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互相垂直的均匀磁场和电场中一维带电谐振子的双波描述

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研究互相垂直的均匀磁场和电场中一维带电谐振子的量子单波及量子双波的描述.

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1 引 言

在均匀电磁场中带电粒子的经典运动是比较容易求解的(经典电动力学),即使是量子性的描述,有关文献[1—3]也作了较多的介绍. 考虑到谐振动广泛地存在于自然界,物理上任何体系在平衡位置附近的小振动如分子振动、晶格振动、原子核表面振动以及热辐射等原则上均可分解为若干一维简谐振动. 因此对谐振子的研究,无论理论上或应用上均有相当的意义. 自双波理论被发现以来,它已被广泛地应用于解决量子力学许多束缚态和非束缚态的问题并取得了相当的成功[4—9]. 因此,作者曾应用双波理论讨论过多维各向同性及各向异性谐振子的双波描述[10]及在均匀磁场中三维带电谐振子的双波描述(该文待发表). 本文着重讨论在相互垂直的均匀磁场和电场中一维带电谐振子的双波描述.

2 经典结果

设一维谐振子 $V(x)=(1/2)\mu\omega^2x^2$ 带电量 q, 均匀磁场 $\boldsymbol{B}(0,0,B)$ 沿 z 方向,均匀电场 $\boldsymbol{E}(E,0,0)$ 沿 x 方向. 取矢势 $\boldsymbol{A}(0,Bx,0)$,则经典哈密顿量为

$$H=\frac{1}{2\mu}\left[p_x^2+\left(p_y-\frac{qB}{c}x\right)^2+p_z^2\right]-qEx+\frac{1}{2}\mu\omega^2x^2,$$

正则方程

$$\dot{x}=\frac{p_x}{\mu},\quad \dot{p}_x=qE-\mu\omega^2+\frac{qB}{c\mu}\left(P_y-\frac{qBx}{c}\right),$$

$$\dot{y}=\frac{p_y}{\mu}-\frac{qB}{c\mu}x,\quad \dot{p}_y=0,\quad \dot{z}=\frac{p_z}{\mu},\quad \dot{p}_z=0.$$

经典解为

$$x=x_0+A\cos\omega'(t-t_0),$$

$$p_x=-\mu A\omega'\sin\omega'(t-t_0),$$

$$y=y_0+\left(\frac{p_y^0}{\mu}-\frac{qB}{c\mu}x_0\right)t-\frac{qBA}{c\mu\omega'}\sin\omega'(t-t_0),$$

$$p_y=p_y^0,\quad z=z_0+\frac{p_z^0}{\mu}t,\quad p_z=p_z^0.$$

其中, x_0,ω' 意义同下, y_0,z_0,t_0 为某种初始值.

3 量子力学单波描述

量子哈密顿量

$$\hat{H}=\frac{1}{2\mu}\left[\hat{p}_x^2+\left(\hat{p}_y-\frac{qB}{c}x\right)^2+\hat{p}_z^2\right]-qEx+\frac{1}{2}\mu\omega^2x^2,$$

$[\hat{p}_y,\hat{H}]=[\hat{p}_z,\hat{H}]=0$, 力学量 p_y,p_z 为守恒量. 取 $\hat{H},\hat{p}_y,\hat{p}_z$ 的共同本征函数 $\Psi(x,y,z)=\frac{1}{2\pi\hbar}e^{i/\hbar(p_yy+p_zz)}\varphi(x)$, $\varphi(x)$ 满足方程

$$\frac{1}{2\mu}\left[\hat{p}_x^2+\left(p_y-\frac{qB}{c}x\right)^2+p_z^2\right]\varphi(x)+\left(\frac{1}{2}\mu\omega^2x^2-qEx\right)\cdot\varphi(x)=E\varphi(x),$$

可求得量子单波解

$$\Psi_{n\cdot p_y\cdot p_z}(x,y,z)=\frac{1}{2\pi\hbar}e^{(i/\hbar)(p_yy+p_zz)}\varphi_{n\cdot p_y\cdot p_z}(x-x_0),$$

$$\varphi_{n\cdot p_y\cdot p_z}(x-x_0)=N_ne^{(-1/2)\alpha^2(x-x_0)^2}H_n[\alpha(x-x_0)],$$

$$E_{n\cdot p_y\cdot p_z}=\left(n+\frac{1}{2}\right)\hbar\omega'+rp_y^2-sp_y+\frac{p_z^2}{2\mu}-\Delta,$$
$$n=0,1,2,3,\cdots\qquad -\infty<p_y,p_z<+\infty,$$
$$\omega'=\left(\omega^2+\frac{q^2B^2}{\mu^2c^2}\right)^{1/2},\quad \alpha=\left(\frac{\mu\omega'}{\hbar}\right)^{1/2},$$
$$x_0=\frac{qBp_y+qE\mu c}{\mu^2\omega'^2c},\quad r=\frac{1}{2\mu}\left(1-\frac{q^2B^2}{\mu^2c^2\omega'^2}\right),$$
$$s=\frac{q^2BE}{\mu^2c\omega'^2},\quad \Delta=\frac{q^2E^2}{2\mu\omega'^2}.$$

4　量子力学双波描述

引入双波函数 $\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)$ 及 $\Psi(x,y,z,t)$,

$$\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)$$
$$=\frac{1}{2\pi\hbar}e^{(i/\hbar)[p_y^0(y-y_0)+p_z^0(z-z_0)]}\varphi_{n\cdot p_y^0\cdot p_z^0}(x-x_0)$$
$$\cdot\exp\left[-\frac{i}{\hbar}E_{n\cdot p_y^0\cdot p_z^0}(t-t_0)\right],$$
$$\Psi(x,y,z,t)$$
$$=\sum_{n'}\iint dp_y dp_z\frac{1}{2\pi\hbar}e^{(i/\hbar)[p_y(y-y_0)+p_z(z-z_0)]}$$
$$\cdot\varphi_{n'\cdot p_y\cdot p_z}(x-x_0)\exp\left[-\frac{i}{\hbar}E_{n'\cdot p_y\cdot p_z}(t-t_0)\right],$$

其中 y_0,z_0,t_0 为双波理论需要引入的三个实参数.

t 时刻任意力学量 $\hat{f}(t)$ 在状态 $\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)$ 中的测量值为

$$\langle f\rangle_{n\cdot p_y^0\cdot p_z^0}=\mathrm{Re}\int\Psi^*(x,y,z,t)\hat{f}\cdot\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)dxdydz.$$

1. $\hat{f}=1$ 归一化条件

$$\mathrm{Re}\int\Psi(x,y,z,t)\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)dxdydz=1.$$

2. $\hat{f}=\hat{H}$

$$\langle H\rangle_{n\cdot p_y^0\cdot p_z^0}=\mathrm{Re}\int\Psi^*(x,y,z,t)\hat{H}\cdot\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)dxdydz$$
$$=E_{n\cdot p_y^0\cdot p_z^0}\text{(能量本征值)}.$$

3. $\hat{f}=\hat{x}$

$$\langle x\rangle_{n\cdot p_y^0\cdot p_z^0}=\mathrm{Re}\int\Psi^*(x,y,z,t)x\cdot\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)dxdydz$$
$$=x_0+\frac{1}{\alpha}\left(\sqrt{\frac{n}{2}}+\sqrt{\frac{n+1}{2}}\right)\cos\omega'(t-t_0).$$

4. $\hat{f}=\hat{p}_x$

$$\langle p_x\rangle_{n\cdot p_y^0\cdot p_z^0}=-\frac{\omega'}{\alpha}\left(\sqrt{\frac{n}{2}}+\sqrt{\frac{n+1}{2}}\right)\sin\omega'(t-t_0).$$

可见,$\langle p_x\rangle_{n\cdot p_y^0\cdot p_z^0}=\mu\frac{d}{dt}\langle x\rangle_{n\cdot p_y^0\cdot p_z^0}$.

5. $\hat{f}=\hat{y}$

$$\langle y\rangle_{n\cdot p_y^0\cdot p_z^0}=\mathrm{Re}\int\Psi^*(x,y,z,t)y\cdot\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)dxdydz$$
$$=\mathrm{Re}\iiint dxdydz\sum_{n'}\iint dp_ydp_z\cdot\frac{1}{2\pi\hbar}e^{(-i/\hbar)[p_y(y-y_0)+p_z(z-z_0)]}$$
$$\cdot\varphi^*_{n'\cdot p_y\cdot p_z}(x-x_0)\exp\left[-\frac{i}{\hbar}E_{n'\cdot p_y\cdot p_z}(t-t_0)\right]$$
$$\cdot y\frac{1}{2\pi\hbar}e^{(i/\hbar)[p_y^0(y-y_0)+p_z^0(z-z_0)]}\varphi_{n\cdot p_y^0\cdot p_z^0}(x-x_0)$$
$$\cdot\exp\left[-\frac{i}{\hbar}E_{n\cdot p_y^0\cdot p_z^0}(t-t_0)\right]$$
$$=y_0-\frac{qB\hbar^{1/2}}{\mu^{3/2}\omega'^{3/2}c}\left[\sqrt{\frac{n+1}{2}}+\sqrt{\frac{n}{2}}\right]\cdot\sin\omega'(t-t_0)+(2rp_y^0-s)(t-t_0).$$

6. $\hat{f}=\hat{p}_y$

$$\langle p_y\rangle_{n\cdot p_y^0\cdot p_z^0}=p_y^0\neq\mu\frac{d}{dt}\langle y\rangle_{n\cdot p_y^0\cdot p_z^0}.$$

这是因为正则动量 $\hat{p}_y$ 与机械动量不同的缘故.

7. $\hat{f}=\hat{z}$

$$\langle z\rangle_{n\cdot p_y^0\cdot p_z^0}=z_0+\frac{p_z^0}{\mu}(t-t_0).$$

8. $\hat{f}=\hat{p}_z$

$$\langle p_z\rangle_{n\cdot p_y^0\cdot p_z^0}=p_z^0=\mu\frac{d}{dt}\langle z\rangle_{n\cdot p_y^0\cdot p_z^0}.$$

5　双波函数描述结果的经典极限

取 $n\to\infty,\hbar\to0,n\hbar\omega'\to(1/2)\mu\omega'^2A^2$,

$$\langle x\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle x\rangle_{c\cdot L},\quad\langle p_x\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle p_x\rangle_{c\cdot L},$$
$$\langle y\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle y\rangle_{c\cdot L},\quad\langle p_y\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle p_y\rangle_{c\cdot L},$$
$$\langle z\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle z\rangle_{c\cdot L},\quad\langle p_z\rangle_{n\cdot p_y^0\cdot p_z^0}\to\langle p_z\rangle_{c\cdot L},$$
$$\langle x\rangle_{c\cdot L}=x_0+A\cos\omega'(t-t_0),$$
$$\langle p_x\rangle_{c\cdot L}=-\mu A\omega'\sin\omega'(t-t_0),$$
$$\langle p_x\rangle_{c\cdot L}=\mu\frac{d}{dt}\langle x\rangle_{c\cdot L},$$

$$\langle y\rangle_{c\cdot L} = y_0 + (2rp_y^0 - s)(t - t_0) - \frac{qBA}{c\mu\omega'}\sin\omega'(t - t_0),$$

$$\langle p_y\rangle_{c\cdot L} = p_y^0 \neq \mu\frac{\mathrm{d}}{\mathrm{d}t}\langle y\rangle_{c\cdot L},$$

$$\langle z\rangle_{c\cdot L} = z_0 + \frac{p_z^0}{\mu}(t - t_0),$$

$$\langle p_z\rangle_{c\cdot L} = p_z^0 = \mu\frac{\mathrm{d}}{\mathrm{d}t}\langle z\rangle_{c\cdot L}.$$

由此可见,双波描述的经典极限结果与纯经典结果形式上完全相同.

6 系综平均

定义系综平均值

$$\overline{\langle f\rangle_{n\cdot p_y^0\cdot p_z^0}} = \lim_{L\to\infty}\frac{1}{(2L)^2}\int_{-L}^{L}\mathrm{d}y_0\int_{-L}^{L}\mathrm{d}z_0 \cdot \frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}(\omega t_0)\langle f\rangle_{n\cdot p_y^0\cdot p_z^0},$$

$$\langle f\rangle_{n\cdot p_y^0\cdot p_z^0} = \mathrm{Re}\int\Psi^*(x,y,z,t)\hat{f} \cdot \Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t)\mathrm{d}x\mathrm{d}y\mathrm{d}z,$$

$$\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z,t) = \frac{1}{2\pi\hbar}\mathrm{e}^{(\mathrm{i}/\hbar)[p_y^0(y-y_0)+p_z^0(z-z_0)]}\varphi_{n\cdot p_y^0\cdot p_z^0}(x - x_0) \cdot \exp\left[-\frac{\mathrm{i}}{\hbar}E_{n\cdot p_y^0\cdot p_z^0}(t - t_0)\right],$$

$$\Psi(x,y,z,t) = \sum_n\iint\mathrm{d}p_y\mathrm{d}p_z \cdot \frac{1}{2\pi\hbar}\mathrm{e}^{(\mathrm{i}/\hbar)[p_y(y-y_0)+p_z(z-z_0)]}\varphi_{n'\cdot p_y\cdot p_z} \cdot \exp\left[-\frac{\mathrm{i}}{\hbar}E_{n'\cdot p_y\cdot p_z}(t - t_0)\right].$$

可以证明

$$\overline{\langle f\rangle_{n\cdot p_y^0\cdot p_z^0}} = \lim_{L\to\infty}\frac{1}{(2L)^2}\int_{-L}^{L}\mathrm{d}y\int_{-L}^{L}\mathrm{d}z\int_{-\infty}^{+\infty}\mathrm{d}x \cdot \mathrm{e}^{-(\mathrm{i}/\hbar)(p_y^0y+p_z^0z)}\varphi^*_{n\cdot p_y^0\cdot p_z^0}(x - x_0) \cdot \hat{f}\mathrm{e}^{(\mathrm{i}/\hbar)(p_y^0y+p_z^0z)}\varphi_{n\cdot p_y^0\cdot p_z^0}(x - x_0)$$

$$= \lim_{L\to\infty}\int_{-L}^{L}\mathrm{d}y\int_{-L}^{L}\mathrm{d}z\int_{-\infty}^{+\infty}\mathrm{d}x \cdot \frac{(2\pi\hbar)^2}{(2L)^2}\Psi^*_{n\cdot p_y^0\cdot p_z^0}(x,y,z)\hat{f} \cdot \Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z).$$

等式右边正是普通量子力学单波描述求力学量 $\hat{f}$ 的平均值公式,这式子对任一力学量 $\hat{f}$ 均成立,说明量子力学单波 $\Psi_{n\cdot p_y^0\cdot p_z^0}(x,y,z)$ 不是描述单个带电谐振子而是描述一个系综,这系综中各带电谐振子具有相同的能量 $E_{n\cdot p_y^0\cdot p_z^0}$ 和动量 p_y^0,p_z^0,但初始值 y_0,z_0,t_0 各不相同.

7 结 论

1. 对互相垂直的均匀磁场和电场中一维带电谐振子求得通常量子描述的单波解(能量及波函数).

2. 应用双波理论描述此带电谐振子,得出了描述单个带电谐振子的运动行为的力学量随时间的演化方程.

3. 双波解的经典极限形式上与纯经典结果完全一致.

4. 定义系综平均值,比较通常量子力学单波解和双波解的不同意义,即单波解描述一个系综,双波解能够描述单个谐振子的运动.

本文讨论的方法同样适用于在相互垂直的均匀磁场和均匀电场中多维带电谐振子的双波描述(另文讨论).

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DOUBLE-WAVE DESCRIPTION OF THE MOTION OF ONE-DIMENSIONAL CHARGED HARMONIC OSCILLATOR IN THE PERPENDICULAR UNIFORM MAGNETIC FIELD AND UNIFORM ELECTRIC FIELD

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ABSTRACT

Double-wave function quantum theory is applied to the description of the motion of one-dimensional charged harmonic oscillator in the uniform magnetic field and uniform electric field which are perpendicular to each other, from which quantum results and classical limit results are derived respectively. A comparison between classical limit results and those of classical mechanics is made.

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