第 49 卷 第 6 期 2000 年 6 月
1000-3290/2000/49(06)/1023-05

物 理 学 报
ACTA PHYSICA SINICA

Vol. 49, No. 6, June, 2000

相对论性 Birkhoff 系统的 Lie 对称性和守恒量*

傅景礼 王新民

(商丘师范学院物理系,商丘 476000)

(1999 年 8 月 15 日收到)

给出相对论性 Birkhoff 系统的 Pfaff-Birkhoff 原理和 Birkhoff 方程. 由微分方程在无限小变换下的不变性,定义相对论性 Birkhoff 系统无限小变换生成元,建立 Lie 对称的确定方程,得到结构方程和守恒量. 并研究该系统的 Lie 对称性逆问题. 给出实例以说明结果的应用.

PACC:0316;0412

1 引 言

1918 年德国数学家 Noether,指出了动力学系统的对称性和守恒量二者之间的紧密联系[1]. 从此,对称性和守恒量的研究在数学、物理、力学等学科变得非常重要. 力学系统的对称性理论包括 Noether 对称性和 Lie 对称性. 关于 Noether 对称性和守恒量的研究已趋于完善[2—8]. 自 70 年代末 Lutzky 等人将上世纪末数学家 Lie 研究微分方程不变性的扩展群方法引入力学领域,提出微分方程不变的 Lie 对称性[9,10]. 20 年来 Lie 对称方法迅速发展,并取得一些结果[11—14]. 本文将 Lie 对称方法应用到高速运动系统中. 研究相对论性 Birkhoff 系统的 Lie 对称性和守恒量问题:一是由系统的 Lie 对称性找到守恒量;二是由系统的守恒量找到 Lie 对称性.

2 相对论性 Pfaff-Birkhoff 原理与 Birkhoff 方程

积分(本文约定重复角码表示求和)

$$A = \int_{t_1}^{t_2} \{R_\nu(t,\boldsymbol{a})\dot{a}^\nu - B(t,\boldsymbol{a})\}\mathrm{d}t \qquad (1)$$

称为 Pfaff 作用量, $B(t,\boldsymbol{a})$ 称为 Birkhoff 函数, $R_\nu(t,\boldsymbol{a})$ $(\nu=1,\cdots,2n)$ 称为 Birkhoff 函数组,则

$$\delta A = 0, \qquad (2)$$

$$\mathrm{d}\delta a^\nu = \delta \mathrm{d}a^\nu \qquad (\nu = 1,\cdots,2n), \qquad (3)$$

$$\delta a^\nu|_{t=t_1} = \delta a^\nu|_{t=t_2} = 0$$

$$(\nu = 1,\cdots,2n), \qquad (4)$$

称为 Pfaff-Birkhoff 原理[15]. 该原理是一个普遍的一阶积分变分原理.

定义相对论性的 Pfaff 作用量

$$A^* = \int_{t_1}^{t_2}\Big\{R_\nu^*(m_i(t,\boldsymbol{a}),t,\boldsymbol{a})\dot{a}^\nu - B^*(m_i(t,\boldsymbol{a}),t,\boldsymbol{a})\Big\}\mathrm{d}t, \qquad (5)$$

式中 B^* 为相对论性 Birkhoff 函数, R_ν^* $(\nu=1,\cdots,2n)$ 为相对论性 Birkhoff 函数组, B^*, R_ν^* 含有相对论性质量

$$m_i = m_{oi}/\sqrt{1-\dot{\boldsymbol{r}}_i^2/c^2}, \qquad (6)$$

m_{oi} 为第 i 个粒子的静止质量, $\boldsymbol{r}_i$ 为第 i 个粒子的位矢, c 为光速.

对(5)式取变分,并考虑(6)式,得

$$\delta A^* = \int_{t_1}^{t_2}\Big\{\Big(\frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial R_\nu^*}{\partial a^\mu} - \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\nu} - \frac{\partial R_\mu^*}{\partial a^\nu}\Big)\dot{a}^\nu - \Big(\frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial B^*}{\partial a^\mu} + \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial R_\mu^*}{\partial t}\Big)\Big\}\delta a^\mu \mathrm{d}t = 0$$

$$(\nu,\mu = 1,\cdots,2n), \qquad (7)$$

(7)式称为相对论性 Pfaff-Birkhoff 原理. 令

$$\widetilde{B}^* = \widetilde{B}^*(t,\boldsymbol{a}) = B^*(m(t,\boldsymbol{a}),t,\boldsymbol{a}), \qquad (8)$$

---

* 国家自然科学基金(批准号:19972010)和河南省自然科学基金(批准号:984053100)资助的课题.

$$\widetilde{R}_\nu^* = \widetilde{R}_\nu^*(t,a) = R_\nu^*(m(t,a),t,a), \quad (9)$$

则有

$$\frac{\partial \widetilde{B}^*}{\partial a^\mu} = \frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial B^*}{\partial a^\mu},$$
$$\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} = \frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial R_\nu^*}{\partial a^\mu}, \quad (10)$$
$$\frac{\partial \widetilde{R}_\nu^*}{\partial t} = \frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial R_\nu^*}{\partial t}.$$

那么原理(7)式可表为

$$\delta A^* = \int_{t_1}^{t_2}\left\{\left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right)\dot{a}^\nu - \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial i}\right)\right\}\delta a^\mu \mathrm{d}t = 0$$
$$(\nu,\mu = 1,\cdots,2n). \quad (11)$$

由积分区间$[t_1,t_2]$的任意性,得到

$$\left\{\left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right)\dot{a}^\nu - \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right)\right\}\delta a^\mu = 0 \quad (\nu,\mu = 1,\cdots,2n). \quad (12)$$

如果 $\delta a^\mu(\mu=1,\cdots,2n)$是彼此独立的,由原理(12)式可得到相对论性 Birkhoff 方程

$$\Omega_{\mu\nu}^*\dot{a}^\nu - \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right) = 0$$
$$(\nu,\mu = 1,\cdots,2n), \quad (13)$$
$$\Omega_{\mu\nu}^* = \left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right).$$

(13)式可写成

$$\dot{a}^\mu = \Omega^{*\mu\nu}\left(\frac{\partial \widetilde{B}^*}{\partial a^\nu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right)$$
$$(\nu,\mu = 1,\cdots,2n), \quad (14)$$
$$\Omega^{*\mu\nu} = \left(\left\|\frac{\partial \widetilde{R}_\beta^*}{\partial a^\alpha} - \frac{\partial \widetilde{R}_\alpha^*}{\partial a^\beta}\right\|^{-1}\right)^{\mu\nu}$$
$$(\alpha,\beta = 1,\cdots,2n),$$

称 $\Omega^{*\mu\nu}$为 Birkhoff 逆变张量. 一般假设

$$\det(\Omega_{\mu\nu}^*) \neq 0. \quad (15)$$

3 Lie 对称的正问题

引入无限小变换

$$t^* = t + \Delta t, q_s^* = q_s + \Delta q_s \quad (16)$$

或写成

$$t^* = t + \varepsilon f(t,a),$$
$$a^{*\mu} = a^\mu + \varepsilon F\mu(t,a), \quad (17)$$

式中 ε 为小参数. 引入无限小变换的生成元向量

$$X^{(0)} = f\frac{\partial}{\partial t} + F_\mu\frac{\partial}{\partial a^\mu} \quad (18)$$

以及它的一次扩展

$$X^{(1)} = f\frac{\partial}{\partial t} + F_\mu\frac{\partial}{\partial a^\mu} + (\dot{F}_\mu - \dot{a}^\mu\dot{f})\frac{\partial}{\partial \dot{a}^\mu}. \quad (19)$$

由微分方程在无限小变换下的不变性,方程(14)在无限小变换(17)下的不变性归为

$$X^{(1)}\left[\dot{a}^\mu - \Omega^{*\mu\nu}\left(\frac{\partial \widetilde{B}^*}{\partial a^\nu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right)\right] = 0. \quad (20)$$

如果

$$\dot{a}^\mu = \Omega^{*\mu\nu}\left(\frac{\partial \widetilde{B}^*}{\partial a^\nu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right),$$

则可得到相对论性 Birkhoff 系统的确定方程

$$\dot{F}_\mu - \Omega^{*\mu\nu}\left(\frac{\partial \widetilde{B}^*}{\partial a^\nu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right)\dot{f} = X^{(0)}\left[\Omega^{*\mu\nu}\left(\frac{\partial \widetilde{B}^*}{\partial a^\nu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right)\right]$$
$$(\nu,\mu = 1,\cdots,2n). \quad (21)$$

如果无限小变换(17)的生成元 $f,F_\mu(\mu=1,\cdots,2n)$满足确定方程(21)式,就称变换(17)是 Lie 对称的. 那么,有

定理 1 对于满足确定方程(21)的元限小变换生成元 f,F_μ,如果存在满足

$$X^{(1)}(\widetilde{R}_\mu^*\dot{a}^\mu - \widetilde{B}^*) + (\widetilde{R}_\mu^*\dot{a}^\mu - \widetilde{B}^*)\dot{f} + \dot{G} = 0 \quad (22)$$

的规范函数 G,那么相对论性 Birkhoff 系统存在如下守恒量

$$I = \widetilde{R}_\mu^*F_\mu - \widetilde{B}^*f + G = \mathrm{const}. \quad (23)$$

证明

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \dot{\widetilde{R}}_\mu^*F_\mu + \widetilde{R}_\mu^*\dot{F}_\mu - \dot{\widetilde{B}}^*f - \widetilde{B}^*\dot{f} - X^{(1)}(\widetilde{R}_\mu^*\dot{a}^\mu - \widetilde{B}^*) - (\widetilde{R}_\mu^*\dot{a}^\mu - \widetilde{B}^*)\dot{f}$$
$$= \left[\left(\frac{\partial R_\mu^*}{\partial a^\nu} + \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\nu} - \frac{\partial R_\nu^*}{\partial a^\mu} - \frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu}\right)\dot{a}^\nu + \left(\frac{\partial B^*}{\partial t} + \frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial B^*}{\partial a^\mu} + \frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu}\right)\right]F_\mu$$
$$- \left(\frac{\partial B^*}{\partial a^\mu} + \frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial R_\mu^*}{\partial t} + \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial t}\right)\dot{a}^\nu f$$

$$= \left[\left(\frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu} - \frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu}\right)\dot{a}^\nu + \frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right]F_\mu - \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right)\dot{a}^\nu f.$$

利用(13)式,有

$$\left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right)\dot{a}^\mu = \left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right)\dot{a}^\nu\dot{a}^\mu = 0,$$

故有

$$\frac{\mathrm{d}I}{\mathrm{d}t} = 0.$$

求解相对论性 Birkhoff 系统 Lie 对称性正问题步骤为:先将给出的相对论性 Birkhoff 方程代入确定方程(21),求得生成元 f,F_μ;再将生成元 f,F_μ 代入结构方程(22)得到 $\dot{G}$,如果 $\dot{G}=0$,或 $\dot{G}$是某个函数的全微分,就可求得规范函数 G;最后将生成元 f,F_μ 和规范函数 G 代入(23)式得到该系统的 Lie 对称守恒量.

4　Lie 对称性逆问题

假定相对论性 Birkhoff 系统有积分

$$I = I(m_i(t,\boldsymbol{a}),t,\boldsymbol{a}) = \text{const}, \tag{24}$$

式中　m_i 为相对论性质量. 试由该积分找出相应的 Lie 对称变换.

(24)式对 t 求导数得

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\partial I}{\partial t} + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial I}{\partial a^\nu}\dot{a}^\nu + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial a^\nu}\dot{a}^\nu = 0 \quad (\nu = 1,\cdots,2n). \tag{25}$$

将相对论性 Birkhoff 方程(13)乘以 $\overline{F}_\mu(=F_\mu - a^\mu f)$并对 μ 求和,再将结果与(25)式相加,得到

$$\frac{\partial I}{\partial t} + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial I}{\partial a^\mu}\dot{a}^\mu + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial a^\mu}\dot{a}^\mu + \left[\left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right)\dot{a}^\nu - \frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right]\overline{F}_\mu = 0. \tag{26}$$

由(26)式中 $\dot{a}^\nu$ 的系数为零得到

$$\frac{\partial I}{\partial a^\nu} + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial a^\nu} + \left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right)F_\mu + \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\nu^*}{\partial t}\right)f = 0. \tag{27}$$

令

$$\det(\Omega_{\mu\nu}^*) = \det\left(\frac{\partial \widetilde{R}_\nu^*}{\partial a^\mu} - \frac{\partial \widetilde{R}_\mu^*}{\partial a^\nu}\right) \neq 0, \tag{28}$$

由(27)式得到

$$F_\mu = -\Omega^{*\mu\nu}\left[\frac{\partial I}{\partial a^\nu} + \frac{\partial I}{\partial m_i}\frac{\partial m_i}{\partial a^\nu} + \left(\frac{\partial \widetilde{B}^*}{\partial a^\mu} + \frac{\partial \widetilde{R}_\mu^*}{\partial t}\right)f\right], \tag{29}$$

其中

$$\Omega^{*\mu\nu}\cdot\Omega_{\nu\rho}^* = \delta_{\mu\rho}, \tag{30}$$

$\frac{\partial \widetilde{B}^*}{\partial a^\mu}$,$\frac{\partial \widetilde{R}_\mu^*}{\partial t}$由(10)式给出. 令积分(24)等于守恒量(23),即

$$I(m(t,\boldsymbol{a}),t,\boldsymbol{a}) = \widetilde{R}_\mu^* F_\mu - \widetilde{B}^* f + G, \tag{31}$$

那么,若已知系统的第一积分,且选定具体的规范函数 G 后,由(29),(31)式即可确定无限小变换的生成元 f,F_μ,它们对应相对论性 Birkhoff 系统的 Noether 对称变换.

将求得的生成元 f,F_μ 代入确定方程(21),若满足,此变换就是该系统的 Lie 对称变换. 否则不是该系统的 Lie 对称变换. 于是有

定理 2　如果已知相对论性 Birkhoff 系统(13)式的 r 个独立的第一积分,由(29),(31)式可求得该系统的无限小变换生成元 f,F_μ. 若 f,F_μ 满足确定方程(21),那么,该变换是系统(13)与积分(24)对应的 Lie 对称变换. 否则不是 Lie 对称变换(证明略).

5　算　　例

例 1　相对论性二阶 Birkhoff 系统的 Birkhoff 函数和 Birkhoff 函数组为

$$B^* = -\left(\frac{m}{Q}c^2 + a^1\right),$$

$$R_1^* = 0, \tag{32}$$

$$R_2^* = \left(\frac{m}{Q}c^2 + a^1\right)\frac{m}{Q}\Big/\left(1 - \frac{(a^2)^2}{c^2}\right),$$

式中 Q 为常力,c 为光速,a 为广义坐标,$m = m_0/\sqrt{1-(a^2)^2/c^2}$,试研究系统的 Lie 对称性和守恒量.

由(32)式可求得

$$(\Omega_{\mu\nu}^*) = \frac{m_0}{Q}\left(1 - \frac{(a^2)^2}{c^2}\right)^{-\frac{3}{2}}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{33}$$

那么

$$(\Omega^{*\mu\nu}) = \frac{Q}{m_0}\left(1-\frac{(a^2)^2}{c^2}\right)^{\frac{3}{2}}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}. \quad (34)$$

将(32),(34)式代入确定方程(21)得

$$\dot{F}_1 - a^2\dot{f} = F_2, \quad (35)$$

$$\dot{F}_2 + \frac{Q}{m_0}\left(1-\frac{(a^2)^2}{c^2}\right)^{\frac{3}{2}}\dot{f} = \frac{3Qa^2}{m_0c^2}\left(1-\frac{(a^2)^2}{c^2}\right)^{\frac{1}{2}}, \quad (36)$$

方程有解

$$f = 1, F_1 = 0, F_2 = 0, \quad (37)$$

$$f = 0, F_1 = a^2,$$

$$F_2 = -\frac{Q}{m_0}\left(1-\frac{(a^2)}{c^2}\right)^{\frac{3}{2}}, \quad (38)$$

将(37)式代入(23)式得到守恒量

$$I = \frac{m}{Q}c^2 + a^1 = \text{const}. \quad (39)$$

将(38)式代入(23)式得到守恒量

$$I = \frac{m}{Q}c^2 + a^1 = \text{const}.$$

例 2　相对论性二阶 Birkhoff 系统的 Birkhoff 函数和 Birkhoff 函数组为(32)式,已知系统有积分(39),试求与之对应的 Lie 对称变换.

因 $\Omega^*_{\mu\nu}$,$\Omega^{*\mu\nu}$可分别由(33),(34)式给出,对积分(39),由(29),(31)式给出为

$$F_1 = a^2(1-f), \quad (40)$$

$$F_2 = -\frac{Q}{m_0}\left(1-\frac{(a^2)^2}{c^2}\right)^{\frac{3}{2}}(1-f), \quad (41)$$

$$F_2\left(\frac{m}{Q}c^2 + a^1\right)\frac{m}{Q}\Big/\left(1-\frac{(a^2)}{c^2}\right) + \left(\frac{m}{Q}c^2 + a^1\right)f + G = \frac{m}{Q}c^2 + a^1. \quad (42)$$

取 $G=0$,由(40),(41),(42)式解得

$$f = 1, F_1 = 0, F_2 = 0,$$

$$f = 0, F_1 = a^2,$$

$$F_2 = \frac{-Q}{m_0}\left(1-\frac{(a^2)^2}{c^2}\right)^{\frac{3}{2}}.$$

二解与(37),(38)式相同,满足确定方程,都是与已知守恒量对应的 Lie 对称变换.

感谢北京理工大学梅凤翔教授的指导与帮助.

[1] A. E. Noether, *Mathematisch-Physicalishe Klasse*, **2**(1918), 235.

[2] Z. P. Li, *Acta Physica Sinica*, **30**(1981), 1659(in Chinese)[李子平,物理学报,**30**(1981),1659].

[3] [梅凤翔、刘端、罗勇,高等分析力学,北京理工大学出版社,北京,1991].

[4] Zi-ping Li, Classical and Quantal Dynamics of Constrained Systems and Their Symmetrical Properties (Beijing Polytechnic University Press, Beijing, 1993)(in Chinese))[李子平,经典和量子约束系统及其对称性质,北京工业大学出版社,北京,1993)].

[5] D. Liu, *Acta Mechanica Sinica*, **21**(1989), 75(in Chinese)[刘端,力学学报,**21**(1989),75].

[6] D. Liu, *Science in China* (*Series A*), **20**(11)(1990), 1189(in Chinese)[刘端,中国科学,A 辑,**20**(11)(1990),1189].

[7] F. X. Mei, *Science in China* (*Series A*), **23**(7)(1993), 709(in Chinese)[梅凤翔,中国科学,A 辑,**23**(7)(1993),709].

[8] Feng-xiang Mei, Rong-chang Shi, Yong-fa Zhang, Hui-bin Wu, Dynamics of Birkhoffian System (Beijing Institute of Technology Press, Beijing, 1996)(in Chinese)[梅凤翔、史荣昌、张永发、吴惠彬,Birkhoff 系统动力学(北京理工大学出版社,北京,1996)].

[9] M. Lutzky, *J. Phys. A: Math. Gen.*, **12**(1979), 973.

[10] G. W. Bluman, S. Kumei, Symmetries and differential equations, Spring-Verlag, New York, 1989.

[11] Y, Y. Zhao, F. X. Mei, *Advances in Machanics*, **23**(3)(1993), 360(in Chinese)[赵跃宇、梅凤翔,力学进展,**23**(3)(1993),360].

[12] J. L. Fu, R. W. Liu, F. X. Mei, *Journal of Beijing Institute of Technology*, **7**(3)(1998), 215.

[13] F. X. Mei, *Applied Mathematics and Mechanics*, **20**(6)(1999), 629.

[14] R. W. Liu, J. L. Fu, *Applied Mathematics and Mechanics*, **20**(6)(1999), 635.

[15] R. M. Santilli, Foundations of theoretical mechanics Ⅱ, Springer-Verlag, New York, 1983.

LIE SYMMETRIES AND CONSERVED QUANTITIES OF RELATIVISTIC BIRKHOFF SYSTEMS*

FU JING-LI　WANG XIN-MIN
(*Shangqiu Teachers College*, *Shangqiu* 476000, *China*)
(Received 15 August 1999)

ABSTRACT

The Pfaff-Birkhoff principle and the Birkhoff equations for relativistic Birkhoff systems are given. The definition of an infinitesimal generator for the relativistic Birkhoff systems is given; by using the invariance of the ordinary differential equations under the infinitesimal transformations, the determining equations of Lie symmetries for the systems are established; the structure equation and the conserved quantities are obtained. And the inverse problem of Lie symmetries of the systems is also studied. Two examples are given to illustrate the application of the result.

PACC: 0316; 0412

* Project supported by the National Natural Science Foundation of Chinese (Grant No. 19972010), and the Natural Science Foundation of Henan Province (Grant No. 984053100).