第 52 卷 第 10 期 2003 年 10 月
1000-3290/2003/52(10)/2359-04
物 理 学 报
ACTA PHYSICA SINICA
Vol.52,No.10,October,2003

(2+1)维破裂孤子方程的新多孤子解*

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(2002 年 12 月 24 日收到;2003 年 1 月 24 日收到修改稿)

Hirota 双线性方法是一种非常有效的直接方法,使得求解非线性演化方程的多孤子解转化为代数求解.将这一方法进一步拓展,求得了(2+1)维破裂孤子方程的新多孤子解.

关键词:双线性方法,多孤子解,(2+1)维破裂孤子方程

PACC:0230,0340

1.引 言

自然科学领域的很多问题的数学模型最终可归结为非线性演化方程(组)来描述.由于这类方程的解析解对于洞察这些问题的物理本质具有很重要的意义,因此寻求非线性演化方程的孤子解一直是物理学和数学工作者的重点课题.十年来,在求解方法方面取得了较好的成果,如逆散射法[1],Bäcklund 变换法[2]、Hirota 变换法[2]、Darboux 变换法[2]、齐次平衡方法[3—5]、Painlevé 展开方法[6]、混合指数法[7]、双曲函数法[8,9]、试探函数法[10,11]、非线性变换法[12]、Sine-cosine 方法[13,14]、Jacobi 椭圆函数展开法[15]、截断展开方法[16]等.最近 Deng,Chen 等[17]对 Hirota 方法进行推广,求出了若干非线性孤子模型的新解.本文对此方法作适当的拓广,进一步研究(2+1)维非线性孤子方程的求解问题.本文以(2+1)维破裂孤子方程

$$u_{xt} = 2u_{xx}u_y + 4u_xu_{xy} - u_{xxxy}, \tag{1}$$

为例进行讨论.方程(1)首先由 Calogero 和 Degasperis 建立[18,19],它可以描述沿着 y 轴传播的 Riemann 波和沿着 x 轴传播的长波的(2+1)维相互作用问题[20,21].而且方程(1)的双 Hamilton 结构和 Lax 对已由李翊神给出[21].在文献[22]利用齐次平衡法,给出方程(1)的类孤子解.

2.(2+1)维破裂孤子方程的新多孤子解

利用 Painlevé 截断展开方法,可以得到方程(1)如下形式的 Bäcklund 变换

$$u(x,y,t) = -2[\ln f(x,y,t)]_x. \tag{2}$$

把(2)式代入(1)式,整理后可得

$$\begin{aligned}&2f_tf_x^2 - 2ff_xf_{xt} - ff_tf_{xx} - 4f_xf_{xy}f_{xx} - 2f_yf_{xx}^2 + f^2f_{xxt}\\&+ 4f_x^2f_{xxy} + 2ff_{xx}f_{xxy} + 4f_yf_xf_{xxx} - 4ff_xf_{xxxy}\\&- ff_yf_{xxxx} + f^2f_{xxxxy} = 0.\end{aligned} \tag{3}$$

设 f 具有以下展开形式

$$\begin{aligned}f(x,y,t) = 1 + \varepsilon f_1(x,y,t) + \varepsilon^2f_2(x,y,t)\\+ \varepsilon^3f_3(x,y,t) + \cdots,\end{aligned} \tag{4}$$

把(4)式代入(3)式,令 ε 的同次幂为零,可得

$$\xi: f_{1xxt} + f_{1xxxxy} = 0, \tag{5}$$

$$\begin{aligned}\xi^2: f_{2xxt} + f_{2xxxxy} =& 2f_{1x}f_{1xt} + f_{1t}f_{1xx} - 2f_1f_{1xxt}\\&- 2f_{1xx}f_{1xxy} + 4f_{1x}f_{1xxxy} + f_{1y}f_{1xxxx}\\&- 2f_1f_{1xxxxy},\end{aligned} \tag{6}$$

$$\begin{aligned}\xi^3: f_{3xxt} + f_{3xxxxy} =& 2f_1f_{1x}f_{1xt} - 2f_{1t}f_{1x}^2 + 2f_{2x}f_{1xt}\\&+ 2f_{1x}f_{2xt} + f_1f_{1t}f_{1xx} + f_{2t}f_{1xx}\\&+ 4f_{1x}f_{1xy}f_{1xx} + 2f_{1y}f_{1xx}^2 + f_{1t}f_{2xx} - f_1^2f_{1xxt}\\&- 2f_2f_{1xxt} - 2f_1f_{2xxt} - 4f_{1x}^2f_{1xxy}\\&- 2f_1f_{1xx}f_{1xxy} - 2f_{2xx}f_{1xxy} - 2f_{1xx}f_{2xxy}\end{aligned}$$

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* 浙江省自然科学基金(批准号:100039)资助的课题.

$$
\begin{aligned}
&-4f_{1y}f_{1x}f_{1xxx}+4f_1f_{1x}f_{1xxxy}+4f_{2x}f_{1xxxy}\\
&+4f_{1x}f_{2xxxy}+f_1f_{1y}f_{1xxxx}+f_{2y}f_{1xxxx}+f_{1y}f_{2xxxx}\\
&-f_1^2f_{1xxxxy}-2f_2f_{1xxxxy}-2f_1f_{2xxxxy},
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\xi^4:\ f_{4xxt}+f_{4xxxxy}={}&2f_2f_{1x}f_{1xt}+2f_1f_{2x}f_{1xt}\\
&-2f_{2t}f_{1x}^2-4f_{1t}f_{1x}f_{2x}+2f_{3x}f_{1xt}\\
&+2f_1f_{1x}f_{2xt}+2f_{2x}f_{2xt}+2f_{1x}f_{3xt}\\
&+f_2f_{1t}f_{1xx}+f_1f_{2t}f_{1xx}+f_{3t}f_{1xx}\\
&+4f_{2x}f_{1xy}f_{1xx}+4f_{1x}f_{2xy}f_{1xx}+2f_{2y}f_{1xx}^2\\
&+f_1f_{1t}f_{2xx}+f_{2t}f_{2xx}+4f_{1x}f_{1xy}f_{2xx}\\
&+4f_{1y}f_{1xx}f_{2xx}+f_{1t}f_{3xx}-2f_1f_2f_{1xxt}\\
&-2f_3f_{1xxt}-f_1^2f_{2xxt}-2f_2f_{2xxt}\\
&-2f_1f_{3xxt}-8f_{1x}f_{2x}f_{1xxy}-2f_2f_{1xx}f_{1xxy}\\
&-2f_1f_{2xx}f_{1xxy}-2f_{3xx}f_{1xxy}-4f_{1x}^2f_{2xxy}\\
&-2f_1f_{1xx}f_{2xxy}-2f_{2xx}f_{2xxy}-2f_{1xx}f_{3xxy}\\
&-4f_{2y}f_{1x}f_{1xxx}-4f_{1y}f_{2x}f_{1xxx}-4f_{1y}f_{1x}f_{2xxx}\\
&+4f_2f_{1x}f_{1xxxy}+4f_1f_{2x}f_{1xxxy}+4f_{3x}f_{1xxxy}\\
&+4f_1f_{1x}f_{2xxxy}+4f_{2x}f_{2xxxy}+4f_{1x}f_{3xxxy}\\
&+f_2f_{1y}f_{1xxxx}+f_1f_{2y}f_{1xxxx}+f_{3y}f_{1xxxx}\\
&+f_1f_{1y}f_{2xxxx}+f_{2y}f_{2xxxx}+f_{1y}f_{3xxxx}\\
&-2f_1f_2f_{1xxxxy}-2f_3f_{1xxxxy}-f_1^2f_{2xxxxy}\\
&-2f_2f_{2xxxxy}-2f_1f_{3xxxxy},
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
\xi^5:\ f_{5xxt}+f_{5xxxxy}={}&2f_3f_{1x}f_{1xt}+2f_2f_{2x}f_{1xt}\\
&+2f_1f_{3x}f_{1xt}-2f_{3t}f_{1x}^2-4f_{2t}f_{1x}f_{2x}\\
&-2f_{1t}f_{2x}^2-4f_{1t}f_{1x}f_{3x}+2f_{4x}f_{1xt}\\
&+2f_2f_{1x}f_{2xt}+2f_1f_{2x}f_{2xt}+2f_{3x}f_{2xt}\\
&+2f_1f_{1x}f_{3xt}+2f_{2x}f_{3xt}+2f_{1x}f_{4xt}\\
&+f_3f_{1t}f_{1xx}+f_2f_{2t}f_{1xx}+f_1f_{3t}f_{1xx}\\
&+f_{4t}f_{1xx}+4f_{3x}f_{1xy}f_{1xx}+4f_{2x}f_{2xy}f_{1xx}\\
&+4f_{1x}f_{3xy}f_{1xx}+2f_{3y}f_{1xx}^2+f_2f_{1t}f_{2xx}\\
&+f_1f_{2t}f_{2xx}+f_{3t}f_{2xx}+4f_{2x}f_{1xy}f_{2xx}\\
&+4f_{1x}f_{2xy}f_{2xx}+4f_{2y}f_{1xx}f_{2xx}+2f_{1y}f_{2xx}^2\\
&+f_1f_{1t}f_{3xx}+f_{2t}f_{3xx}+4f_{1x}f_{1xy}f_{3xx}\\
&+4f_{1y}f_{1xx}f_{3xx}+f_{1t}f_{4xx}-f_2^2f_{1xxt}\\
&-2f_1f_3f_{1xxt}-2f_4f_{1xxt}-2f_1f_2f_{2xxt}\\
&-2f_3f_{2xxt}-f_1^2f_{3xxt}-2f_2f_{3xxt}\\
&-2f_1f_{4xxt}-4f_{2x}^2f_{1xxy}-8f_{1x}f_{3x}f_{1xxy}\\
&-2f_3f_{1xx}f_{1xxy}-2f_2f_{2xx}f_{1xxy}-2f_1f_{3xx}f_{1xxy}\\
&-2f_{4xx}f_{1xxy}-8f_{1x}f_{2x}f_{2xxy}-2f_2f_{1xx}f_{2xxy}\\
&-2f_1f_{2xx}f_{2xxy}-2f_{3xx}f_{2xxy}-4f_{1x}^2f_{3xxy}\\
&-2f_1f_{1xx}f_{3xxy}-2f_{2xx}f_{3xxy}-2f_{1xx}f_{4xxy}\\
&-4f_{3y}f_{1x}f_{1xxx}-4f_{2y}f_{2x}f_{1xxx}-4f_{1y}f_{3x}f_{1xxx}\\
&-4f_{2y}f_{1x}f_{2xxx}-4f_{1y}f_{2x}f_{2xxx}-4f_{1y}f_{1x}f_{3xxx}\\
&+4f_3f_{1x}f_{1xxxy}+4f_2f_{2x}f_{1xxxy}+4f_1f_{3x}f_{1xxxy}\\
&+4f_{4x}f_{1xxxy}+4f_2f_{1x}f_{2xxxy}+4f_1f_{2x}f_{2xxxy}\\
&+4f_{3x}f_{2xxxy}+4f_1f_{1x}f_{3xxxy}+4f_{2x}f_{3xxxy}\\
&+4f_{1x}f_{4xxxy}+f_3f_{1y}f_{1xxxx}+f_2f_{2y}f_{1xxxx}\\
&+f_1f_{3y}f_{1xxxx}+f_{4y}f_{1xxxx}+f_2f_{1y}f_{2xxxx}\\
&+f_1f_{2y}f_{2xxxx}+f_{3y}f_{2xxxx}+f_1f_{1y}f_{3xxxx}\\
&+f_{2y}f_{3xxxx}+f_{1y}f_{4xxxx}-f_2^2f_{1xxxxy}\\
&-2f_1f_3f_{1xxxxy}-2f_4f_{1xxxxy}-2f_1f_2f_{2xxxxy}\\
&-2f_3f_{2xxxxy}-f_1^2f_{3xxxxy}-2f_2f_{3xxxxy}\\
&-2f_1f_{4xxxxy},
\end{aligned}
\tag{9}
$$

…….

为了求解方程(5)—(9),将 f_1 展开成线性函数和指数函数相乘后的叠加形式为

$$
\begin{aligned}
f_1&=\sum_{i=1}^{N}\eta_i e^{\xi_i},\\
\eta_i&=\alpha_i x+\beta_i t+\gamma_i y+\eta_i^0,\\
\xi_i&=k_i x+\omega_i t+p_i y+\xi_i^0,
\end{aligned}
\tag{10}
$$

式中 $\alpha_i,\beta_i,\gamma_i,\eta_i^0,k_i,\omega_i,p_i,\xi_i^0$ 都是实数.

当 $N=1$ 时,设 $\gamma_1=\alpha_1 p_1$,有

$$f_1=\eta_1 e^{\xi_1},\tag{11}$$

把(11)式代入(5)式,得到

$$
\begin{aligned}
\omega_1&=-k_1^2p_1,\\
\beta_1&=-k_1(2+k_1)\alpha_1p_1,
\end{aligned}
\tag{12}
$$

于是有

$$f_2=-\frac{\alpha_1^2}{4k_1^2}e^{2\xi_1},\tag{13}$$

进一步将(11)—(13)式代入(7)—(9)式,可知

$$f_n=0,\ (n\geqslant 3).\tag{14}$$

当 $N=2$ 时,设 $\gamma_i=\alpha_i p_i\quad(i=1,2)$,有

$$f_1=\eta_1 e^{\xi_1}+\eta_2 e^{\xi_2},\tag{15}$$

把(15)式代入(5)式,得到

$$
\begin{aligned}
\omega_1&=-k_1^2p_1,\\
\beta_1&=-k_1(2+k_1)\alpha_1p_1;\\
\omega_2&=-k_2^2p_2,\\
\beta_2&=-k_2(2+k_2)\alpha_2p_2.
\end{aligned}
\tag{16}
$$

将(15),(16)式代入(6)式,可得

$$f_2 = -\frac{\alpha_1^2}{4k_1^2}e^{2\xi_1} - \frac{\alpha_2^2}{4k_2^2}e^{2\xi_2} + \left[\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 \eta_1\eta_2 + \frac{4k_1(k_2 - k_1)\alpha_2}{(k_1 + k_2)^3}\eta_1 + \frac{4k_2(k_1 - k_2)\alpha_1}{(k_1 + k_2)^3}\eta_2 + \frac{4(k_1^2 - 4k_1k_2 + k_2^2)\alpha_1\alpha_2}{(k_1 + k_2)^4}\right]e^{\xi_1+\xi_2}, \quad (17)$$

将(15)—(17)式代入(7)式,可得

$$f_3 = \left[-\frac{\alpha_1^2}{4k_1^2}\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^4 \eta_2 + \frac{2(k_1 - k_2)^3\alpha_1^2\alpha_2}{k_1(k_1 + k_2)^5}\right]e^{2\xi_1+\xi_2} - \left[\frac{\alpha_2^2}{4k_2^2}\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^4 \eta_1 + \frac{2(k_1 - k_2)^3\alpha_1\alpha_2^2}{k_2(k_1 + k_2)^5}\right]e^{\xi_1+2\xi_2}, \quad (18)$$

将(15)—(18)式代入(8)式,可得

$$f_4 = \frac{\alpha_1^2\alpha_2^2}{16k_1^2k_2^2}\left(\frac{k_1 - k_2}{k_1 + k_2}\right)^8 e^{2\xi_1+2\xi_2}, \quad (19)$$

进一步将(15)—(19)式代入(9)式,可知

$$f_n = 0 \quad (n \geqslant 5). \quad (20)$$

由此类推,与 N 孤子解相对应的 f 可表示为

$$f = \sum_{\mu=0,1,2}\left\{\prod_{j=1}^{N}\left(\frac{2\alpha_j}{2\mathrm{i}k_j}\right)^{\mu_j(\mu_j-1)}(\alpha_j\partial_{k_j} + \eta_j^0)^{\mu_j(2-\mu_j)} \times \exp\left[\sum_{j=1}^{N}\mu_j\xi_j + \sum_{1\leqslant j\leqslant l}^{N}\mu_j\mu_l A_{jl}\right]\right\}, \quad (21)$$

其中

$$\omega_j = -k_j^2p_j, \quad \gamma_j = \alpha_jp_j,$$

$$\beta_j = -k_j(2 + k_j)\alpha_jp_j, \quad e^{A_{jl}} = \frac{(k_j - k_l)^2}{(k_j + k_l)^2}.$$

把(21)式代入(2)式,便可得(2+1)维破裂孤子方程的新多孔子解.

3. 结　论

本文在陈登远、邓淑芳和张大军对 Hirota 双线性方法进行推广,求得 KdV 方程、mKdV-sine-Gordon 方程、KP 方程的新多孤子解的基础上,进一步求得了(2+1)维破裂孤子方程的新多孤子解,扩展了本方法的应用.值得指出的是,用 Hirota 双线性方法原得到结果仅是本文结果的特例.

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Novel multi-soliton solutions of the breaking soliton equation*

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(Received 24 December 2002; revised manuscript received 24 January 2003)

Abstract

Hirota bilinear method is a very effective method for solving nonlinear evolution equations. In this paper, by further generalizing this method, we obtain novel multi-soliton solutions of (2+1)-dimensional breaking soliton equation.

Keywords: Hirota bilinear method, multi-soliton solutions, (2+1)-dimensional breaking soliton equation
PACC: 0230, 0340

* Project supported by the Natural Science Foundation of Zhejiang Province, China ((Grant No. 100039).