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黑洞的视界面公式

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从零曲面方程出发,导出了黑洞视界面的普遍公式.利用高登方程验证了该公式的正确性,并利用该公式求出了任意加速含荷黑洞的视界.

关键词:黑洞,视界,零曲面,乌龟坐标变换
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1 引 言

黑洞的视界是黑洞物理的一个重要概念.确定不同类型黑洞的视界是黑洞物理的一项重要任务,通常是通过求解弯曲时空中粒子的场方程[1,2],从而确定黑洞的视界面方程.利用这种方法求黑洞的视界,不仅工作量大,而且只能给出某具体类型黑洞的视界.为了研究黑洞视界所遵循的规律,本文利用零曲面方程,严格推导出黑洞的视界面公式.

2 黑洞的视界面公式

采用超前 Eddington 坐标,弯曲时空中的四维不变线元的度规张量总能化成 $g^{\mu\nu}$ 的形式.在球坐标中,当 $\mu,\nu=0,1,2,3$ 时,分别对应 v,r,θ,φ,则 $g^{\mu\nu}$ 应为 v,r,θ,φ 的函数.若黑洞的源质量为 M,带荷为 Q,单位质量的角动量为 A,对于蒸发黑洞,M,Q 也应为 v 的函数.对于加速黑洞,其北极 $\theta=0$ 始终指向加速度的方向,a 表示源加速度的大小,b 和 c 描述源加速度方向变化的速率,它们也都应该是 v 的函数,于是

$$g^{\mu\nu}=g^{\mu\nu}(v,r,\theta,\varphi,M(v),Q(v),A,a(v),b(v),c(v)). \tag{1}$$

由零曲面方程

$$g^{\mu\nu}\frac{\partial f}{\partial x^{\mu}}\frac{\partial f}{\partial x^{\nu}}=0, \tag{2}$$

将(1)式代入(2)式,可整理为

$$g^{vv}\left(\frac{\partial f}{\partial v}\right)^2+g^{rr}\left(\frac{\partial f}{\partial r}\right)^2+g^{\theta\theta}\left(\frac{\partial f}{\partial \theta}\right)^2+g^{\varphi\varphi}\left(\frac{\partial f}{\partial \varphi}\right)^2+2g^{vr}\frac{\partial f}{\partial v}\frac{\partial f}{\partial r}+2g^{v\theta}\frac{\partial f}{\partial v}\frac{\partial f}{\partial \theta}+2g^{v\varphi}\frac{\partial f}{\partial v}\frac{\partial f}{\partial \varphi}$$
$$+2g^{r\theta}\frac{\partial f}{\partial r}\frac{\partial f}{\partial \theta}+2g^{r\varphi}\frac{\partial f}{\partial r}\frac{\partial f}{\partial \varphi}+2g^{\theta\varphi}\frac{\partial f}{\partial \theta}\frac{\partial f}{\partial \varphi}=0. \tag{3}$$

由

$$f=f(r,\theta,\varphi,v)=0,\quad r=r(\theta,\varphi,v), \tag{4}$$

可得

$$\frac{\partial f}{\partial \theta}+\frac{\partial f}{\partial r}\frac{\partial r}{\partial \theta}=0,\quad \frac{\partial f}{\partial \varphi}+\frac{\partial f}{\partial r}\frac{\partial r}{\partial \varphi}=0,\quad \frac{\partial f}{\partial v}+\frac{\partial f}{\partial r}\frac{\partial r}{\partial v}=0. \tag{5}$$

将(5)式代入(3)式可整理为

$$g^{vv}\left(\frac{\partial r}{\partial v}\right)^2+g^{rr}+g^{\theta\theta}\left(\frac{\partial r}{\partial \theta}\right)^2+g^{\varphi\varphi}\left(\frac{\partial r}{\partial \varphi}\right)^2-2g^{rv}\frac{\partial r}{\partial \theta}-2g^{r\theta}\frac{\partial r}{\partial \theta}-2g^{r\varphi}\frac{\partial r}{\partial \varphi}$$
$$+2g^{v\theta}\frac{\partial r}{\partial v}\frac{\partial r}{\partial \theta}+2g^{v\varphi}\frac{\partial r}{\partial v}\frac{\partial r}{\partial \varphi}+2g^{\theta\varphi}\frac{\partial r}{\partial \theta}\frac{\partial r}{\partial \varphi}=0. \tag{6}$$

令 $i,j=0,2,3$ 分别为 v,θ,φ,则(6)式可以写成为

$$g^{rr}-2g^{rj}r_{,j}+g^{ij}r_{,i}r_{,j}=0, \tag{7}$$

式中 $r_{,i}=\partial r/\partial x^i$，$v=x^0$，$\theta=x^2$，$\varphi=x^3$. 在视界面处，应将(7)式中的 r 改写成 r_{H}，则可得

$$g^{rr}-2g^{rj}r_{\mathrm{H},j}+g^{ij}r_{\mathrm{H},i}r_{\mathrm{H},j}=0, \tag{8}$$

式中 $i, j=0,2,3$ 分别为 v, θ, φ. (8)式便是确定黑洞视界面的公式. 利用该公式可求任何类型黑洞的视界. 对于一些特殊情况，当黑洞具有轴对称性时，视界 r_{H} 与 φ 无关，则 $r_{\mathrm{H},\varphi}=0$；当黑洞具有球对称性时，视界 r_{H} 与 θ,φ 都无关，则 $r_{\mathrm{H},\theta}=0$，$r_{\mathrm{H},\varphi}=0$. 因此只要知道了度规张量的具体形式，代入(8)式，便可得到相应的视界面方程，经过求解就能确定黑洞视界的位置.

3 利用高登方程给出的黑洞视界面公式

为了证实上述公式的正确性，现再从一般的高登方程入手，重新推导黑洞的视界面公式.

Schwinger 给出的含荷黑洞的 Klein-Gordon 方程[3]为

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^\mu}-\mathrm{i}eA_\mu\right)\cdot\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu}-\mathrm{i}eA_\nu\right)\psi\right]=m^2\psi, \tag{9}$$

式中 m 为粒子的质量，e 为粒子所带电荷，$\mu,\nu=0,1,2,3$ 分别为 v,r,θ,φ.

利用 Lorentz 条件

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}A_\nu\right)=0, \tag{10}$$

可将(9)式化简为

$$g^{\mu\nu}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}\cdot\frac{\partial\psi}{\partial x^\nu}+g^{\mu\nu}\frac{\partial^2\psi}{\partial x^\mu\partial x^\nu}+\frac{\partial g^{\mu\nu}}{\partial x^\mu}\frac{\partial\psi}{\partial x^\nu}-2\mathrm{i}eg^{\mu\nu}A_\mu\frac{\partial\psi}{\partial x^\nu}-\left(e^2g^{\mu\nu}A_\mu A_\nu+m^2\right)\psi=0. \tag{11}$$

将(11)式写成

$$\begin{aligned}&g^{rr}\frac{\partial^2\psi}{\partial r^2}+g^{ij}\frac{\partial^2\psi}{\partial x^i\partial x^j}+2g^{rj}\frac{\partial^2\psi}{\partial r\partial x^j}+\left(g^{\mu r}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}+\frac{\partial g^{\mu r}}{\partial x^\mu}-2\mathrm{i}eg^{\mu r}A_\mu\right)\frac{\partial\psi}{\partial r}\\&+\left(g^{\mu j}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}+\frac{\partial g^{\mu j}}{\partial x^\mu}-2\mathrm{i}eg^{\mu j}A_\mu\right)\frac{\partial\psi}{\partial x^j}-\left(e^2g^{\mu\nu}A_\mu A_\nu+m^2\right)\psi=0,\end{aligned} \tag{12}$$

式中 $i, j=0,2,3$.

作广义乌龟坐标变换[4]

$$r_*=r+\frac{1}{2k(v_0,\theta_0,\varphi_0)}\ln[r-r_{\mathrm{H}}(v,\theta,\varphi)],\quad x^j_*=x^j-x^j_0, \tag{13}$$

则可得

$$\begin{aligned}&\frac{\partial}{\partial r}=(1+\rho)\frac{\partial}{\partial r_*},\ \frac{\partial}{\partial x^j}=\frac{\partial}{\partial x^j_*}-\rho r_{\mathrm{H},j}\frac{\partial}{\partial r_*},\ \frac{\partial^2}{\partial r^2}=(1+\rho)^2\frac{\partial^2}{\partial r^2_*}-\rho^2\frac{\partial}{\partial r_*},\\&\frac{\partial^2}{\partial x^i\partial x^j}=\frac{\partial^2}{\partial x^i_*\partial x^j_*}-2\rho r_{\mathrm{H},i}\frac{\partial^2}{\partial r_*\partial x^i_*}+\rho^2r_{\mathrm{H},j}r_{\mathrm{H},i}\frac{\partial^2}{\partial r^2_*}-\left(\rho r_{\mathrm{H},j,i}+\rho^2r_{\mathrm{H},j}r_{\mathrm{H},i}\right)\frac{\partial}{\partial r_*},\\&\frac{\partial^2}{\partial r\partial x^j}=(1+\rho)\frac{\partial^2}{\partial r_*\partial x^j_*}-(1+\rho)\rho r_{\mathrm{H},j}\frac{\partial^2}{\partial r^2_*}+\rho r_{\mathrm{H},\mathrm{j}}\frac{\partial}{\partial r_*},\end{aligned} \tag{14}$$

式中 $\rho=1/2k(r-r_{\mathrm{H}})$，$r_{\mathrm{H},j}=\partial r_{\mathrm{H}}/\partial x^j$，$r_{\mathrm{H},j,i}=\partial r_{\mathrm{H},j}/\partial x^i$. 在乌龟坐标变换下，(12)式可整理为

$$\begin{aligned}&\left[(1+\rho)^2g^{rr}+\rho^2r_{\mathrm{H},j}r_{\mathrm{H},i}g^{ij}-2(1+\rho)\rho r_{\mathrm{H},j}g^{rj}\right]\frac{\partial^2\psi}{\partial r^2_*}+\left[2(1+\rho)g^{rj}\right.\\&\left.-2\rho r_{\mathrm{H},j}g^{ij}\right]\frac{\partial^2\psi}{\partial r_*\partial x^j_*}+g^{ij}\frac{\partial^2\psi}{\partial x^i_*\partial x^j_*}+\left[(1+\rho)\left(g^{\mu r}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}+\frac{\partial g^{\mu r}}{\partial x^\mu}-2\mathrm{i}eg^{\mu r}A_\mu\right)\right.\\&-\rho^2g^{rr}+2\rho r_{\mathrm{H},j}g^{rj}-\left(\rho r_{\mathrm{H},j,i}+\rho^2r_{\mathrm{H},j}r_{\mathrm{H},i}\right)g^{ij}\\&\left.-\rho r_{\mathrm{H},j}\left(g^{\mu j}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}+\frac{\partial g^{\mu j}}{\partial x^\mu}-2\mathrm{i}eg^{\mu j}A_\mu\right)\right]\frac{\partial\psi}{\partial r_*}+\left(g^{\mu j}\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}\right.\\&\left.+\frac{\partial g^{\mu j}}{\partial x^\mu}-2\mathrm{i}eg^{\mu j}A_\mu\right)\frac{\partial\psi}{\partial x^j_*}-\left(e^2g^{\mu r}A_\mu A_v+m^2\right)\psi=0.\end{aligned} \tag{15}$$

按照文献[5]所提供的方法,(15)式中 $\partial^2\psi/\partial r_*^2$ 的系数在 $r\to r_h$ 时应等于一个常数 A,即

$$\lim_{r\to r_H}[(1+\rho)^2 g^{rr}+\rho^2 r_{H,j}r_{H,i}g^{ij}-2(1+\rho)\rho r_{H,j}g^{rj}]=A. \tag{16}$$

将 $\rho=1/2k(r-r_H)$ 代入,可整理为

$$\lim_{r\to r_H}\frac{[2k(r-r_H)+1]^2 g^{rr}+r_{H,j}r_{H,i}g^{ij}-2[1+2k(r-r_H)r_{H,j}g^{rj}]}{[2k(r-r_H)]^2}=A. \tag{17}$$

由于当 $r\to r_H$ 时分母为零,要保证(17)式为常数,必有当 $r\to r_H$ 时分子亦为零,于是便可得

$$g^{rr}-2g^{rj}r_{H,j}+g^{ij}r_{H,i}r_{H,j}=0. \tag{8'}$$

这便是(8)式,由此可见两种方法给出的结果完全一致.

4 加速含荷黑洞的视界

现利用黑洞的视界面公式求任意加速含荷黑洞的视界.采用超前 Eddington 坐标,任意加速含荷黑洞的度规张量的逆变形式为[6]

$$\begin{aligned}
g^{vr}&=-1,\\
g^{rr}&=-1+2Mr^{-1}-Q^2r^{-2}\\
&\quad+2ar\cos\theta+4aQ^2r^{-1}\cos\theta,\\
g^{r\theta}&=-(b\sin\varphi+c\cos\varphi-a\sin\theta),\\
g^{r\varphi}&=-\cot\theta(b\cos\varphi-c\sin\varphi),\\
g^{\theta\theta}&=-r^{-2},\\
g^{\varphi\varphi}&=-r^{-2}\sin^{-2}\theta.
\end{aligned} \tag{18}$$

代入(8)式,可整理为

$$\begin{aligned}
&2a\cos\theta r_H^3+[2r_{H,v}+(b\sin\varphi+c\cos\varphi-a\sin\theta)r_{H,\theta}\\
&+2\cot\theta(b\cos\varphi-c\sin\varphi)r_{H,\varphi}-1]r_H^2\\
&+(2M+4aQ^2\cos\theta)r_H-(Q^2+r_{H,\theta}^2\\
&+r_{H,\varphi}^2\sin^{-2}\theta)=0.
\end{aligned} \tag{19}$$

对应的视界位置

$$\begin{aligned}
r_{H_1}&=2\sqrt{-\frac{p}{3}}\cos\alpha,\\
r_{H_2}&=-2\sqrt{-\frac{p}{3}}\cos\left(\alpha+\frac{\pi}{3}\right),\\
r_{H_3}&=-2\sqrt{-\frac{p}{3}}\cos\left(\alpha-\frac{\pi}{3}\right),
\end{aligned} \tag{20}$$

式中 $\alpha=\frac{1}{3}\arccos\left(-q/2\sqrt{-(\frac{p}{3})^3}\right)$,

$$\begin{aligned}
p&=-\frac{1}{12}[2r_{H,v}+(b\sin\varphi+c\cos\varphi-a\sin\theta)r_{H,\theta}+2\cot\theta(b\cos\varphi\\
&\quad-c\sin\varphi)r_{H,\varphi}-1]^2/a^2\cos^2\theta+(M+2Q^2\cos\theta)/a\cos\theta,\\
q&=\frac{1}{108}[2r_{H,v}+(b\sin\varphi+c\cos\varphi-a\sin\theta)r_{H,\theta}+2\cot\theta(b\cos\varphi-c\sin\varphi)r_{H,\varphi}\\
&\quad-1]^3/a^3\cos^3\theta-\frac{1}{6}(M+2aQ^2\cos\theta)[2r_{H,v}+(b\sin\varphi+c\cos\varphi-a\sin\theta)r_{H,\theta}\\
&\quad-2\cot\theta(b\cos\varphi-c\sin\varphi)r_{H,\varphi}-1]/a^2\cos^2\theta-(Q^2+r_{H,\theta}^2+r_{H,\varphi}^2\sin^2\theta)/2a\cos\theta.
\end{aligned}$$

由引可见,任意加速含荷黑洞的视界不仅仅与 θ 有关,还与 φ 有关,完全失去了对称性.对于不含荷直线加速黑洞,由于 b,c 和 Q 都等于零,黑洞具有轴对称性,且 $r_{H,\varphi}=0$,代入(19)式可化为

$$2a\cos\theta r_H^3+(2r_{H,v}-a\sin\theta r_{H,\theta}-1)r_H^2+2Mr_H-r_{H,\theta}^2=0. \tag{21}$$

这与文献[7]的结果相同.

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THE EVENT HORIZON FORMULA OF BLACK HOLE

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ABSTRACT

The general event horizon formula of black hole is given from null hypersurface equation. We prove that this formula is correct by Klein-Gordon equation, and we obtain yet the event horizon equation of arbitrarily accelerated black hole containing charges by using of this formula.

Keywords: blank hole, event horizon, null surface, tortoise transformation

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