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直线加速动态黑洞 Dirac 场的熵*

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采用薄层模型 brick-wall 方法,计算出了直线加速动态黑洞视界面上 Dirac 场的熵以及 Rindler 视界面上 Dirac 场的熵密度,通过适当选择时间依赖的截断因子 ε 和薄层厚度 δ,仍可得出熵与面积成正比的结论.

关键词:熵,加速动态黑洞,薄层模型,Dirac 场,Dirac 方程
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1. 引 言

自从 Bekenstein 提出黑洞熵与黑洞视界面积成正比的建议以来[1],有关研究取得了很大的进展. 1985 年,'t Hooft 提出 brick-wall 模型[2],这一方法被人们用来计算静态球对称黑洞的熵,并取得了令人满意的结果[3—9].然而,使用 brick-wall 模型必须先假定黑洞与外界在大尺度范围内存在热平衡,动态黑洞显然不属于这种情况,因而不能用 brick-wall 方法来计算动态黑洞的熵.李翔、高长军、赵峥等人针对上述不完美之处,把 brick-wall 模型改进为薄层模型[10,11].在改进的方案中,只要求在黑洞外薄层范围内局部存在热平衡,黑洞的熵被认为是来自视界附近一个薄层的贡献.他们利用这一模型计算了各种动态黑洞标量场的熵[10—12],计算结果获得了预期的成功.而对于 Dirac 场的熵,以往的文献只计算了动态球对称的情况[13].主要的困难是在非球对称背景时空中 Dirac 方程不易退耦和分离变量,因而不能严格求解.本文考虑直线加速动态黑洞,在小质量近似下,通过进行适当变换,成功地将 Dirac 方程退耦.采用薄层模型和 WKB 近似,算出了视界面上的熵密度和熵.

2. 视界面上 Dirac 场的熵密度的计算

直线加速动态黑洞的时空度规用超前 Eddington-Finkelstein 坐标表示为[14,15]

$$\mathrm{d}s^2 = g_{00}\mathrm{d}v^2 + 2g_{01}\mathrm{d}v\mathrm{d}r + 2g_{02}\mathrm{d}v\mathrm{d}\theta + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2, \tag{1}$$

其中

$$g_{00} = 1 - \frac{2M(v)}{r} - 2a(v)r\cos\theta - a^2(v)r^2\sin^2\theta,$$

$$g_{01} = -1, g_{02} = ar^2\sin\theta,$$

$$g_{22} = -r^2, g_{33} = -r^2\sin^2\theta.$$

利用零曲面条件和时空对称性,不难求出局部事件视界面方程为[15]

$$2r_{\mathrm{h},\nu} - \left(1 - \frac{2M}{r} - 2ar\cos\theta\right) + 2a\sin\theta r_{\mathrm{h},\theta} - \frac{r^2_{\mathrm{h},\theta}}{r^2_{\mathrm{h}}} = 0, \tag{2}$$

式中 $r_{\mathrm{h},\nu} = \dfrac{\partial r_{\mathrm{h}}}{\partial v}, r_{\mathrm{h},\theta} = \dfrac{\partial r_{\mathrm{h}}}{\partial \theta}$.显然,$r_{\mathrm{h}}$ 是 v 和方向角 θ 的函数,即视界位置随时间 v 和方向角 θ 而变化.此外,(2)式下标 h 适用于黑洞视界和 Rindler 视界.

引入如下坐标变换[16]

$$R = r - r_{\mathrm{h}}(v,\theta), V = v - v_0, \Theta = \theta - \theta_0, \tag{3}$$

则有

$$\mathrm{d}R = \mathrm{d}r - r_{\mathrm{h},\nu}\mathrm{d}\nu - r_{\mathrm{h}\theta}\mathrm{d}\theta, \tag{4}$$

$$\frac{\partial}{\partial R} = \frac{\partial}{\partial r}, \frac{\partial}{\partial V} = r_{\mathrm{h},\nu}\frac{\partial}{\partial r} + \frac{\partial}{\partial \nu}, \frac{\partial}{\partial \Theta} = r_{\mathrm{h},\theta}\frac{\partial}{\partial r} + \frac{\partial}{\partial \theta}. \tag{5}$$

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变换后的度规形式为

$$\begin{aligned}\mathrm{d}s^2 &= (g_{00}+2r_{\mathrm{h}\nu}g_{01})\mathrm{d}V^2+2g_{01}\mathrm{d}V\mathrm{d}R\\ &\quad+2(r_{\mathrm{h},\theta}g_{01}+g_{02})\mathrm{d}V\mathrm{d}\Theta\\ &\quad+g_{22}\mathrm{d}\Theta^2+g_{33}\mathrm{d}\varphi^2.\end{aligned}\tag{6}$$

(6)式可以写为

$$\begin{aligned}\mathrm{d}s^2 &= \left(g_{00}+2r_{\mathrm{h},\nu}g_{01}-\frac{(r_{\mathrm{h},\theta}g_{01}+g_{02})^2}{g_{22}}\right)\mathrm{d}V^2\\ &\quad+2g_{01}\mathrm{d}V\mathrm{d}R+g_{22}(-\Omega\mathrm{d}V+\mathrm{d}\Theta)^2+g_{33}\mathrm{d}\varphi^2,\end{aligned}\tag{7}$$

其中

$$\Omega=-\frac{r_{\mathrm{h},\theta}g_{01}+g_{02}}{g_{22}}=a\sin\theta-\frac{r_{\mathrm{h},\theta}}{r^2}\tag{8}$$

为类似于旋转黑洞而定义的拖曳角速度[12].如果令$\frac{\mathrm{d}V}{\mathrm{d}\Theta}=\Omega$,则(7)式将变为拖曳系中的线元.

下面以变换后的时空度规为背景时空进行讨论,写成显式则为

$$\begin{aligned}\mathrm{d}s^2 &= (g_{00}-2r_{\mathrm{h},\nu})\mathrm{d}V^2-2\mathrm{d}V\mathrm{d}R\\ &\quad+2(ar^2\sin\theta-r_{\mathrm{h},\theta})\mathrm{d}V\mathrm{d}\Theta-r^2\mathrm{d}\Theta^2\\ &\quad-r^2\sin^2\theta\mathrm{d}\varphi^2,\end{aligned}\tag{9}$$

其不为零的逆变分量可求出为

$$\begin{aligned}&g^{01}=g^{10}=-1,\\ &g^{11}=2r_{\mathrm{h},v}+2ar_{\mathrm{h},\theta}\sin\theta-\left(1-\frac{2M}{r}-2ar\cos\theta\right)-\frac{r_{\mathrm{h},\theta}^2}{r^2},\\ &g^{12}=g^{21}=-a\sin\theta+\frac{r_{\mathrm{h},\theta}}{r^2},\\ &g^{22}=-\frac{1}{r^2},g^{33}=-\frac{1}{r^2\sin^2\theta}.\end{aligned}\tag{10}$$

显然,$g^{11}=0$ 即为视界面方程.

取零标架

$$\begin{aligned}&l^\mu=(0,1,0,0),\\ &n^\mu=\left(-1,\frac{1}{2}g^{11},-a\sin\theta+\frac{r_{\mathrm{h},\theta}}{r^2},0\right),\\ &m^\mu=\frac{1}{\sqrt{2}r}\left(0,0,1,\frac{\mathrm{i}}{\sin\theta}\right),\\ &\bar{m}^\mu=\frac{1}{\sqrt{2}r}\left(0,0,1,\frac{-\mathrm{i}}{\sin\theta}\right).\end{aligned}\tag{11}$$

由(11)式可求出其协变分量,易验证满足伪正交条件和坐标条件.由度规和零标架的表示式,并利用(5)式,可求得不为零的旋系数为

$$\begin{aligned}&\pi=\tau=-\frac{r_{\mathrm{h},\theta}}{\sqrt{2}r^2},\rho=-\frac{1}{r},\\ &\beta=\frac{\cot\theta}{2\sqrt{2}r},\alpha=-\frac{r_{\mathrm{h},\theta}}{\sqrt{2}r^2}-\frac{\cot\theta}{2\sqrt{2}r},\\ &\gamma=\frac{1}{4}\frac{\partial\Sigma}{\partial r}-\frac{r_{\mathrm{h},\theta}^2}{2r^3},\\ &\lambda=\frac{r_{\mathrm{h},\theta\theta}}{2r^2}-\frac{r_{\mathrm{h},\theta}^2}{r^3}-\frac{\cot\theta}{2r^2}r_{\mathrm{h},\theta},\\ &\mu=\frac{1}{2r}\Big(-2ar\cos\theta+\frac{r_{\mathrm{h},\theta\theta}}{r}\\ &\qquad+\frac{\cot\theta}{r}r_{\mathrm{h},\theta}-\Sigma-\frac{r_{\mathrm{h},\theta}^2}{r^2}\Big),\\ &\nu=\frac{1}{\sqrt{2}r}\Big(\frac{1}{2}r_{\mathrm{h},\theta}\frac{\partial\Sigma}{\partial r}+\frac{1}{2}\frac{\partial\Sigma}{\partial\theta}-r_{\mathrm{h},\nu\theta}\\ &\qquad-a\sin\theta r_{\mathrm{h},\theta\theta}+ar\sin\theta r_{\mathrm{h},\nu}-a\cos\theta r_{\mathrm{h},\theta}\\ &\qquad+\frac{r_{\mathrm{h},\theta}}{r^2}r_{\mathrm{h},\theta\theta}-\frac{r_{\mathrm{h},\theta}r_{\mathrm{h},\nu}}{r}-\frac{r_{\mathrm{h},\theta}^3}{r^3}\Big),\end{aligned}$$

式中 $r_{\mathrm{h},\theta\theta}=\frac{\partial^2r_{\mathrm{h}}}{\partial\theta^2}$, $r_{\mathrm{h},\nu\theta}=\frac{\partial^2r_{\mathrm{h}}}{\partial\theta\partial\nu}$, $\Sigma=1-\frac{2M}{r}-2ar\cos\theta$.对应的四个方向导数为

$$\begin{aligned}&D=\frac{\partial}{\partial R},\\ &\Delta=-\frac{\partial}{\partial V}+\frac{1}{2}g^{11}\frac{\partial}{\partial R}-\left(a\sin\theta-\frac{r_{\mathrm{h},\theta}}{r^2}\right)\frac{\partial}{\partial\Theta},\\ &\delta=\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\Theta}+\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right),\\ &\bar{\delta}=\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\Theta}-\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right).\end{aligned}\tag{12}$$

对应的 Dirac 方程的形式为

$$\left(\frac{\partial}{\partial R}+\frac{1}{r}\right)F_1+\left(\frac{1}{\sqrt{2}r}\frac{\partial}{\partial\Theta}-\frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cos\theta}{2\sqrt{2}r}\right)F_2=\mathrm{i}\mu G_1,\tag{13a}$$

$$\begin{aligned}&\left[-\frac{\partial}{\partial V}+\frac{1}{2}g^{11}\frac{\partial}{\partial R}-\left(a\sin\theta-\frac{r_{\mathrm{h},\theta}}{r^2}\right)\frac{\partial}{\partial\Theta}+\frac{1}{2r}\left(-2ar\cos\theta+\frac{r_{\mathrm{h},\theta\theta}}{r}+\frac{\cot\theta}{r}r_{\mathrm{h},\theta}-\Sigma\right)-\frac{1}{4}\frac{\partial\Sigma}{\partial r}\right]F_2\\ &+\left[\frac{1}{\sqrt{2}r}\frac{\partial}{\partial\Theta}+\frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{r_{\mathrm{h},\theta}}{\sqrt{2}r^2}+\frac{\cot\theta}{2\sqrt{2}r}\right]F_1=\mathrm{i}\mu G_2,\end{aligned}\tag{13b}$$

$$\left(\frac{\partial}{\partial R}+\frac{1}{r}\right)G_2-\left(\frac{1}{\sqrt{2}r}\frac{\partial}{\partial\Theta}+\frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{\cot\theta}{2\sqrt{2}r}\right)G_1=\mathrm{i}\mu F_2,\tag{13c}$$

$$\left[-\frac{\partial}{\partial V}+\frac{1}{2}g^{11}\frac{\partial}{\partial R}-\left(a\sin\theta-\frac{r_{h,\theta}}{r^2}\right)\frac{\partial}{\partial\Theta}+\frac{1}{2r}\left(-2ar\cos\theta+\frac{r_{h,\theta\theta}}{r}+\frac{\cot\theta}{r}r_{h,\theta}-\Sigma\right)-\frac{1}{4}\frac{\partial\Sigma}{\partial r}\right]G_1$$
$$-\left[\frac{1}{\sqrt{2}r}\frac{\partial}{\partial\Theta}-\frac{\mathrm{i}}{\sqrt{2}r\sin\theta}\frac{\partial}{\partial\varphi}+\frac{r_{h,\theta}}{\sqrt{2}r^2}+\frac{\cot\theta}{2\sqrt{2}r}\right]G_2=\mathrm{i}\mu F_1. \tag{13d}$$

在上面的方程组中,变量 φ 不出现在求导算符的系数中,因而可以把此变量分离成 $\exp(\mathrm{i}m\varphi/\hbar)$ 的形式.对于动态黑洞,变量 V 出现在 g^{11} 中,一般不能分离为 $\exp(-\mathrm{i}EV/\hbar)$,但我们采用薄层模型求黑洞的熵,考虑的是视界附近的渐近行为,对于不是剧烈演化的黑洞,这种处理在精度上是足够的[10].因此可以作如下变换

$$F_1=\frac{1}{r}f_1\exp(\mathrm{i}m\varphi-\mathrm{i}EV)/\hbar,$$
$$F_2=f_2\exp(\mathrm{i}m\varphi-\mathrm{i}EV)/\hbar,$$
$$G_1=g_1\exp(\mathrm{i}m\varphi-\mathrm{i}EV)/\hbar,$$
$$G_2=\frac{1}{r}g_2\exp(\mathrm{i}m\varphi-\mathrm{i}EV)/\hbar. \tag{14}$$

在上式中,我们有意保留 $\hbar$,这是为了后面采用 WKB 近似时,保留方程中的主要项.此外,在求熵时,一般都要作小质量近似,为了简便起见,令(13)式中 $\mu=0$.

将(14)式代入(13)式,并令(13)式中 $\mu=0$,可得

$$\frac{\partial f_1}{\partial R}+\frac{1}{\sqrt{2}}\frac{\partial f_2}{\partial\Theta}+A_+f_2=0, \tag{15a}$$

$$\frac{\partial f_2}{\partial R}+\frac{\sqrt{2}}{r^2g^{11}}\frac{\partial f_1}{\partial\Theta}-\frac{2}{g^{11}}\left(a\sin\theta-\frac{r_{h,\theta}}{r^2}\right)\frac{\partial f_2}{\partial\Theta}$$
$$+C_-f_1+Bf_2=0, \tag{15b}$$

$$\frac{\partial g_1}{\partial R}-\frac{2}{g^{11}}\left(a\sin\theta-\frac{r_{h,\theta}}{r^2}\right)\frac{\partial g_1}{\partial\Theta}-\frac{\sqrt{2}}{r^2g^{11}}\frac{\partial g_2}{\partial\Theta}$$
$$+Bg_1-C_+g_2=0, \tag{15c}$$

$$\frac{\partial g_2}{\partial R}-\frac{1}{\sqrt{2}}\frac{\partial g_1}{\partial\Theta}+A_-g_1=0. \tag{15d}$$

其中

$$A_\pm=\frac{m}{\hbar\sqrt{2}\sin\theta}\pm\frac{\cot\theta}{2\sqrt{2}},$$
$$C_\pm=\frac{1}{\hbar}\left(\frac{\pm\sqrt{2}m}{r^2g^{11}\sin\theta}\right)+\frac{\cot\theta}{\sqrt{2}r^2g^{11}},$$
$$B=\frac{\mathrm{i}}{\hbar}\left(\frac{2E}{g^{11}}\right)+\frac{1}{g^{11}}\left(-2a\cos\theta+\frac{r_{h,\theta\theta}}{r^2}\right.$$
$$\left.+\frac{r_{h,\theta}\cot\theta}{r^2}-\frac{\Sigma}{r}-\frac{1}{2}\frac{\partial\Sigma}{\partial r}\right). \tag{16}$$

因为我们仅考虑视界面附近的量子行为,对于薄层模型有

$$r_h+\varepsilon<r<r_h+\varepsilon+\delta. \tag{17}$$

其中 ε 为截断因子,δ 为薄层厚度.因此,在薄层中 $g^{11}\neq0$,方程组(15)的各项系数是充分光滑的,对应的方程组为双曲型方程组[17].采用文献[17]中的方法对波函数的四个分量(f_1,f_2,g_1,g_2)作如下变换:

$$\begin{pmatrix}f_1\\f_2\\g_1\\g_2\end{pmatrix}=\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\begin{pmatrix}\sqrt{2}\lambda_2&-\sqrt{2}\lambda_1&0&0\\-1&1&0&0\\0&0&1&-1\\0&0&\sqrt{2}\lambda_2&-\sqrt{2}\lambda_1\end{pmatrix}\begin{pmatrix}\nu_1\\\nu_2\\\nu_3\\\nu_4\end{pmatrix}, \tag{18}$$

则方程组(15)可以对角化为

$$\frac{\partial\nu_1}{\partial R}+\lambda_1\frac{\partial\nu_1}{\partial\Theta}+d_{11}\nu_1+d_{12}\nu_2=0, \tag{19a}$$

$$\frac{\partial\nu_2}{\partial R}+\lambda_2\frac{\partial\nu_2}{\partial\Theta}+d_{21}\nu_1+d_{22}\nu_2=0, \tag{19b}$$

$$\frac{\partial\nu_3}{\partial R}+\lambda_1\frac{\partial\nu_3}{\partial\Theta}+d_{33}\nu_3+d_{34}\nu_4=0, \tag{19c}$$

$$\frac{\partial\nu_4}{\partial R}+\lambda_2\frac{\partial\nu_4}{\partial\Theta}+d_{43}\nu_3+d_{44}\nu_4=0. \tag{19d}$$

其中

$$\lambda_1 = -\frac{1}{g^{11}}\left(a\sin\theta - \frac{r_{h,\theta}}{r^2}\right) + \frac{1}{g^{11}}\sqrt{\left(a\sin\theta - \frac{r_{h,\theta}}{r^2}\right)^2 + \frac{g^{11}}{r^2}} = -\frac{1}{g^{11}}\left(\Omega - \sqrt{\Omega^2 + \frac{g^{11}}{r^2}}\right), \tag{20a}$$

$$\lambda_2 = -\frac{1}{g^{11}}\left(a\sin\theta - \frac{r_{h,\theta}}{r^2}\right) - \frac{1}{g^{11}}\sqrt{\left(a\sin\theta - \frac{r_{h,\theta}}{r^2}\right)^2 + \frac{g^{11}}{r^2}} = -\frac{1}{g^{11}}\left(\Omega + \sqrt{\Omega^2 + \frac{g^{11}}{r^2}}\right), \tag{20b}$$

为双曲型方程组(15)中对 Θ 求偏导数的系数矩阵的特征值.(19)式中 d_{ij} 的具体形式为

$$\begin{aligned} d_{11} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(-\frac{2\sqrt{2}\lambda_1\lambda_2 m}{r^2 g^{11}\sin\theta} - \frac{m}{\sqrt{2}\sin\theta} - \mathrm{i}\frac{2\sqrt{2}\lambda_1 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(\frac{4\lambda_1\lambda_2}{r^2 g^{11}} - 1\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. - \frac{\sqrt{2}\lambda_1}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) + \sqrt{2}\lambda_{1r} + \sqrt{2}\lambda_1(r_{h,\theta}\lambda_{1r} + \lambda_{1\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{12} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(\frac{2\sqrt{2}\lambda_1^2 m}{r^2 g^{11}\sin\theta} + \frac{m}{\sqrt{2}\sin\theta} + \mathrm{i}\frac{2\sqrt{2}\lambda_1 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(1 - \frac{4\lambda_1^2}{r^2 g^{11}}\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. + \frac{\sqrt{2}\lambda_1}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) - \sqrt{2}\lambda_{1r} - \sqrt{2}\lambda_1(r_{h,\theta}\lambda_{1r} + \lambda_{1\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{21} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(-\frac{2\sqrt{2}\lambda_2^2 m}{r^2 g^{11}\sin\theta} - \frac{m}{\sqrt{2}\sin\theta} - \mathrm{i}\frac{2\sqrt{2}\lambda_2 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(\frac{4\lambda_2^2}{r^2 g^{11}} - 1\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. - \frac{\sqrt{2}\lambda_2}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) + \sqrt{2}\lambda_{2r} + \sqrt{2}\lambda_2(r_{h,\theta}\lambda_{2r} + \lambda_{2\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{22} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(\frac{2\sqrt{2}\lambda_1\lambda_2 m}{r^2 g^{11}\sin\theta} + \frac{m}{\sqrt{2}\sin\theta} + \mathrm{i}\frac{2\sqrt{2}\lambda_2 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(1 - \frac{4\lambda_1\lambda_2}{r^2 g^{11}}\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. + \frac{\sqrt{2}\lambda_2}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) - \sqrt{2}\lambda_{2r} - \sqrt{2}\lambda_2(r_{h,\theta}\lambda_{2r} + \lambda_{2\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{33} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(\frac{2\sqrt{2}\lambda_1\lambda_2 m}{r^2 g^{11}\sin\theta} + \frac{m}{\sqrt{2}\sin\theta} - \mathrm{i}\frac{2\sqrt{2}\lambda_1 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(\frac{4\lambda_1\lambda_2}{r^2 g^{11}} - 1\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. - \frac{\sqrt{2}\lambda_1}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) + \sqrt{2}\lambda_{1r} + \sqrt{2}\lambda_1(r_{h,\theta}\lambda_{1r} + \lambda_{1\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{34} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(-\frac{2\sqrt{2}\lambda_1^2 m}{r^2 g^{11}\sin\theta} - \frac{m}{\sqrt{2}\sin\theta} + \mathrm{i}\frac{2\sqrt{2}\lambda_1 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(1 - \frac{4\lambda_1^2}{r^2 g^{11}}\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. + \frac{\sqrt{2}\lambda_1}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) - \sqrt{2}\lambda_{1r} - \sqrt{2}\lambda_1(r_{h,\theta}\lambda_{1r} + \lambda_{1\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{43} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(\frac{2\sqrt{2}\lambda_2^2 m}{r^2 g^{11}\sin\theta} + \frac{m}{\sqrt{2}\sin\theta} - \mathrm{i}\frac{2\sqrt{2}\lambda_2 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(\frac{4\lambda_2^2}{r^2 g^{11}} - 1\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. - \frac{\sqrt{2}\lambda_2}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) + \sqrt{2}\lambda_{2r} + \sqrt{2}\lambda_2(r_{h,\theta}\lambda_{2r} + \lambda_{2\theta})\right], \end{aligned}$$

$$\begin{aligned} d_{44} = &\frac{1}{\hbar}\frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left(-\frac{2\sqrt{2}\lambda_1\lambda_2 m}{r^2 g^{11}\sin\theta} - \frac{m}{\sqrt{2}\sin\theta} + \mathrm{i}\frac{2\sqrt{2}\lambda_2 E}{g^{11}}\right) + \frac{1}{\sqrt{2}(\lambda_2-\lambda_1)}\left[\left(1 - \frac{4\lambda_1\lambda_2}{r^2 g^{11}}\right)\frac{\cot\theta}{2\sqrt{2}}\right.\\ &\left. + \frac{\sqrt{2}\lambda_2}{g^{11}}\left(-2a\cos\theta + \frac{r_{h,\theta\theta}}{r^2} + \frac{\cot\theta}{r^2}r_{h,\theta} - \frac{\Sigma}{r} - \frac{1}{2}\frac{\partial\Sigma}{\partial r}\right) - \sqrt{2}\lambda_{2r} - \sqrt{2}\lambda_2(r_{h,\theta}\lambda_{2r} + \lambda_{2\theta})\right], \end{aligned}$$

式中 $\lambda_{1r}=\frac{\partial\lambda_1}{\partial r}$,$\lambda_{1\theta}=\frac{\partial\lambda_1}{\partial\theta}$,$\lambda_{2r}=\frac{\partial\lambda_2}{\partial r}$,$\lambda_{2\theta}=\frac{\partial\lambda_2}{\partial\theta}$.

将方程组(19)用代入法可得退耦后的方程组为

$$\frac{\partial^2\nu_1}{\partial R^2}+(\lambda_1+\lambda_2)\frac{\partial^2\nu_1}{\partial R\partial\Theta}+\lambda_1\lambda_2\frac{\partial^2\nu_1}{\partial\Theta^2}+D_{11}\frac{\partial\nu_1}{\partial R}+D_{12}\frac{\partial\nu_1}{\partial\Theta}+D_{13}\nu_1=0, \tag{21a}$$

$$\frac{\partial^2\nu_2}{\partial R^2}+(\lambda_1+\lambda_2)\frac{\partial^2\nu_2}{\partial R\partial\Theta}+\lambda_1\lambda_2\frac{\partial^2\nu_2}{\partial\Theta^2}+D_{21}\frac{\partial\nu_2}{\partial R}+D_{22}\frac{\partial\nu_2}{\partial\Theta}+D_{23}\nu_2=0, \tag{21b}$$

$$\frac{\partial^2\nu_3}{\partial R^2}+(\lambda_1+\lambda_2)\frac{\partial^2\nu_3}{\partial R\partial\Theta}+\lambda_1\lambda_2\frac{\partial^2\nu_3}{\partial\Theta^2}+D_{31}\frac{\partial\nu_3}{\partial R}+D_{32}\frac{\partial\nu_3}{\partial\Theta}+D_{33}\nu_3=0, \tag{21c}$$

$$\frac{\partial^2\nu_4}{\partial R^2}+(\lambda_1+\lambda_2)\frac{\partial^2\nu_4}{\partial R\partial\Theta}+\lambda_1\lambda_2\frac{\partial^2\nu_4}{\partial\Theta^2}+D_{41}\frac{\partial\nu_4}{\partial R}+D_{42}\frac{\partial\nu_4}{\partial\Theta}+D_{43}\nu_4=0, \tag{21d}$$

其中

$$\begin{aligned}
D_{11}&=d_{11}+d_{22}-d_{12,r}/d_{12}-\lambda_2 d_{12,\Theta}/d_{12},\\
D_{12}&=\lambda_2 d_{11}+\lambda_1 d_{22}+\lambda_{1r}-\lambda_1 d_{12,r}/d_{12}+\lambda_2\lambda_{1\Theta}-\lambda_1\lambda_2 d_{12,\Theta}/d_{12},\\
D_{13}&=d_{11}d_{22}-d_{12}d_{21}+d_{11,r}-d_{11}d_{12,r}/d_{12}+\lambda_2 d_{11,\Theta}-\lambda_2 d_{11}d_{12,\Theta}/d_{12},\\
D_{21}&=d_{11}+d_{22}-d_{21,r}/d_{21}-\lambda_1 d_{21,\Theta}/d_{21},\\
D_{22}&=\lambda_2 d_{11}+\lambda_1 d_{22}+\lambda_{2r}-\lambda_2 d_{21,r}/d_{21}+\lambda_1\lambda_{2\Theta}-\lambda_1\lambda_2 d_{21,\Theta}/d_{21},\\
D_{23}&=d_{11}d_{22}-d_{12}d_{21}+d_{22,r}-d_{22}d_{21,r}/d_{21}+\lambda_1 d_{22,\Theta}-\lambda_1 d_{22}d_{21,\Theta}/d_{21},\\
D_{31}&=d_{33}+d_{44}-d_{34,r}/d_{34}-\lambda_2 d_{34,\Theta}/d_{34},\\
D_{32}&=\lambda_2 d_{33}+\lambda_1 d_{44}+\lambda_{1r}-\lambda_1 d_{34,r}/d_{34}+\lambda_2\lambda_{1\Theta}-\lambda_1\lambda_2 d_{34,\Theta}/d_{34},\\
D_{33}&=d_{33}d_{44}-d_{34}d_{43}+d_{33,r}-d_{33}d_{34,r}/d_{34}+\lambda_2 d_{33,\Theta}-\lambda_2 d_{33}d_{34,\Theta}/d_{34},\\
D_{41}&=d_{33}+d_{44}-d_{43,r}/d_{43}-\lambda_1 d_{43,\Theta}/d_{43},\\
D_{42}&=\lambda_2 d_{33}+\lambda_1 d_{44}+\lambda_{2r}-\lambda_2 d_{43,r}/d_{43}+\lambda_1\lambda_{2\Theta}-\lambda_1\lambda_2 d_{43,\Theta}/d_{43},\\
D_{43}&=d_{33}d_{44}-d_{34}d_{43}+d_{44,r}-d_{44}d_{43,r}/d_{43}+\lambda_1 d_{44,\Theta}-\lambda_1 d_{44}d_{43,\Theta}/d_{43}.
\end{aligned} \tag{22}$$

式中 $d_{ij,r}=\frac{\partial d_{ij}}{\partial r}$,$d_{ij,\Theta}=\frac{\partial d_{ij}}{\partial\Theta}=r_{\mathrm{h},\theta}\frac{\partial d_{ij}}{\partial r}+\frac{\partial d_{ij}}{\partial\theta}$,($i=1$,2,3,4;$j=1$,2,3).

下面利用 WKB 近似,求自由能和熵.设波函数的四个分量

$$\nu_j=\exp(\mathrm{i}S_j(R,\Theta)/\hbar)\quad(j=,1,2,3,4), \tag{23}$$

则径向动量和角向动量分别为

$$K_{jR}=\frac{\partial S_j}{\partial R},$$

$$K_{j\Theta}=\frac{\partial S_j}{\partial\Theta}\qquad(j=,1,2,3,4). \tag{24}$$

将(23)式代入(21)式,方程组两边同乘 $\hbar^2$,并令 $\hbar\to0$,可求得

$$-g^{11}K_{1R}^2-2(E-\Omega K_{1\Theta})K_{1R}+\frac{K_{1\Theta}^2}{r^2}+\frac{m^2}{r^2\sin^2\theta}=0, \tag{25a}$$

$$-g^{11}K_{2R}^2-2(E-\Omega K_{2\Theta})K_{2R}+\frac{K_{2\Theta}^2}{r^2}+\frac{m^2}{r^2\sin^2\theta}=0, \tag{25b}$$

$$-g^{11}K_{3R}^2-2(E-\Omega K_{3\Theta})K_{3R}+\frac{K_{3\Theta}^2}{r^2}+\frac{m^2}{r^2\sin^2\theta}=0, \tag{25c}$$

$$-g^{11}K_{4R}^2-2(E-\Omega K_{4\Theta})K_{4R}+\frac{K_{4\Theta}^2}{r^2}+\frac{m^2}{r^2\sin^2\theta}=0. \tag{25d}$$

显然,四个分量对自由能的贡献相同.下面以第一分量为例来计算.为了方便,在此去掉 K_{1R},$K_{1\Theta}$ 中的下标 1.求解方程(25a),可得

$$\begin{aligned}
K_R^{\pm}&=\frac{E-\Omega K_\Theta}{-g^{11}}\\
&\pm\frac{\sqrt{(E-\Omega K_\Theta)^2+g^{11}\left(\frac{K_\Theta^2}{r^2}+\frac{m^2}{r^2\sin^2\theta}\right)}}{-g^{11}}\\
&=\frac{E-\Omega K_\Theta}{-g^{11}}\pm k_R.
\end{aligned} \tag{26}$$

根据量子统计理论,波函数第一分量对系统自由能的贡献可以表示为[10,18]

$$F_1 = -\int \mathrm{d}E \frac{\Gamma_1(E)}{\mathrm{e}^{\beta(E-\Omega K_\Theta)}-1}, \tag{27}$$

其中 $\Gamma(E)$是能量小于或等于 E 的微观态的数目.根据半经典量子化条件和薄层模型有[10]

$$\begin{aligned}\Gamma_1(E) &= \frac{1}{4\pi^3}\int \mathrm{d}\theta\mathrm{d}\varphi\int \mathrm{d}m\int \mathrm{d}K_\Theta\left(\int_\varepsilon^{\varepsilon+\delta}K_R^+\mathrm{d}R + \int_{\varepsilon+\delta}^{\varepsilon}K_R^-\mathrm{d}R\right)\\ &= \frac{1}{2\pi^3}\int \mathrm{d}m\int \mathrm{d}\theta\mathrm{d}\varphi\int \mathrm{d}K_\Theta\int_\varepsilon^{\varepsilon+\delta}k_R\mathrm{d}R.\end{aligned} \tag{28}$$

引入 $\tilde{E} = E - \Omega K_\Theta$,则

$$k_R = \frac{\sqrt{\tilde{E}^2 + g^{11}\left(\frac{K_\Theta^2}{r^2} + \frac{m^2}{r^2\sin^2\theta}\right)}}{-g^{11}},$$

$$F_1 = -\int \mathrm{d}\tilde{E}\frac{\Gamma_1(\tilde{E})}{\mathrm{e}^{\beta\tilde{E}}-1}. \tag{29}$$

视界面上的自由能的面密度可以表示为

$$\sigma_{F_1} = -\int \mathrm{d}\tilde{E}\frac{\sigma_{\Gamma_1}}{\mathrm{e}^{\beta\tilde{E}}-1}, \tag{30}$$

其中 σ_{F_1} 和 σ_{Γ_1} 定义为

$$F_1 = \int \sigma_{F_1}\mathrm{d}A, \Gamma(\tilde{E}) = \int \sigma_{\Gamma_1}\mathrm{d}A, \tag{31}$$

将(29)式中的 k_R 代入(28)式,先求出对 m 和 K_Θ 的积分,其积分上下限由 k_R 中根式定出.得

$$\Gamma(\tilde{E}) = \frac{\tilde{E}^3}{6\pi^2}\int \mathrm{d}A\int_\varepsilon^{\varepsilon+\delta}(g^{11})^{-2}\mathrm{d}R, \tag{32}$$

式中 $\mathrm{d}A = r^2\sin\theta\mathrm{d}\theta\mathrm{d}\varphi$,对于视界面上的观者,$\mathrm{d}A$ 表示面元.把(32)式与(31)式比较得

$$\sigma_{\Gamma_1} = \frac{\tilde{E}^3}{6\pi^2}\int_\varepsilon^{\varepsilon+\delta}(g^{11})^{-2}\mathrm{d}R. \tag{33}$$

自由能的面密度为

$$\begin{aligned}\sigma_{F_1} &= -\frac{1}{6\pi^2}\int_0^{+\infty}\mathrm{d}\tilde{E}\frac{\tilde{E}^3}{\mathrm{e}^{\beta\tilde{E}}-1}\int_\varepsilon^{\varepsilon+\delta}(g^{11})^{-2}\mathrm{d}R\\ &= -\frac{\pi^2}{90\beta^4}\int_\varepsilon^{\varepsilon+\delta}(g^{11})^{-2}\mathrm{d}R.\end{aligned} \tag{34}$$

由

$$S = \beta^2\left(\frac{\partial F_1}{\partial\beta}\right)_{\beta_\mathrm{h}}, \tag{35}$$

可得波函数第一分量产生的熵的面密度为

$$\sigma_{S_1} = \frac{4\pi^2}{90\beta_\mathrm{h}^3}\int_\varepsilon^{\varepsilon+\delta}(g^{11})^{-2}\mathrm{d}R. \tag{36}$$

把 g^{11}在 r_h 附近展开成泰勒级数

$$\begin{aligned}g^{11} &\approx (g^{11})|_{r=r_\mathrm{h}} + \left(\frac{\partial g^{11}}{\partial r}\right)|_{r=r_\mathrm{h}}(r-r_h)\\ &= \left(\frac{\partial g^{11}}{\partial r}\right)|_{r=r_\mathrm{h}}(r-r_\mathrm{h}).\end{aligned} \tag{37}$$

令

$$f_\mathrm{h} = \left(\frac{\partial g^{11}}{\partial r}\right)|_{r=r_\mathrm{h}}, \tag{38}$$

完成对 R 的积分得

$$\sigma_{S_1} = \frac{4\pi^2}{90\beta_\mathrm{h}^3 f_\mathrm{h}^2}\int_\varepsilon^{\varepsilon+\delta}\frac{1}{R^2}\mathrm{d}R = \frac{4\pi^2}{90\beta_\mathrm{h}^3 f_\mathrm{h}^2}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{39}$$

由参考文献[15]提供的方法,不难求得下面的关系式

$$\frac{f_\mathrm{h}}{2\kappa} = -\left(1 - 2r_{\mathrm{h},\nu} - 2a\sin\theta r_{\mathrm{h},\theta} + \frac{2r_{\mathrm{h},\theta}^2}{r_\mathrm{h}^2}\right). \tag{40}$$

将(40)式代入(39)式,并考虑到 Dirac 粒子波函数其他三个分量的贡献,得总熵密度为

$$\begin{aligned}\sigma_S &= 4\sigma_{S_1} = \frac{1}{90\beta_\mathrm{h}}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\\ &\times\frac{1}{(1-2r_{\mathrm{h},\nu} - 2a\sin\theta r_{\mathrm{r},\theta} + 2r_{\mathrm{h},\theta}^2/r_\mathrm{h}^2)^2}.\end{aligned} \tag{41}$$

3. 讨 论

在上面的计算中,对能量的积分已换成对 $\tilde{E}$ 的积分,实际求出的是热辐射的熵密度.对于直线加速动态黑洞存在黑洞视界和 Rindler 视界,且(2)式适用于二种视界面,因此,熵密度公式(41)式适用于二种视界.下面分别进行讨论.

1)黑洞视界面上的总熵

由(41)式,得黑洞视界面上的总熵为

$$\begin{aligned}S &= \int\sigma_s\mathrm{d}A = \int\frac{1}{90\beta_\mathrm{h}}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\\ &\times\frac{1}{(1-2r_{\mathrm{h},\nu} - 2a\sin\theta r_{\mathrm{h},\theta} + 2r_{\mathrm{h},\theta}^2/r_h^2)^2}\mathrm{d}A.\end{aligned} \tag{42}$$

a)若 $a=0$,则黑洞退化为球对称的情况.此时 $r_{\mathrm{h},\theta}=0$,$r_{\mathrm{h},\nu}$与 θ 无关.积分(42)式得

$$S = \frac{1}{90\beta_\mathrm{h}}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\frac{A_\mathrm{h}}{(1-2r_{\mathrm{h},\nu})^2}, \tag{43}$$

式中 A_h 为黑洞视界面积.若选定

$$\varepsilon' = \frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_\mathrm{h}(1-2r_{\mathrm{h},\nu})^2, \tag{44}$$

得

$$S = A_\mathrm{h}, \tag{45}$$

此时热辐射的熵正比于黑洞视界面积,但显然此时的截断因子 ε'是时间依赖的.

b)若 $a\neq 0$,选定

$$\varepsilon' = \frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\mathrm{h}}(1 - 2r_{\mathrm{h},\nu} - 2a\sin\theta r_{\mathrm{r},\theta} + 2r_{\mathrm{h},\theta}^2/r_{\mathrm{h}}^2)^2, \quad (46)$$

则仍可得出 $S = A_h$,此时的截断因子 ε' 显然也是各点不同,而且是时间依赖的.

2)Rindler 视界面上的熵密度

由(41)式,若选截断因子 ε 和薄层厚度 δ,使得 ε' 满足(46)式,则有

$$\sigma_{SR} = 1. \quad (47)$$

当 $M = 0$,$a = \mathrm{const}.$,则因[15]

$$\beta = \frac{2\pi}{a}, r_{\mathrm{h}} = \frac{1}{a(1+\cos\theta)},$$
$$r_{\mathrm{h},\nu} = 0, r_{\mathrm{h},\theta} = \frac{\sin\theta}{a(1+\cos\theta)^2}, \quad (48)$$

则此时的截断因子

$$\varepsilon' = \frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\mathrm{h}} \quad (49)$$

是与 ν 和 θ 无关的.与文献[19]结果相同.

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Entropy of Dirac field in a straightly accelerating non-stationary black hole*

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Abstract

Using the thin film model which is based on the brick – wall method, we have calculated the entropy of Dirac field on event horizon and on unit area of Rindler horizon to a straightly accelerating black hole. The conclusion that the black hole entropy is proportional to its area can still be valid by regulating the cut-off ε and the film's thickness δ, which are time dependent.

Keywords: entropy, accelerating non-stationary black hole, thin film model, Dirac field, Dirac equations
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