第 49 卷 第 6 期 2000 年 6 月
1000-3290/2000/49(06)/1035-04

物 理 学 报
ACTA PHYSICA SINICA

Vol. 49, No. 6, June, 2000

轴对称荷电动态黑洞的量子热效应

卢卯旺

(零陵师范专科学校物理系,湖南 425006)

(1999 年 10 月 15 日收到;1999 年 11 月 30 日收到修改稿)

讨论了轴对称荷电动态黑洞的 Hawking 辐射,得到了局部事件视界面方程和温度. 结果显示黑洞的形状和温度不仅随时间变化而且随角度变化.

PACC:0420;9760L

黑洞热辐射是天体物理中一种重要的量子效应[1],近年来引起了人们的关注[2—6]. 最近,荆继良等人获得了一个轴对称荷电动态黑洞解[7],其度规为

$$\begin{aligned} \mathrm{d}s^2 = {} & [1-(2mr-Q^2)\rho\bar{\rho}]\mathrm{d}\nu^2 - 2\mathrm{d}\nu\mathrm{d}r \\ & + 2a(2mr-Q^2)\rho\bar{\rho}\sin^2\theta\mathrm{d}\nu\mathrm{d}\varphi \\ & + 2a\sin^2\theta\mathrm{d}r\mathrm{d}\varphi - \frac{1}{\rho\bar{\rho}}\mathrm{d}\theta^2 - \Big[(2mr \\ & - Q^2)a^2\rho\bar{\rho} + \frac{r^2+a^2}{\sin^2\theta}\Big]\sin^4\theta\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

其中 ν 为超前爱丁顿坐标,质量 $m=m(\nu)$,电荷 $Q=Q(\nu)$,$\rho=-(r-\mathrm{i}a\cos\theta)^{-1}$,$a$ 为比角动量常数. 文献[8]讨论了此黑洞的特征曲面,本文以此黑洞为时空前景,研究该时空中标量粒子的动力学行为,按照赵峥的方法考察该黑洞的 Hawking 效应. 容易得出度规(1)式的行列式及不为零的逆变分量

$$g = -(r^2+a^2\cos^2\theta)^2\sin^2\theta; \tag{2}$$

$$\begin{aligned} & g^{00} = -a^2\rho\bar{\rho}\sin^2\theta, \\ & g^{01} = -(r^2+a^2)\rho\bar{\rho}, \\ & g^{02} = g^{13} = -a\rho\bar{\rho}, \\ & g^{11} = \rho\bar{\rho}(2mr-Q^2-r^2-a^2), \\ & g^{22} = -\rho\bar{\rho}, g^{33} = -\frac{\rho\bar{\rho}}{\sin^2\theta}. \end{aligned} \tag{3}$$

现在,用零曲面条件[5] $g^{\mu\nu}\frac{\partial F}{\partial x^\mu}\frac{\partial F}{\partial x^\nu}=0$,来寻找上述时空的局部事件视界,式中 $F=F(\nu,r,\theta,\varphi)=0$ 为零曲面方程,将(3)式代入得

$$\begin{aligned} & a^2\rho\bar{\rho}\sin^2\theta\dot{r}_{\mathrm{H}}^2 - 2(r^2+a^2)\rho\bar{\rho}\dot{r}_{\mathrm{H}} - [2mr-Q^2 \\ & -(r^2+a^2)]\rho\bar{\rho} + \rho\bar{\rho}r_{\mathrm{H}}'^2 = 0, \end{aligned} \tag{4}$$

这里利用了时空的轴对称性,即 $F(\nu,r,\theta)=0$,$\frac{\partial F}{\partial\varphi}=0$,并记 $\dot{r}_{\mathrm{H}}=\left(\frac{\partial r}{\partial\nu}\right)_{r=r_{\mathrm{H}}}$,$r_{\mathrm{H}}'=\left(\frac{\partial r}{\partial\theta}\right)_{r=r_{\mathrm{H}}}$.

下面我们考查此时空 Klein-Gordon 粒子的运动,K-G 方程为

$$\frac{1}{\sqrt{-g}}\left[\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\Phi\right)\right] - \mu^2\Phi = 0, \tag{5}$$

μ 是 K-G 粒子的质量. 把(2)和(3)式代入(5)式展开得到

$$\begin{aligned} & -a^2\sin^2\theta\frac{\partial^2\Phi}{\partial\nu^2} + [2mr-Q^2-(r^2+a^2)]\frac{\partial^2\Phi}{\partial r^2} \\ & -\frac{\partial^2\Phi}{\partial\theta^2} - \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\varphi^2} - 2(r^2+a^2)\frac{\partial^2\Phi}{\partial\nu\partial r} \\ & -2a\frac{\partial^2\Phi}{\partial\nu\partial\varphi} - 2a\frac{\partial^2\Phi}{\partial r\partial\varphi} - 2r\frac{\partial\Phi}{\partial\nu} + 2(m-r)\frac{\partial\Phi}{\partial r} \\ & + \mathrm{ctg}\theta\frac{\partial\Phi}{\partial\theta} - \frac{\mu^2}{\rho\bar{\rho}}\Phi = 0. \end{aligned} \tag{6}$$

从(4)式可知,视界位置 r_{H} 不仅与 ν 有关,而且与方位角 θ 和角动量 a 有关,所以我们推广乌龟坐标变换[2]

$$\begin{aligned} & r_* = r + \frac{1}{2k(\nu_0,\theta_0)}\ln[r-r_{\mathrm{H}}(\nu,\theta)], \\ & \nu_* = \nu-\nu_0, \theta_* = \theta-\theta_0, \end{aligned} \tag{7}$$

式中 k 是温度函数,ν_0,θ_0 是固定的参数,即我们计算 $\nu=\nu_0$ 时刻,$\theta=\theta_0$ 方向上视界温度. 从(7)式可以得到

$$\begin{aligned} & \frac{\partial}{\partial\nu} = \frac{\partial}{\partial\nu_*} - \frac{\dot{r}_{\mathrm{H}}}{2k(r-r_{\mathrm{H}})}\frac{\partial}{\partial r_*}, \\ & \frac{\partial}{\partial r} = \left[1+\frac{1}{2k(r-r_{\mathrm{H}})}\right]\frac{\partial}{\partial r_*}, \\ & \frac{\partial}{\partial\theta} = \frac{\partial}{\partial\theta_*} - \frac{r_{\mathrm{H}}'}{2k(r-r_{\mathrm{H}})}\frac{\partial}{\partial r_*}, \end{aligned}$$

$$\frac{\partial^2}{\partial \nu^2}=\frac{\partial^2}{\partial \nu_*^2}-\frac{2\dot{r}_H}{2k(r-r_H)}\frac{\partial}{\partial \nu_*}\frac{\partial}{\partial r_*}+\left[\frac{\dot{r}_H}{2k(r-r_H)}\right]^2\frac{\partial^2}{\partial r_*^2}-\frac{r''_H(r-r_H)+r'^2_H}{2k(r-r_H)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial r^2}=\left[1+\frac{1}{2k(r-r_H)}\right]^2\frac{\partial^2}{\partial r_*^2}-\frac{1}{2k(r-r_H)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial \theta^2}=\frac{\partial^2}{\partial \theta_*^2}-\frac{2r'_H}{2k(r-r_H)}\frac{\partial^2}{\partial \theta_*\partial r_*}+\left[\frac{r'_H}{2k(r-r_H)}\right]^2\frac{\partial^2}{\partial r_*^2}-\frac{r''_H(r-r_H)+r'^2_H}{2k(r-r_H)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial \nu\partial r}=\left[1+\frac{1}{2k(r-r_H)}\right]\frac{\partial^2}{\partial \nu_*\partial r_*}-\frac{\dot{r}_H}{2k(r-r_H)}\left[1+\frac{1}{2k(r-r_H)}\right]\frac{\partial^2}{\partial r_*^2}+\frac{\dot{r}_H}{2k(r-r_H)}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial \nu\partial \varphi}=\frac{\partial^2}{\partial \nu_*\partial \varphi}-\frac{\dot{r}_H}{2k(r-r_H)}\frac{\partial}{\partial r_*}\frac{\partial}{\partial \varphi},$$

$$\frac{\partial^2}{\partial \gamma\partial \varphi}=\left[1+\frac{1}{2k(r-r_H)}\right]\frac{\partial^2}{\partial r_*\partial \varphi},\tag{8}$$

于是(6)式可化为

$$\begin{aligned}&-a^2\sin^2\theta\frac{\partial^2\Phi}{\partial \nu_*^2}+\Bigg\{-a^2\sin^2\theta\left[\frac{\dot{r}_H}{2k(r-r_H)}\right]^2+(2mr-Q^2-a^2-r^2)\left[1+\frac{1}{2k(r-r_H)}\right]^2\\&-\left[\frac{r'_H}{2k(r-r_H)}\right]^2+(r^2+a^2)\frac{\dot{r}_H}{k(r-r_H)}\cdot\left[1+\frac{1}{2k(r-r_H)}\right]\Bigg\}\frac{\partial^2\Phi}{\partial r_*^2}-\frac{\partial^2\Phi}{\partial \theta_*^2}\\&-\frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial \varphi^2}+\left\{\frac{a^2\sin^2\theta\dot{r}_H}{k(r-r_H)}-2(r^2+a^2)\left[1+\frac{1}{2k(r-r_H)}\right]\right\}\frac{\partial^2\Phi}{\partial \nu_*\partial r_*}\\&-2a^2\frac{\partial^2\Phi}{\partial \nu_*\partial \varphi}+\frac{r'_H}{k(r-r_H)}\frac{\partial^2\Phi}{\partial r_*\partial \theta_*}+\left\{\frac{a\dot{r}_H}{k(r-r_H)}-2a\left[1+\frac{1}{2k(r-r_H)}\right]\right\}\frac{\partial^2\Phi}{\partial r_*\partial \varphi}\\&-2r\frac{\partial \Phi}{\partial \nu_*}-(2r-\mathrm{ctg}\theta)\frac{\partial \Phi}{\partial \theta_*}+\Bigg\{a^2\sin^2\theta\left[\frac{\dot{r}_H^2+\ddot{r}_H(r-r_H)}{2k(r-r_H)}\right]\\&-\frac{1}{2k(r-r_H)^2}[2mr-Q^2-(r^2+a^2)]+\frac{r''_H(r-r_H)+r'^2_H}{2k(r-r_H)^2}-\frac{(r^2+a^2)\dot{r}_H}{k(r-r_H)^2}\\&+\frac{r\dot{r}_H}{k(r-r_H)}+2(m-r)\left[1+\frac{1}{2k(r-r_H)}\right]+\frac{r'_H\mathrm{ctg}\theta}{2k(r-r_H)}\Bigg\}\frac{\partial \Phi}{\partial r_*}-\frac{\mu^2}{\rho\bar{\rho}}\Phi=0.\end{aligned}\tag{9}$$

根据文献[3—6]中提出的方法,在视界附近,K-G 方程应具有典型的波动方程形式,我们可用 Damour 和 Ruffini[9] 的方法得到辐射谱. 为此我们首先令$\frac{\partial^2\Phi}{\partial \nu_*\partial r_*}$的系数为 -2,并讨论在视界附近的行为,即在 $\nu=\nu_o,\theta=\theta_o,r=r_H$ 处取值,此时$\frac{\partial^2\Phi}{\partial r_*^2}$的系数应为

$$\lim_{r\to r_H}\left\{\frac{a^2\sin^2\theta\dot{r}_H^2+r'^2_H-2\dot{r}_H(r^2+a^2)[1+2k(r-r_H)]-[2mr-Q^2-(r^2+a^2)][1+2k(r-r_H)]^2}{2k(r-r_H)\{a^2\sin^2\theta\dot{r}_H-(r^2+a^2)[1+2k(r-r_H)]\}}\right\}=-1,$$

由于上式分母在 $r\to r_H$ 时为零,所以分子在 $r\to r_H$ 时也趋于零,由此也可得到和前面相同的局部视界位置的方程(4),再用洛必达法则,可得温度函数 k,

$$k=\frac{2\dot{r}_Hr_H+m-r_H}{a^2\dot{r}_H\sin^2\theta+(r_H^2+a^2)(1-2\dot{r}_H)-2(2mr_H-Q^2)},\tag{10}$$

这样,在 $r\to r_H$ 的极限下,K-G 方程(9)可化为波动方程

$$\frac{\partial^2\Phi}{\partial r_*^2}+2\frac{\partial^2\Phi}{\partial \nu_*\partial r_*}+A(u_o,\theta_o)\frac{\partial^2\Phi}{\partial r_*\partial \theta_*}+B(u_o,\theta_*)\frac{\partial^2\Phi}{\partial r_*\partial \varphi}+C(u_*,\theta_*)\frac{\partial \Phi}{\partial r_*}=0,\tag{11}$$

其中

$$A(\nu_o,\theta_o)=\frac{2r'_H}{a^2\sin^2\theta_o\dot{r}_H-(a^2+r_H^2)},$$

$$B(\nu_o,\theta_o)=\frac{2a(\dot{r}_H-1)}{a^2\sin^2\theta_o\dot{r}_H-(a^2+r_H^2)},$$

$$C(\nu_o,\theta_o)=\frac{a^2\sin^2\theta\ddot{r}_H+r''_H-2r_H\dot{r}_H+r'_H\mathrm{ctg}\theta}{a^2\sin^2\theta_o\dot{r}_H-(r_H^2+a^2)}. \tag{12}$$

引入变换 $\Phi=R(r_*)\Theta(\theta_*)\exp(\mathrm{i}m\varphi-\mathrm{i}\omega\nu_*)$**，其中** ω **为** K-G **粒子的能量，**m **为粒子角动量在** φ **轴上的投影，经分离变量(11)式化为**

$$\begin{aligned}&\Theta'=\lambda\Theta,\\&R''+(\lambda A+C+\mathrm{i}mB-2\mathrm{i}\omega)R'=0,\end{aligned} \tag{13}$$

其中 λ **为常数，(13)式的解为**

$$\begin{aligned}&\Theta=C_1\exp(\lambda\theta_*),\\&R=C_2\exp[(2\mathrm{i}\omega-\mathrm{i}mB-\lambda A-C)r_*]+C_3,\end{aligned} \tag{14}$$

式中 C_1,C_2,C_3 **是积分常数，考虑到时空从轴对称过渡到球对称时，**Θ **应成为勒让德函数的一部分，我们选择** λ **为实数. 因此，其中径向分量为**

$$\begin{aligned}\psi_{in}&=\exp(-\mathrm{i}\omega\nu_*),\\\psi_{out}&=\exp[-\mathrm{i}\omega\nu_*+2\mathrm{i}(\omega-\omega_o)r_*\\&\quad-(\lambda A+C)r_*],\end{aligned} \tag{15}$$

式中 $\omega_o=\frac{1}{2}mB$. **由于在视界附近有** $r_*\sim\frac{1}{2k}\ln(r-r_H)$**，于是出射波可写为**

$$\psi_{out}=(r-r_H)^{\mathrm{i}(\omega-\omega_o)/k}(r-r_H)^{-(\lambda A+C)/2k}\mathrm{e}^{-\mathrm{i}\omega\nu_*}, \tag{16}$$

显然，ψ_{out}**在** $r=r_H$ **处非解析，只能通过下半复** r **平面将** ψ_{out} **解析延拓到视界内部，即** $(r-r_H)\to|r-r_H|\mathrm{e}^{-\mathrm{i}\pi}=(r_H-r)\mathrm{e}^{-\mathrm{i}\pi}$**，于是**

$$\begin{aligned}\psi_{out}\to\widetilde{\psi}_{out}&=\exp[-\mathrm{i}\omega\nu_*+2\mathrm{i}(\omega-\omega_o)r_*\\&\quad-(\lambda A+C)r_*+\mathrm{i}\pi(\lambda A+C)/2k\\&\quad+\pi(\omega-\omega_o)/k].\end{aligned} \tag{17}$$

出射波在视界处的散射概率为 $\left|\frac{\psi_{out}}{\widetilde{\psi}_{out}}\right|^2=\mathrm{e}^{-2\pi(\omega-\omega_o)/k}$**，根据** Sannan **建议的方法**[10]**，得出出射波的黑体谱**

$$N_\omega=\frac{1}{\exp[(\omega-\omega_o)/(k_BT)]\pm1}, \tag{18}$$

$$T=k/2\pi k_B, \tag{19}$$

其中"+"号对应着费米子，"−"号对应着玻色子，k_B **是玻尔兹曼常数. 可见，辐射温度和辐射谱与方位角有关.**

[1] G. W. Gibbons, S. W. Hawking, *Phys. Rev.*, **D15** (1977), 2738.

[2] Z. Zhao, X. X. Dai, *Chinese Science Bulletin*, **36** (1991), 1870.

[3] Z. Zhao, *Chinese Science*, **A23** (1993), 178.

[4] Z. Q. Luo, Z. Zhao, *Acta Physica Sinica*, **42** (1993), 506 (in Chinese) [罗志坚、赵峥，物理学报，**42**(1993)，506].

[5] S. Wu, Z. Zhao, *Acta Astronomica Sinica*, **34** (1993), 17 (in Chinese) [吴思，赵峥，天文学报，**34**(1993)，17].

[6] M. C. Sun *et al.*, *Acta Physica Sinica*, **44** (1995), 1018 (in Chinese) [孙鸣超、赵仁、赵峥，物理学报，**44**(1995)，1018].

[7] J. L. Ji, Y. J. Wang, *International Journal of Theoretical Physics*, **35** (1996), 1481.

[8] J. L. Lü, Y. J. Wang, *Acta Physica Sinica*, **48** (1999), 389 (in Chinese) [吕君丽、王永久，物理学报，**48**(1999)，389].

[9] T. Damour, R. Ruffini, *Phys. Rev.*, **D14** (1976), 332.

[10] S. Sannon, *Gen. Rel. Grav.*, **20** (1988), 239.

THE QUANTUM THERMAL EFFECT OF THE CHARGED, AXIALLY SYMMETRIC NONSTATIONARY BLACH HOLE

LU MAO-WANG
(*Department of Physics*, *Lingling Teacher's College*, *Hunan* 425006, *China*)
(Received 15 October 1999; revised manuscript received 30 November 1999)

ABSTRACT

The Hawking radiation for a charged axially symmetric nonstationary black hole is studied. We obtain the event horizon equation and the Hawking thermal spectrum formula. Both the shape and the temperature of the black hole depend on the time and the angle. They can reduce to the well-known results when there is no electric charge.

PACC: 0420; 9760L