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(2+1)维 Camassa-Holm 方程的相似约化与解析解*

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将 Clarkson 和 Krushal 引入的直接约化方法推广并应用到(2+1)维 Camassa-Holm 方程组,获得了该方程的若干相似约化和解析解,其中包括 Logistic 方程和 Bernoulli 方程.约化结果得到了 Peakon 解、Cuspon 解和关于时间 t 的奇异解.该方法也适用于其他有重要物理背景的非线性演化方程.

关键词:Camassa-Homl 方程,相似约化,直接方法,解析解

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1. 引　言

随着科学技术的不断发展和人们对自然现象的不断深入了解,很多有重要意义的自然科学与工程技术常常要归结为非线性方程,特别是孤子方程,然而求解非线性方程较困难,这需要首先对方程进行约化.目前存在三种有效的偏微分方程相似约化方法:(1)经典法——无穷小变量的 Lie 群法[1,2];(2)非经典法——条件对称法[3,4];(3)CK 法——直接约化方法[5,6].直接约化方法没有引入群的概念,在一定程度上简化了繁琐的数学运算,并且用该方法很可能得到经典 Lie 群法所不能得到的约化结果.这种方法已被广泛地运用于(1+1)维、(2+1)维非线性演化方程,如 Boussinesq 方程、Burgers 方程、KP 方程、KdV 方程、mKdV 方程、WBK 方程、ZK 方程及 SKdV方程等[7—22].但在(2+1)维非线性方程组中的应用,特别是系统的方程个数大于 2 的推广甚少.本文以(2+1)维的 Camassa-Holm 方程组(CHEs)

$$v_t - uv_x + 2vu_x = 0, \tag{1}$$

$$u_y + vu_x - vu_{xx} - 2vw_x = 0, \tag{2}$$

$$w_x + (vw)_x + (vw_x)_x = 0 \tag{3}$$

为例,进行进一步研究.Camassa-Holm 方程组是 Kraenkel, Senthilvelan 和 Zenchuk[2] 在对浅水波进行多尺度分析研究时首先导出的,并用 Lie 对称法将系统(1)—(3)约化为(1+1)的偏微分方程组(PDEs),求得了一些特解,如 Peakon 解,然后通过提出该系统的所有 Lax 对证明了它们的可积性.本文将用 Clarkson 和 Krushal 的直接约化方法推广并应用于系统(1)—(3)中,将它们约化为常微分方程(ODE),获得了若干相似约化和解析解,其中包括 Logistic 方程,Peakon 解、Cuspon 解及显含时间 t 的奇异解.

2. Camassa-Holm 方程的相似约化与解析解

类似于文献[1,5]的方法,对于方程(1)—(3),寻求下述形式的相似解:

$$u(x,y,t) = \alpha(x,y,t) + \beta(x,y,t)H(z), \tag{4}$$

$$v(x,y,t) = a(x,y,t) + b(x,y,t)Q(z), \tag{5}$$

$$w(x,y,t) = \xi(x,y,t) + \eta(x,y,t)F(z),$$
$$z = z(x,y,t), \tag{6}$$

其中 $\alpha,\beta,a,b,\xi,\eta,z$ 分别为上述表示式中变量 x, y, t 的函数,$H(z)$, $Q(z)$, $F(z)$ 只是 z 的函数且满足常微分方程.将(4)—(6)式代入系统(1)—(3)

$$\begin{aligned} & v_t - uv_x + 2vu_x \\ &= R_1 + R_2 Q + R_3 H + R_4 HQ + R_5 Q' \\ &\quad + R_6 H' - b\beta z_x HQ' + 2b\beta z_x H'Q = 0, \end{aligned} \tag{7}$$

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$$
\begin{aligned}
&u_y + vu_x - vu_{xx} - 2vw_x \\
&= S_7 - S_8 F + S_9 H + S_{10} Q - S_{11} FQ \\
&\quad + S_{12} HQ + S_{13} H' - S_{14} F' + S_{15} H'Q \\
&\quad - S_{16} F'Q - S_{17} H'' - b\beta z_x^2 H''Q \\
&= 0,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
&w_x + (vw)_x + (vw_x)_x \\
&= T_{18} + T_{19} F + T_{20} Q + T_{21} FQ + T_{22} F' \\
&\quad + T_{23} Q' + T_{24} FQ' + T_{25} F'Q + T_{26} F'' \\
&\quad + b\eta z_x^2 (F'Q)' \\
&= 0,
\end{aligned} \tag{9}
$$

其中“$'$”和“$''$”分别为 $\mathrm{d}/\mathrm{d}z$ 和 $\mathrm{d}^2/\mathrm{d}z^2$，

$$R_1 = 2a\alpha_x - \alpha a_x + a_t,$$
$$R_2 = 2b\alpha_x - \alpha b_x + b_t,$$
$$R_3 = 2a\beta_x - \beta a_x,$$
$$R_4 = 2b\beta_x - \beta b_x,$$
$$R_5 = bz_t - \alpha bz_x,$$
$$R_6 = 2a\beta z_x,$$
$$S_7 = a\alpha_x - 2a\xi_x - a\alpha_{xx} + \alpha_y,$$
$$S_8 = 2a\eta_x,$$
$$S_9 = a\beta_x - a\beta_{xx} + \beta_y,$$
$$S_{10} = b\alpha_x - 2b\xi_x - b\alpha_{xx},$$
$$S_{11} = 2b\eta_x,$$
$$S_{12} = b\beta_x - b\beta_{xx},$$
$$S_{13} = a\beta z_x + \beta z_y - 2a\beta_x z_x - a\beta z_{xx},$$
$$S_{14} = 2a\eta z_x,$$
$$S_{15} = b\beta z_x - 2b\beta_x z_x - b\beta z_{xx},$$
$$S_{16} = 2b\eta z_x,$$
$$S_{17} = a\beta z_x^2,$$
$$T_{18} = \xi_y + (a\xi + a\xi_x)_x,$$
$$T_{19} = \eta_y + (a\eta + a\eta_x)_x,$$
$$T_{20} = (b\xi + b\xi_x)_x,$$
$$T_{21} = (b\eta + b\eta_x)_x,$$
$$T_{22} = \eta z_y + a\eta(z_x + z_{xx}) + \eta a_x z_x + 2a\eta_x z_x,$$
$$T_{23} = b(\xi + \xi_x)z_x,$$
$$T_{24} = b(\eta + \eta_x)z_x,$$
$$T_{25} = b\eta(z_x + z_{xx}) + \eta b_x z_x + 2b\eta_x z_x,$$
$$T_{26} = a\eta z_x^2.$$

根据假设，方程(7)—(9)为 H，Q，F 关于 z 的常微分方程组，所以 H，Q，F 及其导数乘积的系数之比只是关于 z 的函数，即要满足：

$$R_i = b\beta z_x \Gamma_i(z), \qquad i = 1,2,\cdots,6, \tag{10}$$

$$S_j = b\beta z_x^2 \Gamma_j(z), \qquad j = 7,8,\cdots,17, \tag{11}$$

$$T_k = b\eta z_x^2 \Gamma_k(z), \qquad k = 18,19,\cdots,26, \tag{12}$$

其中 $\Gamma_i(z)$ $(i=1,2,\cdots,26)$ 为 z 的任意函数.

类似于文献[5,6]，为了求解方程(10)—(12)，决定 $\alpha,\beta,a,b,\xi,\eta,H,Q,F,z$，并注意到在设定相似解(4)—(6)式时存在一定的自由度，为了固定其自由度，应作一定规则[1,5]，因这些规则已在许多文献中表述，在此不再赘述. 但须注意，每个规则使用相应的自由度，且只能使用一次，过多地使用规则将失去一般性. 运用这些规则，可以求得

$$
\begin{aligned}
\Gamma_6 &= \Gamma_1 = \Gamma_3 = \Gamma_{14} = \Gamma_{17} = \Gamma_{26} = \Gamma_5 \\
&= \Gamma_{16} - 2 = \Gamma_{13} - 1 = \Gamma_{22} - 1 = \Gamma_{12} = \Gamma_4 \\
&= \Gamma_{11} = \Gamma_{15} - 1 = \Gamma_{24} - 1 = \Gamma_{25} - 1 = \Gamma_{24} \\
&= \Gamma_{18} = \Gamma_{20} = \Gamma_{21} = \Gamma_{22} - 1 = \Gamma_{23} = \Gamma_7 \\
&= \Gamma_2 = 0,
\end{aligned} \tag{13}
$$

$$\Gamma_9 = \Gamma_{19} = N, \tag{14}$$

$$a = \xi = 0, \tag{15}$$

$$z(x,y,t) = x + \lambda y + \theta(t), \tag{16}$$

$$\alpha = \frac{z_t}{z_x} = \theta_t(t), \tag{17}$$

$$b = \frac{z_y}{z_x^2} = \lambda, \tag{18}$$

$$\beta = \eta = \sigma(t)\exp(N\lambda y), \tag{19}$$

其中 N，λ 为任意常数，$\theta(t)$，$\sigma(t)$ 为 t 的任意函数.

下面通过(13)—(19)式，讨论系统(1)—(3)约化的一般情形及其特解.

当 $N \neq 0$ 时，可得系统(1)—(3)的相似约化.

$$u(x,y,t) = \theta_t(t) + \sigma(t)\exp(N\lambda y)H(z), \tag{20}$$

$$v(x,y,t) = \lambda Q(z), \tag{21}$$

$$w(x,y,t) = \sigma(t)\exp(N\lambda y)H(z), \tag{22}$$

$$2H'Q - HQ' = 0, \tag{23}$$

$$H''Q + 2F'Q - H'Q - H' - NH = 0, \tag{24}$$

$$(F'Q)' + (FQ)' + F' + NF = 0. \tag{25}$$

当 $N=0$ 时，则可得方程(23)—(25)的一些特解，如 Peakon 解、Cuspon 解.

$$
\begin{aligned}
u(x,y,t) &= \theta_t + \sigma(t)H(z), \\
v(x,y,t) &= \lambda Q(z), \\
w(x,y,t) &= \sigma(t)H(z),
\end{aligned} \tag{26}
$$

$$2H'Q - HQ' = 0, \tag{27}$$

$$H''Q + 2F'Q - H'Q - H' = 0, \tag{28}$$

$$(F'Q)' + (FQ)' + F' = 0. \tag{29}$$

方程(27)—(29)可以转化为椭圆积分方程. 对方程(27)—(29)积分一次，得

$$Q = kH^2, \tag{30}$$

$$F = \frac{1}{2}\left(-H' + H - \frac{1}{kH} - b_1\right), \tag{31}$$

$$F' + F + \frac{F}{Q} - \frac{b_2}{Q} = 0, \tag{32}$$

其中 k，b_1 和 b_2 为任意积分常数. 将方程(30)和(31)代入方程(32)，乘以 H' 后积分一次，得

$$(H')^2 = \frac{H^4 + C_1 H^3 + C_2 H^2 + C_3 H^3 + C_4}{H^2}, \tag{33}$$

其中 $C_1 = 2b$，$C_2 = -4b_3$，$C_3 = -\dfrac{2}{k(b_1 - b_2)}$，$C_4 = \dfrac{1}{k^2}$，$b_3 = \mathrm{const.}$，当选取适当的常数后，方程(33)就是文献[2]中的方程(45)，(63)，(67)，(69)的情况. 而 Camassa-Holm 方程中具有重要意义的物理量只是 $u(x,y,t)$[2]. 若取 $\theta_t(t) = \sigma(t) = \mathrm{const.}$ 时，$u(x,y,t) = C_0 H(z)$，所以他们对 $H(z)$ 作了详细的讨论. 这里只写出方程(33)的结果.

若 $C_3 = C_4 = 0$，从方程(33)得 Peakon 解

$$H(z) = c\exp(-|z|) + H_0. \tag{34}$$

上式图示见图 1.

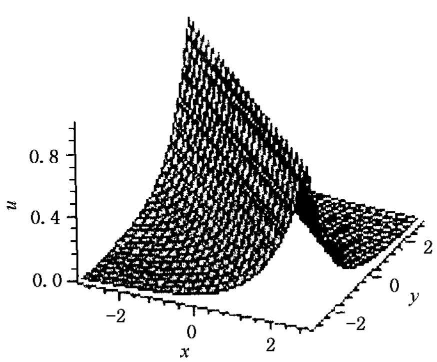

图 1 $u(x,y,t) = \exp(-|x+y+t|)$，$t = 0$，$x = -3\cdots3$，$y = -3\cdots3$ 时的 Peakon 解

若在 $z = 0$ 处 $H = 0$，并且 $C_3 > 0$，$C_4 = 0$，方程(33)有 Cuspon 解.

$$H(z) = \gamma z^{2/3}\,|_{z\to 0},$$

$$\gamma = \left(\frac{3}{2}\sqrt{C_3}\right)^{2/3}. \tag{35}$$

对方程(27)—(29)求解时，如果积分常数全取为零，还可以进一步将系统(1)—(3)约化为

$$Q = H^2, \tag{36}$$

$$2F = (H - H^{-1} - H'), \tag{37}$$

$$H'' = H - H^{-3}. \tag{38}$$

对(38)式积分二次，得

$$H^2 + \sqrt{H^4 + 1} = C\exp(\pm 2z), \tag{39}$$

其显式精确解由 Maple 或 Mathematica 容易解得

$$H(z) = \pm\frac{\sqrt{2C\exp(\pm z)(-1 + C^2\exp(\pm 4z))}}{2C\exp(\pm z)}. \tag{40}$$

特别当 $H^4 \geqslant 1$ 时，

$$H = C_0\exp(\pm z), \tag{41}$$

$$Q = H^2 = C_0^2\exp(\pm 2z), \tag{42}$$

$$F_1 = -\frac{C_0}{2}\exp(-z),$$

$$F_2 = -\frac{C_0}{2}\exp(z) + C_0\exp(-z). \tag{43}$$

从而得到系统(1)—(3)二组解

$$u_1(x,y,t) = \theta_t(t) + C_0\sigma(t)\exp[x + \lambda y + \theta(t)], \tag{44}$$

$$v_1(x,y,t) = C_0^2\lambda\exp[2(x + \lambda y + \theta(t))], \tag{45}$$

$$w_1(x,y,t) = -\frac{C_0}{2}\sigma(t)\exp[-(x + \lambda y + \theta(t))] \tag{46}$$

和

$$\begin{aligned} u_2(x,y,t) &= \theta_t(t) + C_0\sigma(t) \\ &\quad\times\exp[-(x + \lambda y + \theta(t))], \end{aligned} \tag{47}$$

$$v_2(x,y,t) = C_0^2\lambda\exp[-2(x + \lambda y + \theta(t))], \tag{48}$$

$$\begin{aligned} w_2(x,y,t) &= -\frac{C_0}{2}\sigma(t)\exp(x + \lambda y + \theta(t)) \\ &\quad + C_0\sigma(t)\exp[-(x + \lambda y + \theta(t))], \end{aligned} \tag{49}$$

其中 C，C_0 为任意常数.

上述的讨论仅对 $z_x \neq 0$ 的情况成立. 若 $z_x = 0$，即 $z = z(y,t)$，这时系统(1)—(3)变为

$$\begin{aligned} &bz_t Q' + (2b\beta_x - \beta b_x)HQ \\ &+ (2a\beta_x - \beta a_x)H + (2b\alpha_x - \alpha b_x + b_t)Q \\ &+ 2a\alpha_x - \alpha a_x + a_t = 0, \end{aligned} \tag{50}$$

$$\begin{aligned} &\beta z_y H' + (b\beta_x - b\beta)HQ - (2b\eta_x)FQ \\ &+ (b\alpha_x - 2b\xi_x - b\alpha_{xx})Q - (2a\eta_x)F \\ &+ (a\beta_x - \alpha\beta_{xx} + \beta_y)H + a\alpha_x - 2a\xi_x \\ &- a\alpha_{xx} + \alpha_y = 0, \end{aligned} \tag{51}$$

$$\begin{aligned} &\eta z_y F' + (b\eta + b\eta_x)_x FQ + (b\xi + b\xi_x)_x Q \\ &+ [\eta_y + (a\eta + a\eta_x)_x]F + \xi_y + (a\xi + a\xi_x)_x \\ &= 0. \end{aligned} \tag{52}$$

根据假设，方程(50)—(52)也是 H，Q，F 关于 z 的常

微分方程组，所以 H,Q,F 及其导数乘积的系数之比只是关于 z 的函数，即要满足：

$$2a\alpha_x - \alpha a_x + a_t = bz_tP_1(z), \tag{53}$$

$$2b\alpha_x - \alpha b_x + b_t = bz_tP_2(z), \tag{54}$$

$$2a\beta_x - \beta a_x = bz_tP_3(z), \tag{55}$$

$$2b\beta_x - \beta b_x = bz_tP_4(z), \tag{56}$$

$$a\alpha_x - 2a\xi_x - a\alpha_{xx} + \alpha_y = \beta z_yP_5(z), \tag{57}$$

$$2a\eta_x = \beta z_yP_6(z), \tag{58}$$

$$a\beta_x - \beta a_{xx} + \beta_y = \beta z_yP_7(z), \tag{59}$$

$$b\alpha_x - 2b\xi_x - b\alpha_{xx} = \beta z_yP_8(z), \tag{60}$$

$$2b\eta_x = \beta z_yP_9(z), \tag{61}$$

$$b\beta_x - bb_{xx} = \beta z_yP_{10}(z), \tag{62}$$

$$\xi_y + (a\xi + a\xi_x)_x = \eta z_yP_{11}(z), \tag{63}$$

$$\eta_y + (a\eta + a\eta_x)_x = \eta z_yP_{12}(z), \tag{64}$$

$$(b\xi + b\xi_x)_x = \eta z_yP_{13}(z), \tag{65}$$

$$(b\eta + b\eta_x)_x = \eta z_yP_{14}(z). \tag{66}$$

以上各式中 $P_i(z)$ $(i=1,2,\cdots,14)$ 为 $z(y,t)$ 的任意待定函数．根据类似的规则，可作与 $z_x\neq 0$ 时完全相同的分析，从方程(53)—(66)可得

$$\alpha = a = \xi = 0, \tag{67}$$

$$b = b(x,y), \tag{68}$$

$$\beta = \beta(x,t), \tag{69}$$

$$\eta = \eta(x,t), \tag{70}$$

$$\begin{aligned} P_6 &= P_1 = P_3 = P_5 = P_{11} = P_{12} \\ &= P_{13} = P_2 = P_7 = P_8 \\ &= P_9 - M = 0. \end{aligned} \tag{71}$$

这时方程(1)—(3)约化为

$$Q' + P_4(z)HQ = 0, \tag{72}$$

$$H' - MFQ + P_{10}(z)HQ = 0, \tag{73}$$

$$F' + P_{14}(z)FQ = 0, \tag{74}$$

其中 $M=\mathrm{const.}$，

$$P_4(z) = \frac{2b\beta_x - \beta b_x}{\beta z_t},$$

$$P_{10}(z) = \frac{b\beta_x - b\beta_{xx}}{\beta z_y},$$

$$P_{14}(z) = \frac{(b\eta + b\eta_x)_x}{\eta z_y}.$$

显然方程(72)—(74)存在不同类型的相似约化和相似解．这里给出一些特解，如取 $\alpha=a=\xi=0, \beta=b=x+\beta_0.\ \eta=1, z=y+t$ 时，有 $P_4=P_{10}=P_{14}=1$，则方程(72)—(74)化为

$$Q' + HQ = 0, \tag{75}$$

$$H' + HQ = 0, \tag{76}$$

$$F' + FQ = 0. \tag{77}$$

这里有两种情况的解：

1)当 $H=Q$ 时，从方程(75)—(77)可求得系统(1)—(3)的解析解

$$u(x,y,t) = \frac{x+\beta_0}{y+t+c_1}, \tag{78}$$

$$v(x,y,t) = \frac{x+\beta_0}{y+t+c_1}, \tag{79}$$

$$w(x,y,t) = \frac{c_2}{y+t+c_1}. \tag{80}$$

2)当 $H=Q-c_3, c_3>0$ 时，方程(75)—(77)将变为 Logistic 方程

$$Q' = c_3Q - Q^2, \tag{81}$$

可求得系统(1)—(3)的解析解．

当 $c_4<c_3$ 时

$$u(x,y,t) = \frac{c_3}{2}(x+\beta_0)\left[-1+\tanh\frac{c_3}{2}(y+t+c_4)\right], \tag{82}$$

$$v(x,y,t) = \frac{c_3}{2}(x+\beta_0)\left[1+\tanh\frac{c_3}{2}(y+t+c_4)\right], \tag{83}$$

$$w(x,y,t) = \frac{c_5}{1-\frac{c_4}{c_3}+\frac{c_4}{c_3}\exp[c_3(y+t)]}. \tag{84}$$

当 $c_4>c_3$ 时

$$u(x,y,t) = \frac{c_3}{2}(x+\beta_0)\left[-1+\coth\frac{c_3}{2}(y+t+c_4)\right], \tag{85}$$

$$v(x,y,t) = \frac{c_3}{2}(x+\beta_0)\left[1+\coth\frac{c_3}{2}(y+t+c_4)\right], \tag{86}$$

$$w(x,y,t) = \frac{c_5}{1-\frac{c_4}{c_3}+\frac{c_4}{c_3}\exp[c_3(y+t)]}, \tag{87}$$

其中 c_i $(i=1,2,\cdots,5)$，β_0 为任意常数．

3. 结　论

本文将有效的直接约化法成功地推广并应用于(2+1)维的 Camassa-Holm 方程组，获得了若干不同类型的相似约化和解析解，其中包括 Riccati 方程、Logistic 方程，Peakon 解、Cuspon 解及显含时间的奇异解．方程(33)与文献[2]经典 Lie 群法所得结果相

同.相信这种方法可进一步推广并应用于其他有重要物理背景的非线性微分方程.但是如何运用条件对称主法,即非经典 Lie 群法得到本文所给出的所有结果或者得到其他新的结果等问题,有待于进一步研究.

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Similarity reductions and analytic solutions for the (2+1)-dimensional Camassa-Holm equations*

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Abstract

In this paper, the direct method proposed by Clarkson and Kruskal (CK) is extended and applied to the (2+1)-dimensional Camassa-Holm equations. As a result, several types of similarity reductions and analytic solutions are obtained, which contain Riccati equation, Logistic equation, Peakon solution, Cuspon solution and singularity solutions of t. The CK method can be applied to other nonlinear physics models.

Keywords: Camassa-Holm equations, direct method, similarity reduction, analytic solution

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