轨道的相互作用,电子的哈密顿算符为

$$\hat{H}=\frac{1}{2\mu}\left[\hat{p}_x^2+\left(\hat{p}_y+\frac{eBx}{c}\right)^2+\hat{p}_z^2\right]+eEx+\frac{1}{2}\mu\omega^2(x^2+z^2)+M_B B\hat{\sigma}_z,\quad(1)$$

$M_B=\frac{e\hbar}{2\mu c}$,$\hat{\sigma}_z$ 为自旋算符.在 $\hat{\sigma}_z$ 表象中 $\hat{H}$ 本征函数及本征值分别为

$$\Psi_{n_x,p_y,n_z,\sigma}(x,y,z)=\varphi_{n_x}(x-x_0)\varphi_{p_y}(y)\varphi_{n_z}(z)\chi_\sigma,$$
$$E_{n_x,p_y,n_z,\sigma}=(n_x+1/2)\hbar\omega'+(n_z+1/2)\hbar\omega+rp_y^2-sp_y-\Delta+\sigma M_B B,\quad(2)$$

其中 $\sigma=+1,-1$ 对应电子不同的自旋方向,$\hat{\sigma}_z$ 本征函数 χ_σ 为

$$\chi_1=\begin{pmatrix}1\\0\end{pmatrix},\quad\chi_{-1}=\begin{pmatrix}0\\1\end{pmatrix}.$$

$$\varphi_{n_x}(x-x_0)=N_{n_x}\exp\left(\frac{1}{2}\alpha^2(x-x_0)^2\right)\cdot H_{n_x}[\alpha'(x-x_0)],$$

$$\varphi_{p_y}(y)=\frac{1}{\sqrt{2\pi\hbar}}\exp\left(\frac{\mathrm{i}}{\hbar}p_y y\right),$$

$$\varphi_{n_z}(z)=N_{n_z}\exp\left(-\frac{1}{2}\alpha^2z^2\right)H_{n_z}[\alpha z],$$

$$\alpha'=\sqrt{\frac{\mu\omega'}{\hbar}},\ \alpha=\sqrt{\frac{\mu\omega}{\hbar}},$$

$$\omega'=\left(\omega^2+\frac{e^2B^2}{\mu^2c^2}\right)^{1/2},$$

$$x_0=-\frac{eBp_y+eEc\mu}{\mu^2\omega'^2c},\ r=\frac{1}{2\mu}\left(1-\frac{e^2B^2}{\mu^2c^2\omega'^2}\right),$$

$$S=\frac{e^2BE}{\mu^2c\omega'^2},\ \Delta=\frac{e^2E^2}{2\mu\omega'^2}.$$

3 量子双波描述

引入双波函数

$$\psi_{n_x,p_y^0,n_z,\sigma}(x,y,z,t)=\varphi_{n_x}(x-x_0')\varphi_{p_y^0}(y-y_0)$$

$$\cdot \varphi_{n_z}(z)\chi_\sigma \exp\left[-\mathrm{i}n_x\beta' - \mathrm{i}n_z\beta - \frac{\mathrm{i}}{\hbar}E_{n_x,p_y^0,n_z,\sigma}t\right],$$

$$\psi(x,y,z,t) = \sum_{\sigma'=-1}^{1}\sum_{n_x',n_z'}\int \mathrm{d}p_y\varphi_{n_x'}(x-x_0)\varphi_{p_y}(y-y_0) \cdot \varphi_{n_z'}(z)\chi_{\sigma'}\exp\left[-\mathrm{i}n_x'\beta' - \mathrm{i}n_z'\beta - \frac{\mathrm{i}}{\hbar}E_{n_x',p_y,n_z',\sigma'}t\right], \tag{3}$$

其中 $x_0' = -\dfrac{eBp_y^0 + eE\mu c}{\mu^2\omega'^2 c}$,而 y_0,β,β' 为双波理论引入的特定参数.任意力学量算符 $\hat{f}(t)$ 对应的力学量 $f(t)$ 在这状态中的测量值为

$$\begin{aligned}\langle f(t)\rangle_{n_x,p_y^0,n_z,\sigma} &= \mathrm{Re}\iiint \psi^*(x,y,z,t) \cdot \hat{f}(t)\psi_{n_x,p_y^0,n_z,\sigma}(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \mathrm{Re}\iiint \mathrm{d}x\mathrm{d}y\mathrm{d}z\sum_{\sigma'=-1}^{1}\sum_{n_x',n_z'}\int \mathrm{d}p_y\varphi_{n_x'}^* \\ &\cdot (x-x_0)\varphi_{p_y}^*(y-y_0)\varphi_{n_z'}^*(z)x_{\sigma'}^+ \\ &\cdot \exp\left[\mathrm{i}n_x'\beta' + \mathrm{i}n_z'\beta + \frac{\mathrm{i}}{\hbar}E_{n_x',p_y,n_z',\sigma'}t\right]\hat{f}(t) \\ &\cdot \varphi_{n_x}(x-x_0')\varphi_{p_y^0}(y-y_0)\varphi_{n_z}(z)x_\sigma \\ &\cdot \exp\left[-\mathrm{i}n_x\beta' - \mathrm{i}n_z\beta - \frac{\mathrm{i}}{\hbar}E_{n_x,p_y^0,n_z,\sigma}t\right].\end{aligned} \tag{4}$$

1. $\hat{f}(t) = 1$ 归一化

$$\begin{aligned}&\mathrm{Re}\iiint \mathrm{d}x\mathrm{d}y\mathrm{d}z\sum_{\sigma'=-1}^{1}\sum_{n_x'n_z'}\int \mathrm{d}p_y\varphi_{n_x'}^*(x-x_0)\varphi_{p_y}^* \\ &.(y-y_0)\varphi_{n_z'}^*(z)x_{\sigma'}^+\exp\left[\mathrm{i}n_x'\beta' + \mathrm{i}n_z'\beta\right. \\ &\left.+\frac{\mathrm{i}}{\hbar}E_{n_x',p_y,n_z',\sigma'}t\right]\varphi_{n_x}(x-x_0')\varphi_{p_y^0} \\ &\cdot (y-y_0)\varphi_{n_z}(z)x_\sigma\exp\left[-\mathrm{i}n_x\beta' - \mathrm{i}n_z\beta\right. \\ &\left.-\frac{\mathrm{i}}{\hbar}E_{n_x,p_y^0,n_z,\sigma}t\right] = 1.\end{aligned} \tag{5}$$

2. $\hat{f}(t) = \hat{H}$ 能量本征值

$$\langle \hat{H}\rangle_{n_x,p_y^0,n_z,\sigma} = (n_x + 1/2)\hbar\omega' + (n_z + 1/2)\hbar\omega + r(p_y^0)^2 - sp_y^0 - \Delta + \sigma M_B B. \tag{6}$$

3. $\hat{f}(t) = x$

$$\langle x\rangle_{n_x,p_y^0,n_z,\sigma} = x_0' + 1/\alpha'\left(\sqrt{\frac{n_x}{2}} + \sqrt{\frac{n_x+1}{2}}\right) \cdot \cos(\omega' t + \beta'). \tag{7}$$

4. $\hat{f}(t) = \hat{p}_x = -\mathrm{i}h\dfrac{\partial}{\partial x}$

$$\langle p_x\rangle_{n_x,p_y^0,n_z,\sigma} = \mu\frac{\mathrm{d}\langle x\rangle_{n_x,p_y^0,n_z,\sigma}}{\mathrm{d}t} = -\frac{\mu\omega'}{\alpha'}\left(\sqrt{\frac{n_x}{2}} + \sqrt{\frac{n_x+1}{2}}\right)\sin(\omega' t + \beta'), \tag{8}$$

说明 x 方向正则动量与机械动量等价.

5. $\hat{f} = y$

$$\langle y\rangle_{n_x,p_y^0,n_z,\sigma} = y_0 + \frac{eB\hbar^{1/2}}{\mu^{3/2}\omega'^{3/2}c}\left(\sqrt{\frac{n_x}{2}} + \sqrt{\frac{n_x+1}{2}}\right) \cdot \sin\omega' t + (2rp_y^0 - s)t. \tag{9}$$

6. $\hat{f} = \hat{p}_y = -\mathrm{i}h\dfrac{\partial}{\partial y}$

$$\langle p_y\rangle_{n_x,p_y^0,n_z,\sigma} = p_y^0. \tag{10}$$

可见 $\langle p_y\rangle_{n_x,p_y^0,n_z,\sigma} \neq \mu\dfrac{\mathrm{d}\langle y\rangle_{n_x,p_y^0,n_z,\sigma}}{\mathrm{d}t}$,说明 y 方向正则动量与机械动量不等价.

7. $\hat{f} = z$

$$\langle z\rangle_{n_x,p_y^0,n_z,\sigma} = \frac{1}{\alpha}\left(\sqrt{\frac{n_z+1}{2}} + \sqrt{\frac{n_z}{2}}\right)\cos(\omega t + \beta). \tag{11}$$

8. $\hat{f} = \hat{p}_z$

$$\langle p_z\rangle_{n_x,p_y^0,n_z,\sigma} = \mu\frac{\mathrm{d}\langle z\rangle_{n_x,p_y^0,n_z,\sigma}}{\mathrm{d}t} = -\frac{\mu\omega}{\alpha}\left(\sqrt{\frac{n_z+1}{2}} + \sqrt{\frac{n_z}{2}}\right)\sin(\omega t + \beta). \tag{12}$$

9. $\hat{f} = \hat{\sigma}_z$

$$\langle \sigma_z\rangle_{n_x,p_y^0,n_z,\sigma} = \sigma = \pm 1. \tag{13}$$

10. $\hat{f} = \hat{\sigma}_x$

$$\langle \sigma_x\rangle_{n_x,p_y^0,n_z,\sigma} = \cos\frac{2\sigma M_B B}{\hbar}t = \cos\frac{2M_B B}{\hbar}t. \tag{14}$$

11. $\hat{f} = \hat{\sigma}_y$

$$\langle \sigma_y\rangle_{n_x,p_y^0,n_z,\sigma} = \sin\frac{2M_B B}{\hbar}t. \tag{15}$$

4　讨论与结论

1. 本文讨论各向同性谐振子势 $U(x,z) = \dfrac{1}{2}\mu\omega^2(x^2 + z^2)$,如果针对各向异性谐振子势

$U(x,z)=\frac{1}{2}\mu\omega_x^2x^2+\frac{1}{2}\mu\omega_z^2z^2$，只需作变换：$\omega=\omega_z$，$\omega'=\left(\omega_x^2+\frac{e^2B^2}{\mu^2c^2}\right)^{1/2}$，则上面各种结果形式不变.

2. 若进一步考虑均匀系综或时间系综[16]，可以证明本文的双波函数定义的状态中各力学量测量值的系综平均值正是通常量子力学单波描述的状态平均值，说明双波函数可以描述单个电子，而量子力学单个波函数只能描述一个系综，在该系综中代表不同初位相和振动模式的电子的三个参数 y_0,β,β' 各不相同.

3. 退化问题

(1)如果不考虑电子自旋，单波解及双波解分别退化为

单波

$$\varphi_{n_x,p_y,n_z}(x,y,z)=\varphi_{n_x}(x-x_0)\varphi_{p_y}(y)\varphi_{n_z}(z),$$
$$E_{n_x,p_y,n_z}=(n_x+1/2)h\omega'+(n_z+1/2)h\omega+rp_y^2-sp_y-\Delta. \tag{16}$$

双波

$$\psi_{n_x,p_y^0,n_z}(x,y,z,t)=\varphi_{n_x}(x-x_0')\varphi_{p_y^0}(y-y_0)\cdot\varphi_{n_z}(z)\exp\left[-\mathrm{i}n_x\beta'-\mathrm{i}n_z\beta-\frac{\mathrm{i}}{\hbar}E_{n_x,p_y^0,n_z}t\right],$$
$$\psi(x,y,z,t)=\sum_{n_x'n_z'}\int\mathrm{d}p_y\varphi_{n_x'}(x-x_0)\cdot\varphi_{p_y}(y-y_0)\varphi_{n_z'}(z)\exp\left[-\mathrm{i}n_x'\beta'-\mathrm{i}n_z'\beta-\frac{\mathrm{i}}{\hbar}E_{n_x',p_y,n_z'}t\right]. \tag{17}$$

各力学量测量值为

$$\mathrm{Re}\iiint\mathrm{d}x\mathrm{d}y\mathrm{d}z\varphi^*(x,y,z,t)\varphi_{n_x,p_y^0,n_z}(x,y,z,t)=1\quad(\text{归一化}).$$

$$\langle H\rangle_{n_x,p_y^0,n_z}=E_{n_x,p_y^0,n_z}=\left(n_x+\frac{1}{2}\right)\hbar\omega'+\left(n_z+\frac{1}{2}\right)\hbar\omega+rp_y^{0^2}+sp_y^0-\Delta, \tag{18}$$

$$\langle x\rangle_{n_x,p_y^0,n_z}=x_0'+\frac{1}{\alpha'}\left(\sqrt{\frac{n_x}{2}}+\sqrt{\frac{n_x+1}{2}}\right)\cdot\cos(\omega't+\beta'), \tag{19}$$

$$\langle p_x\rangle_{n_x,p_y^0,n_z}=\mu\frac{\mathrm{d}\langle x\rangle_{n_x,p_y^0,n_z}}{\mathrm{d}t}, \tag{20}$$

$$\langle y\rangle_{n_x,p_y^0,n_z}=y_0+\frac{eB\hbar^{1/2}}{\mu^{3/2}\omega'^{3/2}c}\left[\sqrt{\frac{n_x}{2}}+\sqrt{\frac{n_x+1}{2}}\right]\cdot\sin\omega't+(2rp_y^0-s)t, \tag{21}$$

$$\langle p_y\rangle_{n_x,p_y^0,n_z}=p_y^0\neq\mu\frac{\mathrm{d}\langle y\rangle_{n_x,p_y^0,n_z}}{\mathrm{d}t}, \tag{22}$$

$$\langle z\rangle_{n_x,p_y^0,n_z}=\frac{1}{\alpha}\left[\sqrt{\frac{n_z+1}{2}}+\sqrt{\frac{n_z}{2}}\right]\cos(\omega t+\beta), \tag{22'}$$

$$\langle p_z\rangle_{n_x,p_y^0,n_z}=\mu\frac{\mathrm{d}\langle z\rangle_{n_x,p_y^0,n_z}}{\mathrm{d}t}. \tag{23}$$

可见，如不考虑电子自旋，除了能量本征值 $\langle H\rangle_{n_x,p_y^0,n_z}$ 没有自旋附加项 σM_BB 以外，其他各力学量相应的表达式不变. 当然此时没有 $\langle\sigma_x\rangle,\langle\sigma_y\rangle,\langle\sigma_z\rangle$.

(2)经典极限 c.L

取极限条件：$n_x\to\infty,n_z\to\infty,n_x\hbar\omega'\to\frac{1}{2}\mu\omega'^2c_x^2,n_z\hbar\omega\to\frac{1}{2}\mu\omega^2c_z^2$，

$$\langle x\rangle_{n_x,p_y^0,n_z}\to\langle x\rangle_{\mathrm{c.L}}=x_0'+c_x\cos(\omega't+\beta'), \tag{24}$$

$$\langle p_x\rangle_{n_x,p_y^0,n_z}\to\langle p_x\rangle_{\mathrm{c.L}}=-\mu\omega'c_x\sin(\omega't+\beta'), \tag{25}$$

$$\langle y\rangle_{n_x,p_y^0,n_z}\to\langle y\rangle_{\mathrm{c.L}}=y_0+\frac{eBc_x}{\mu c\omega'}\sin\omega't+(2rp_y^0-s)t=y_0+\frac{eBc_x}{\mu c\omega'}\sin\omega't+\left(\frac{p_y^0}{\mu}+\frac{eBx_0'}{c\mu}\right)t, \tag{26}$$

$$\langle p_y\rangle_{n_x,p_y^0,n_z}\to\langle p_y\rangle_{\mathrm{c.L}}=p_y^0\neq\mu\frac{\mathrm{d}\langle y\rangle_{\mathrm{c.L}}}{\mathrm{d}t}, \tag{27}$$

$$\langle z\rangle_{n_x,p_y^0,n_z}\to\langle z\rangle_{\mathrm{c.L}}=c_z\cos(\omega t+\beta), \tag{28}$$

$$\langle p_z\rangle_{n_x,p_y^0,n_z}\to\langle p_z\rangle_{\mathrm{c.L}}=-\mu\omega c_z\sin(\omega t+\beta). \tag{29}$$

(3)可以证明

$$\langle x^2\rangle_{\mathrm{c.L}}=\langle x\rangle_{\mathrm{c.L}}^2,\ \langle p_x^2\rangle_{\mathrm{c.L}}=\langle p_x\rangle_{\mathrm{c.L}}^2,$$
$$\langle y^2\rangle_{\mathrm{c.L}}=\langle y\rangle_{\mathrm{c.L}}^2,\ \langle p_y^2\rangle_{\mathrm{c.L}}=\langle p_y\rangle_{\mathrm{c.L}}^2,$$
$$\langle z^2\rangle_{\mathrm{c.L}}=\langle z\rangle_{\mathrm{c.L}}^2,\ \langle p_z^2\rangle_{\mathrm{c.L}}=\langle p_z\rangle_{\mathrm{c.L}}^2. \tag{30}$$

用归纳法可以进一步证明

$$\langle x^n\rangle_{\mathrm{c.L}}=\langle x\rangle_{\mathrm{c.L}}^n,\ \langle p_x^n\rangle_{\mathrm{c.L}}=\langle p_x\rangle_{\mathrm{c.L}}^n,$$

$$\langle y^n\rangle_{\mathrm{c.L}} = \langle y\rangle^n_{\mathrm{c.L}},\ \langle p_y^n\rangle_{\mathrm{c.L}} = \langle p_y\rangle^n_{\mathrm{c.L}},$$
$$\langle z^n\rangle_{\mathrm{c.L}} = \langle z\rangle^n_{\mathrm{c.L}},\ \langle p_z^n\rangle_{\mathrm{c.L}} = \langle p_z\rangle^n_{\mathrm{c.L}}, \quad (31)$$

n 为正整数.

显然,在双波函数定义的状态中所有力学量测量值在经典极限下均为无弥散(dispersion-free)的,这是普通量子力学所无法给出的一般性的结论.

(4)经典描述

电子在互相垂直的均匀电磁场及二维谐振子场中的经典哈密顿量为

$$H = \frac{1}{2\mu}\left[p_x^2 + \left(p_y + \frac{eB}{c}x\right)^2\right] + \frac{p_z^2}{2\mu} + eEx + \frac{1}{2}\mu\omega^2(x^2 + z^2). \quad (32)$$

正则方程为

$$\dot{x} = \frac{\partial H}{\partial p_x} = \frac{p_x}{\mu},$$
$$\dot{p}_x = -\frac{\partial H}{\partial x} = -\left[\frac{eB}{\mu c}\left(p_y + \frac{eB}{c}x\right) + eE + \mu\omega^2 x\right],$$
$$\dot{y} = \frac{\partial H}{\partial p_y} = \frac{1}{\mu}\left(p_y + \frac{eB}{c}x\right),\ \dot{p}_y = -\frac{\partial H}{\partial y} = 0,$$
$$\dot{z} = \frac{\partial H}{\partial p_z} = \frac{p_z}{\mu},\ \dot{p}_z = -\frac{\partial H}{\partial z} = -\mu\omega^2 z. \quad (33)$$

经典解为

$$x = x_0' + c_x\cos(\omega' t + \beta'),$$
$$p_x = -\mu\omega' c_x\sin(\omega' t + \beta'),$$
$$y = y_0 + \left(\frac{p_y^0}{\mu} + \frac{eB}{c\mu}x_0'\right)t + \frac{eBc_x}{c\mu\omega'}\sin\omega' t,$$
$$p_y = p_y^0,$$
$$z = c_z\cos(\omega t + \beta),\ p_z = -\mu\omega c_z\sin(\omega t + \beta). \quad (34)$$

由此可见,在互相垂直的均匀电磁场及二维谐振子场中非旋电子的双波描述所给出的力学量测量值的经典极限与纯经典描述的力学量测量值完全一致.

(5)上述量子双波解及经典解中$\langle y\rangle$出现匀速运动项 $(2rp_y^0 - s)t = \left(\frac{p_y^0}{\mu} + \frac{eB}{c\mu}x_0'\right)t$,这是由于外加电场和谐振子场对电子产生附加作用的结果. 容易证明:

1)当没有电场和谐振子场即 $E = \omega = 0$ 时,该匀速运动项 $(2rp_y^0 - s)t = \left(\frac{p_y^0}{\mu} + \frac{eB}{c\mu}x_0'\right)t$,将消失,所有结果与文献[6]完全相同.

2)只要存在电场和谐振子场或二者之一,一般情况下必然出现匀速运动项,即 $(2rp_y^0 - s)t = \left(\frac{p_y^0}{\mu} + \frac{eB}{c\mu}x_0'\right)t \neq 0$.

3)特例:$\omega \neq 0, E \neq 0$,但$\frac{E}{\omega^2} = \frac{\mu c}{e^2 B}p_y^0$ 时,$\langle y\rangle$中匀速运动项同样消失.

本文讨论有旋电子的双波描述时忽略自旋与轨道的相互作用. 如果计及这种作用,电子的双波描述应该是什么结果,这个问题作者正在探讨中.

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DOUBLE-WAVE DESCRIPTION OF THE MOTION OF SPINNING ELECTRON IN BOTH ELECTROMAGNETIC FIELD AND TWO-DIMENSIONAL HARMONIC OSCILLATOR POTENTIAL FIELD

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ABSTRACT

Quantum theory of double-wave function description (DWFD) is applied to describe the motion of spinning electron in both perpendicular uniform electromagnetic field and two-dimensional harmonic oscillator potential field. The time evolution equations of related mechanical quantities are derived. Furthermore, DWFD of spin-zero electron in the same fields is also given, and it is found that their classical limit results correspond to those of classical mechanics.

Keywords: double wave description, spinning electron, electromagnetic field, harmonic oscillator potential field

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