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匀加速直线运动的 Kerr 黑洞的非热效应*

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研究了匀加速直线运动的 Kerr 黑洞周围时空中的自发辐射.得到了发生自发辐射的能量条件，它不仅依赖于黑洞的角动量和加速度，而且还与黑洞的变化有关.

关键词：动态 Kerr 黑洞，非热辐射
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1. 引 言

Kerr 度规描述一个匀角速转动球体的外部引力场，它是一个稳态轴对称度规.文献[1]通过复坐标变换得到了作任意加速运动的动态 Kerr 解，文献[2]和[3]分别对这类动态黑洞的特征曲面和热辐射进行了研究.本文将进一步研究作匀加速直线运动的动态 Kerr 黑洞的非热辐射问题.

匀加速直线运动的 Kerr 黑洞用超前爱丁顿坐标表示为[1]

$$\begin{aligned} \mathrm{d}s^2 = &(1-F)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + 2f\rho^2\mathrm{d}v\mathrm{d}\theta \\ &+ 2AF\sin^2\theta\mathrm{d}v\mathrm{d}\phi - 2A\sin^2\theta\mathrm{d}r\mathrm{d}\phi \\ &- \rho^2\mathrm{d}\theta^2 - 2Af\rho^2\sin^2\theta\mathrm{d}\theta\mathrm{d}\phi \\ &- (A^2F\sin^2\theta + A^2 + r^2)\sin^2\theta\mathrm{d}\phi^2, \end{aligned} \tag{1}$$

其中

$$\begin{aligned} F &= 2ar\cos\theta + \frac{2Mr}{\rho^2} + \rho^2 f^2, \\ f &= -a\sin\theta, \\ \rho^2 &= r^2 + A^2\cos^2\theta. \end{aligned} \tag{2}$$

转动轴的北极($\theta=0$)指向加速度的方向.A 为单位质量的角动量，a 为加速度，A 和 a 都是常量.M 为源质量，是 v 的函数.

由(1)式可算出度规行列式

$$g = -\rho^4\sin^2\theta \tag{3}$$

和逆变度规

$$(g^{\mu\nu}) = \begin{bmatrix} -\frac{A^2}{\rho^2}\sin^2\theta & \frac{A^2+r^2}{\rho^2} & 0 & -\frac{A}{\rho^2} \\ \frac{A^2+r^2}{\rho^2} & 2ar\cos\theta + \frac{1}{\rho^2}(2Mr - A^2 - r^2) & f & \frac{A}{\rho^2} \\ 0 & f & -\frac{1}{\rho^2} & 0 \\ -\frac{A}{\rho^2} & \frac{A}{\rho^2} & 0 & -\frac{1}{\rho^2\sin^2\theta} \end{bmatrix}. \tag{4}$$

2. 视界面方程

我们主要对事件视界附近的非热效应感兴趣.由零曲面条件

$$n_\mu n^\mu = g^{\mu\nu}\frac{\partial G}{\partial x^\mu}\frac{\partial G}{\partial x^\nu} = 0 \tag{5}$$

来寻找上述时空的局部事件视界.由于对称性，曲面方程应与 ϕ 无关，

$$G(v, r, \theta) = 0. \tag{6}$$

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由(4),(5)和(6)式,得视界面方程

$$-A^2\sin^2\theta\left(\frac{\partial G}{\partial v}\right)^2+\left(2ar\rho^2\cos\theta+2Mr-A^2-r^2\right)\left(\frac{\partial G}{\partial r}\right)^2-\left(\frac{\partial G}{\partial\theta}\right)^2+2\left(A^2+r^2\right)\frac{\partial G}{\partial v}\frac{\partial G}{\partial r}+2f\rho^2\frac{\partial G}{\partial r}\frac{\partial G}{\partial\theta}=0. \tag{7}$$

对(7)式作广义乌龟变换[4]

$$r_*=r+\frac{1}{2\kappa}\ln[r-r_H(v,\theta)],\quad v_*=v-v_0,\quad \theta_*=\theta-\theta_0, \tag{8}$$

$$\frac{\partial}{\partial r}=\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\quad \frac{\partial}{\partial v}=\frac{\partial}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*},\quad \frac{\partial}{\partial\theta}=\frac{\partial}{\partial\theta_*}-\frac{r_H'}{2\kappa(r-r_H)}\frac{\partial}{\partial r_*}, \tag{9}$$

并且令 $r=r_H$,可得[5]

$$A^2\dot r_H^2\sin^2\theta+r_H'^2+2[(A^2+r_H^2)\dot r_H+f\rho_H^2r_H']-(2ar_H\rho^2\cos\theta+2Mr_H-A^2-r_H^2)=0, \tag{10}$$

式中 $\rho_H^2=r_H^2+A^2\cos^2\theta$,$\dot r_H=\partial r_H/\partial v$,$r_H'=\partial r_H/\partial\theta$.

(10)式就是匀加速直线运动的 Kerr 黑洞的视界面方程.显然,在视界面上 r_H 不是常数,它与 v 和 θ 都有关.这是一个动态的视界面.当 $a=0$,$\dot r_H=0$,$r_H'=0$ 时,由(10)式可得

$$r_H^{\pm}=M\pm\sqrt{M^2-A^2}. \tag{11}$$

这就是稳态 Kerr 黑洞的视界面方程.

3. 自发辐射的能量条件

质量为 μ 的标量粒子在弯曲时空中的运动方程是 Hamilton-Jacobin 方程[6]

$$g^{\mu\nu}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu}-\mu^2=0, \tag{12}$$

式中 $S=\int L\mathrm{d}\tau$ 是 Hamilton 主函数.

将(4)式代入(12)式,得

$$-A^2\sin^2\theta\left(\frac{\partial S}{\partial v}\right)^2+\left(2ar\rho^2\cos\theta+2Mr-A^2-r^2\right)\left(\frac{\partial S}{\partial r}\right)^2-\left(\frac{\partial S}{\partial\theta}\right)^2-\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial\phi}\right)^2+2(A^2+r^2)\frac{\partial S}{\partial v}\frac{\partial S}{\partial r}-2A\frac{\partial S}{\partial v}\frac{\partial S}{\partial\phi}+2f\rho^2\frac{\partial S}{\partial r}\frac{\partial S}{\partial\theta}+2A\frac{\partial S}{\partial r}\frac{\partial S}{\partial\phi}-\mu^2\rho^2=0. \tag{13}$$

在(13)式中引入广义乌龟变换,得

$$\begin{aligned}&-A^2\sin^2\theta\left[\frac{\partial S}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]^2+\left(2ar\rho^2\cos\theta+2Mr-A^2-r^2\right)\left[\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]^2\\&-\left[\frac{\partial S}{\partial\theta_*}-\frac{r_H'}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]^2-\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial\phi}\right)^2+2A\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\frac{\partial S}{\partial\phi}+2(A^2+r^2)\\&\times\left[\frac{\partial S}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}-2A\left[\frac{\partial S}{\partial v_*}-\frac{\dot r_H}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\frac{\partial S}{\partial\phi}\\&+2f\rho^2\left[\frac{2\kappa(r-r_H)+1}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]\times\left[\frac{\partial S}{\partial\theta_*}-\frac{r_H'}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*}\right]-\mu^2\rho^2=0.\end{aligned} \tag{14}$$

可以证明,该时空存在一个 Killing 矢量 $(\partial/\partial\phi)^a$,故有

$$\frac{\partial S}{\partial\phi}=m(\mathrm{const.}). \tag{15}$$

定义

$$\omega=-\frac{\partial S}{\partial v_*}, \tag{16}$$

$$l=\frac{\partial S}{\partial\theta_*}, \tag{17}$$

ω 和 l 分别是粒子的能量和角动量[7-9].引进(15)式、(16)式和(17)式,(14)式可化为

$$D\left(\frac{\partial s}{\partial r_*}\right)^2-4\kappa(r-r_H)E\left(\frac{\partial s}{\partial r_*}\right)+4\kappa^2(r-r_H)^2R=0, \tag{18}$$

式中

$$\begin{aligned}D=&A^2\dot r_H^2\sin^2\theta+r_H'^2+2\left[(A^2+r^2)\dot r_H+f\rho^2r_H'\right][2\kappa(r-r_H)+1]-(2ar\rho^2\cos\theta+2Mr\\&-A^2-r^2)[2\kappa(r-r_H)+1]^2,\end{aligned} \tag{19}$$

$$E=(Am-A^2\omega\sin^2\theta)\dot r_H+lr_H'+[f\rho^2l+Am-(A^2+r^2)\omega][2\kappa(r-r_H)+1], \tag{20}$$

$$R=A^2\omega^2\sin^2\theta+l^2+m^2/\sin^2\theta-2A\omega m+\mu^2\rho^2. \tag{21}$$

(18)式有解

$$\frac{\partial S}{\partial r_*}=\frac{2\kappa(r-r_H)}{D}\left(E\pm\sqrt{E^2-DR}\right). \tag{22}$$

因为 S 和 $\partial S/\partial r_*$ 都必须是实数，即必须满足条件

$$E^2 - DR \geqslant 0. \tag{23}$$

这就是匀加速直线运动的 Kerr 黑洞周围时空中粒子能量 ω 的方程. 在上式中取等号后解得

$$\omega^{\pm} = \frac{B_2 \pm \sqrt{B_2^2 - B_1 B_3}}{B_1}, \tag{24}$$

式中

$$B_1 = (A^2 + r^2)^2[2\kappa(r - r_H) + 1]^2 + 2\dot{r}_H A^2(A^2 + r^2)\sin^2\theta[2\kappa(r - r_H) + 1] + A^4\dot{r}_H^2\sin^4\theta - A^2 D\sin^2\theta, \tag{25}$$

$$B_2 = (f\rho^2 l + Am)(A^2 + r^2)[2\kappa(r - r_H) + 1]^2 + (Am\dot{r}_H + lr_H')A^2\dot{r}_H\sin^2\theta - AmD + \Big[(Am\dot{r}_H + lr_H')(A^2 + r^2) + A^2\dot{r}_H(f\rho^2 l + Am)\sin^2\theta\Big][2\kappa(r - r_H) + 1], \tag{26}$$

$$B_3 = (f\rho^2 l + Am)^2[2\kappa(r - r_H) + 1]^2 + 2(Am\dot{r}_H + lr_H')(f\rho^2 l + Am)[2\kappa(r - r_H) + 1] + (Am\dot{r}_H + lr_H')^2 - (l^2 + m^2/\sin^2\theta + \mu^2\rho^2)D. \tag{27}$$

(24)式给出了粒子的正能态

$$\omega \geqslant \omega^+ \tag{28}$$

和负能态

$$\omega \leqslant \omega^-. \tag{29}$$

粒子能量的禁区是

$$\omega^- < \omega < \omega^+, \tag{30}$$

禁区宽度为

$$\Delta\omega = \omega^+ - w^- = 2\sqrt{(B_2/B_1)^2 - B_3}. \tag{31}$$

4. 讨　论

1. 当 r 很大时，时空应渐近平直，即

$$F \to 0,\ f \to 0,\ A \to 0.$$

由(24)式可得

$$\omega^{\pm} \to \pm\mu, \tag{32}$$

这正是 Minkowski 时空中真空 Dirace 能级的分布. 其禁区宽度为 $\Delta\omega = 2\mu$.

2. 在视界附近，$r \to r_H$，由(19)式，得

$$\lim_{r\to r_H} D = A^2\dot{r}_H^2\sin^2\theta + r_H'^2 + 2\Big[(r_H^2 + A^2)\dot{r}_H + f\rho_H^2 r_H'\Big] - (2ar_H\rho_H^2\cos\theta + 2Mr_H - A^2 - r_H^2). \tag{33}$$

这正是零曲面方程(10)式. 因此，

$$\lim_{r\to r_H} D = 0. \tag{34}$$

又由(24)式到(27)式，得

$$\lim_{r\to r_H}(B_2^2 - B_1 B_3) = 0, \tag{35}$$

$$\omega_0 = \lim_{r\to r_H}\omega^+ = \lim_{r\to r_H}\omega^- = \frac{Am\dot{r}_H + lr_H' + f\rho_H^2 l + Am}{A^2 + r_H^2 + A^2\dot{r}_H\sin^2\theta}. \tag{36}$$

这表明，在视界上粒子能量禁区宽度 $\Delta\omega = 0$，最低正能级 ω^+ 与最高负能级 ω^- 重合. 只要 $\omega_0 > \mu$，正负能级就会发生交错现象. 在交错区域，其能量 ω 满足

$$\mu < \omega \leqslant \omega_0 \tag{37}$$

的处于负能态的粒子，将由于量子隧道效应而穿过禁区，成为正能粒子出射到远方. 这是一种与温度无关的自发辐射过程，这种非热辐射称为 Starobinsky-Unruh 过程[10].

从(36)式可以看到，ω_0 不仅与黑洞视界的变化($\dot{r}_H$ 和 r_H')有关，而且与黑洞的加速度 a 也有关. 加速度 a 对 ω_0 的影响是使得 ω_0 减少. 若黑洞的视界变化很小，$\dot{r}_H \approx 0$，$r_H' \approx 0$，则

$$\omega_0 = \frac{Am - a\rho_H^2 l\sin\theta}{A^2 + r_H^2}. \tag{38}$$

可见，当黑洞的加速度 a 大到 $\omega_0 < 0$ 时，$\omega_0 > \mu$ 不能满足，不可能有粒子自视界上出射到远方. 这种自发辐射将会导致黑洞动能(包括转动动能和平动动能)和动量的减少，同时也导致动态黑洞的变化减缓，最终成为 Schwarzschild 黑洞，不再有非热辐射.

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The nonthermal effects of a uniformly rectilinearly accelerating Kerr black hole*

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Abstract

The spontaneous radiation of a uniformly rectilinearly accelerating Kerr black hole is studied. The energy condition for the occurrence of the spontaneous radiation is obtained. It not only depends on the angular momentum and acceleration of the black hole, but also the variations of the black hole.

Keywords: nonstationary Kerr black hole, nonthermal radiation

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