第 52 卷 第 11 期 2003 年 11 月
1000-3290/2003/52(11)/2664-07

物 理 学 报
ACTA PHYSICA SINICA

Vol.52, No.11, November, 2003

相对论性 Birkhoff 系统的对称性摄动及其逆问题

傅景礼[1)2)] 陈立群[2)] 谢凤萍[1)]

[1)]（商丘师范学院数学力学和数学物理研究所，商丘 476000）

[2)]（上海大学上海市应用数学和力学研究所，上海 200072）

（2002 年 12 月 10 日收到；2003 年 2 月 10 日收到修改稿）

研究在小干扰力作用下相对论性 Birkhoff 系统的对称性摄动问题.建立了相对论性 Birkhoff 系统的基本原理、运动方程和小扰动方程.讨论该系统的 Lie 对称性变换和守恒量.研究在无限小变换下该系统的对称性摄动，构造了 s 阶绝热不变量.给出了绝热不变量存在的条件和形式.研究该系统的对称性摄动逆问题，当系统存在 s 阶绝热不变量时，得到了该系统的无限小变换的对称性摄动.研究相对论性 Birkhoff 系统和经典 Birkhoff 系统对称性摄动之间的关系.

关键词：Lie 对称性，摄动，绝热不变量，相对论

PACC：0320，0414

1. 引 言

对称性是数学、力学和物理学中的一个十分重要而又普遍的性质[1—3].近年来该方面的研究十分活跃[4—9].当力学或物理系统受到小扰动时，其行为要受到微小的缓慢的影响，小扰动作用下力学系统的对称性的微小改变称之为对称性摄动.经典的绝热不变量是指当系统的某参量缓慢变化时，相对该参量的变化而改变更缓慢的某一物理量[10]，绝热不变量又称缓渐不变量或浸渐不变量[11].绝热不变量的研究始于 1911 年爱因斯坦曾就缓变长度的单摆问题中提出 E/ω 是一个绝热不变量.1917 年 Burgers 证明了对于非简谐振动 E/ω 不是一个绝热不变量，但作用量仍是一个绝热不变量[12].1962 年 Kruskal 研究了哈密顿系统的绝热不变量[13].1981 年 Djukic 研究了非保守哈密顿系统的小干扰力作用下绝热不变量[14]之后，绝热不变量的研究已成为一个热门课题，且在力学、原子与分子物理学、天体物理学、工程等诸多方面得到广泛应用[15—18].1996 年 Zhao 研究了一般力学系统的精确不变量和绝热不变量[19]，2000 年 Chen 等人研究了 Birkhoff 系统、变质量系统、包含伺服约束的非完整系统的对称性摄动与绝热不变量，并提出了 Birkhoff 系统的高阶绝热不变量的概念，揭示了该系统高阶绝热不变量和无限小 Lie 对称变换的摄动之间的关系[20—22].最近，Zhang 研究了约束哈密顿系统和单面约束 Birkhoff 系统的对称性摄动问题[23,24].但是，所有这些研究仅限于经典动力学系统.

经过 Birkhoff，Santilli 和 Mei 的相继工作，建立的比哈密顿力学更为一般的 Birkhoff 系统动力学已在近代物理中逐步扮演重要角色[25—29].近年来我们已将 Birkhoff 系统动力学研究推广到高速运动的相对论领域，得到了相对论性 Birkhoff 系统动力学的基本原理和运动方程[30]、对称性理论和代数结构[31,32]，以及平衡稳定性理论[33,34].本文基于 Lie 变换与不变量理论，研究相对论性 Birkhoff 系统受到小干扰力作用时的对称性摄动和绝热不变量问题，包括正问题和逆问题.

2. 相对论性 Pfaff-Birkhoff 原理和 Birkhoff 方程

考虑高速运动的相对论系统，第 i 个粒子的相对论性质量为[35,36]

$$m_i = m_i(t, \boldsymbol{a}) = m_{0i}/\sqrt{1 - \dot{\boldsymbol{r}}_i{}^2/c^2},$$

式中 $\boldsymbol{a} = \boldsymbol{a}(t, a^{\mu})$ 为广义坐标，m_{0i} 为第 i 个粒子的静止质量，$\boldsymbol{r}_i(t, \boldsymbol{a})$ 为第 i 个粒子的位矢，c 为光速.

利用嵌入质量的方法，定义相对论性 Pfaff 作用量为

$$A^{\cdot} = \int_{t_1}^{t_2} \{R^{\cdot}_{\nu}(m_i(t, \boldsymbol{a}), t, \boldsymbol{a})\dot{a}^{\nu} - B^{\cdot}(m_i(t, \boldsymbol{a}), t, \boldsymbol{a})\}\mathrm{d}t$$

$$(\nu = 1,\cdots,2n;i = 1,\cdots,N),\tag{1}$$

式中 B^*，R_ν^* 含有相对论性质量，分别称为相对论性 Birkhoff 函数和相对论性 Birkhoff 函数组，可以利用 Santilli 方法将它们首先构造出来.(1)式中含有相对论性质量.

对(1)式取变分，得

$$\begin{aligned}\delta A^* = \int_{t_1}^{t_2}\Big\{&\Big(\frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial R_\nu^*}{\partial a^\mu} - \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\nu} - \frac{\partial R_\mu^*}{\partial a^\nu}\Big)\dot a^\nu \\ &- \Big(\frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial B^*}{\partial a^\mu} + \frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial R_\mu^*}{\partial t}\Big)\Big\} \\ &\times \delta a^\nu \mathrm{d}t = 0 \qquad (\mu,\nu = 1,\cdots,2n).\end{aligned}\tag{2}$$

(2)式与交换关系和端点条件

$$\mathrm{d}\delta a^\nu = \delta \mathrm{d} a^\nu,\tag{3}$$

$$\delta a^\nu|_{t=t_1} = \delta a^\nu|_{t=t_2}\tag{4}$$

称为相对论性 Pfaff-Birkhoff 原理.该原理与非相对论性 Pfaff-Birkhoff 原理的区别在于 B^*，R_ν^* 含有相对论性质量和相对论效应项 $\frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu}$，$\frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\nu}$，$\frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu}$和$\frac{\partial R_\mu^*}{\partial m_i}\frac{\partial m_i}{\partial t}$.为使公式简化，令

$$\tilde B^* = \tilde B^*(t,\boldsymbol a) = B^*(m_i(t,\boldsymbol a),t,\boldsymbol a),$$
$$\tilde R_\nu^* = \tilde R_\nu^*(t,\boldsymbol a) = R_\nu^*(m_i(t,\boldsymbol a),t,\boldsymbol a),$$

则

$$\frac{\partial \tilde R_\nu^*}{\partial a^\mu} = \frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial R_\nu^*}{\partial a^\mu},\quad \frac{\partial \tilde B^*}{\partial a^\mu} = \frac{\partial B^*}{\partial m_i}\frac{\partial m_i}{\partial a^\mu} + \frac{\partial B^*}{\partial a^\mu},$$
$$\frac{\partial \tilde R_\nu^*}{\partial t} = \frac{\partial R_\nu^*}{\partial m_i}\frac{\partial m_i}{\partial t} + \frac{\partial R_\nu^*}{\partial t}.\tag{5}$$

原理中(2)式可表为

$$\begin{aligned}\delta A^* = \int_{t_1}^{t_2}\Big\{&\Big(\frac{\partial \tilde R_\nu^*}{\partial a^\mu} - \frac{\partial \tilde R_\mu^*}{\partial a^\nu}\Big)\dot a^\nu \\ &- \Big(\frac{\partial \tilde B^*}{\partial a^\mu} + \frac{\partial \tilde R_\mu^*}{\partial t}\Big)\Big\}\delta a^\mu \mathrm{d}t = 0.\end{aligned}\tag{6}$$

根据 δa^μ 的独立性，积分区间$[t_1,t_2]$的任意性，由(6)式可得到该系统的相对论性 Birkhoff 方程

$$\tilde\omega_{\mu\nu}^*\dot a^\nu - \Big(\frac{\partial \tilde B^*}{\partial a^\mu} + \frac{\partial \tilde R_\mu^*}{\partial t}\Big) = 0 \qquad (\mu,\nu = 1,\cdots,2n),\tag{7}$$

式中

$$\tilde\omega_{\mu\nu}^* = \Big(\frac{\partial \tilde R_\nu^*}{\partial a^\mu} - \frac{\partial \tilde R_\mu^*}{\partial a^\nu}\Big)\tag{8}$$

称为相对论性 Birkhoff 协变张量.(7)式可写成逆变形式

$$\dot a^\nu = \tilde\omega^{*\mu\nu}\Big(\frac{\partial \tilde B^*}{\partial a^\mu} + \frac{\partial \tilde R_\mu^*}{\partial t}\Big) \qquad (\mu,\nu = 1,\cdots,2n).\tag{9}$$

假定 $\tilde\omega_{\mu\nu}^*$ 非奇异，且

$$\tilde\omega^{*\mu\nu} = (|\tilde\omega_{\mu\nu}^*|^{-1})^{*\mu\nu} = \Big(\Big|\frac{\partial \tilde R_\beta^*}{\partial a^\alpha} - \frac{\partial \tilde R_\alpha^*}{\partial a^\beta}\Big|^{-1}\Big)^{\mu\nu}\tag{10}$$

称为相对论性 Birkhoff 逆变张量.

利用(5)式后，尽管(6)—(10)式与非相对论的情形有相同的形式，但是它们含有相对论效应项.

3. 相对论性 Birkhoff 系统的 Lie 对称性

引入时间 t 和变量 a^μ 的单参数变换

$$t^* = t + \Delta t,\quad a^{*\mu} = a^\mu + \Delta a^\mu\tag{11}$$

或写成

$$t^* = t + \varepsilon \tilde f^0(t,\boldsymbol a),\quad a^{*\mu} = a^\mu + \varepsilon \tilde F_\mu^0(t,\boldsymbol a),\tag{12}$$

式中 ε 为无限小参数，$\tilde f^0$，$\tilde F_\mu^0$ 为无限小变换生成元，这里

$$\tilde f^0(t,\boldsymbol a) = f^0(m_i(t,\boldsymbol a),t,\boldsymbol a),$$
$$\tilde F_\mu^0(t,\boldsymbol a) = F_\mu^0(m_i(t,\boldsymbol a),t,\boldsymbol a)$$

含有相对论性质量.再引入无限小变换的生成元向量

$$\tilde X^0 = \tilde f^0\frac{\partial}{\partial t} + \tilde F_\mu^0\frac{\partial}{\partial a^\mu}\tag{13}$$

和它的一次扩展

$$\tilde X^{(1)} = \tilde X^{(0)} + (\dot{\tilde F}_\mu^0 - \dot a^\mu \dot{\tilde f}^0)\frac{\partial}{\partial \dot a^\mu},\tag{14}$$

根据微分方程在无限小变换下的不变性理论，方程(9)的不变性归结为如下确定方程[31]：

$$\dot{\tilde F}_\mu^0 - \dot{\tilde f}^0\tilde\omega^{*\mu\nu}\Big(\frac{\partial \tilde B^*}{\partial a^\mu} + \frac{\partial \tilde R_\mu^*}{\partial t}\Big) = \tilde X^{(0)}\Big[\tilde\omega^{*\mu\nu}\Big(\frac{\partial \tilde B^*}{\partial a^\mu} + \frac{\partial \tilde R_\mu^*}{\partial t}\Big)\Big]$$
$$(\mu,\nu = 1,\cdots,2n).\tag{15}$$

(15)式不同于非相对论的相应公式，$\frac{\partial \tilde B^*}{\partial a^\mu}$，$\frac{\partial \tilde R_\mu^*}{\partial t}$中含有相对论效应项，于是有

定义 1　如果相对论性 Birkhoff 系统的无限小变换生成元 $\tilde f^0$，$\tilde F_\mu^0$ 满足确定方程(15)，则相应的对称性称为相对论性 Birkhoff 系统的 Lie 对称性.

相对论性 Birkhoff 系统的 Lie 对称性不一定导致守恒量.该系统存在 Noether 型守恒量的条件及守恒量的形式由下面的定理给出.

定理 1　对于满足相对论性 Birkhoff 系统确定方程(15)的无限小生成元 $\tilde f^0$，$\tilde F_\mu^0$，如果存在规范函数 $\tilde G^0 = \tilde G^0(t,\boldsymbol a) = G^0(m_i(t,\boldsymbol a),t,\boldsymbol a)$满足结构方程

$$\tilde{X}^{(1)}(\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot}) + (\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot})\dot{\tilde{f}}^{0} + \dot{\tilde{G}}^{0} = 0, \tag{16}$$

则相对论性 Birkhoff 系统存在 Noether 型守恒量

$$I^{\cdot} = \tilde{R}^{\cdot}_{\mu}\tilde{F}^{0}_{\mu} - \tilde{B}^{\cdot}\tilde{f}^{0} + \tilde{G}^{0} = \mathrm{const}. \tag{17}$$

(17)式也称为 Lie 对称性的 Noether 型精确不变量.(16)和(17)式也不同于非相对论的相应公式,它们含有相对论性质量.

4. 相对论性 Birkhoff 系统的对称性摄动与绝热不变量

当系统受到小干扰力作用时,系统的对称性要产生微小的变化,称之为系统的对称性摄动.同时与对称性相对应的守恒量也要发生相应的变化,本文用绝热不变量来描述.下面给出绝热不变量的定义.

定义 2 如果 $I_s(t, \boldsymbol{a}, \varepsilon)$是力学系统的一个含有 ε 的最高次幂为 s 的物理量,其对时间 t 的一阶导数正比于ε^{s+1},则称 I_s 为力学系统的 s 阶绝热不变量.

假设相对论性 Birkhoff 系统受到一个小扰动 $\varepsilon Q_{\mu}(\mu = 1,\cdots,2n)$的作用,则系统的运动方程为

$$\tilde{\omega}^{\cdot}_{\mu\nu}\dot{a}^{\nu} - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^{\mu}} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t}\right) + \varepsilon Q_{\mu} = 0$$

$$(\mu, \nu = 1,\cdots,2n). \tag{18}$$

由于 εQ_{μ} 的影响,系统原有的对称性与不变量相应地会发生改变,假设这种变化是在系统无扰动的 Lie 对称性变换的基础上发生的小摄动,如果

$$\tilde{f}(t, \boldsymbol{a}) = f(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}),$$
$$\tilde{F}_{\mu}(t, \boldsymbol{a}) = F_{\mu}(m_i(t, \boldsymbol{a}), t, \boldsymbol{a})$$

表示与扰动后的时间和空间对应的无限小生成元,则

$$\tilde{f} = \tilde{f}^{0} + \varepsilon\tilde{f}^{1} + \varepsilon^{2}\tilde{f}^{2} + \cdots,$$
$$\tilde{F}_{\mu} = \tilde{F}^{0}_{\mu} + \varepsilon\tilde{F}^{1}_{\mu} + \varepsilon^{2}\tilde{F}^{2}_{\mu} + \cdots, \tag{19}$$

式中 $\tilde{f}^{1}, \tilde{f}^{2}, \cdots; \tilde{F}^{1}_{\mu}, \tilde{F}^{2}_{\mu}, \cdots$为无限小生成元各阶小摄动项,它们也是相对论性的.无限小生成元向量及其一次扩展为

$$\tilde{X}^{(0)} = \tilde{f}\frac{\partial}{\partial t} + \tilde{F}_{\mu}\frac{\partial}{\partial a^{\mu}} \tag{20}$$

和

$$\tilde{X}^{(1)} = \tilde{X}^{(0)} + (\dot{\tilde{F}}_{\mu} - \dot{a}^{\mu}\dot{\tilde{f}})\frac{\partial}{\partial \dot{a}^{\mu}}, \tag{21}$$

满足

$$\tilde{X}^{(1)}(\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot}) + (\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot})\dot{\tilde{f}} + \varepsilon Q_{\mu}(\tilde{F}_{\mu} - \dot{a}^{\mu}\tilde{f}) + \dot{\tilde{G}} = 0, \tag{22}$$

式中 $\tilde{G}$ 为规范函数,记作

$$\tilde{G} = \tilde{G}^{0} + \varepsilon\tilde{G}^{1} + \varepsilon^{2}\tilde{G}^{2} + \cdots, \tag{23}$$

式中 $\tilde{G}, \tilde{G}^{0}, \tilde{G}^{1}, \tilde{G}^{2}, \cdots$也是相对论性的,并且

$$\tilde{X}^{(1)} = \varepsilon^{k}\tilde{X}^{(1)}_{k} \qquad (k = 0,1,2,\cdots), \tag{24}$$

式中

$$\tilde{X}^{(1)}_{k} = \tilde{f}^{k}\frac{\partial}{\partial t} + \tilde{F}^{k}_{\mu}\frac{\partial}{\partial a^{\mu}} + (\dot{\tilde{F}}^{k}_{\mu} - \dot{a}^{\mu}\dot{\tilde{f}}^{k})\frac{\partial}{\partial \dot{a}^{\mu}}. \tag{25}$$

将(19)和(23)—(25)式代入(22)式,得到结构方程为

$$\tilde{X}^{(1)}_{k}(\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot}) + (\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot})\dot{\tilde{f}}^{k} + Q_{\mu}(\tilde{F}^{k-1}_{\mu} - \dot{a}^{\mu}\tilde{f}^{k-1}) + \dot{\tilde{G}}^{k} = 0, \tag{26}$$

式中 $k = 0$ 时,约定 $Q_{\mu} = 0$,那么有

定理 2 对于受到小扰动 εQ_{μ} 作用的相对论性 Birkhoff 系统,如果存在规范函数 $\tilde{G}^{k}(t, \boldsymbol{a})$,使得无限小变换生成元 $\tilde{f}^{k}(t, \boldsymbol{a})$,$\tilde{F}^{k}_{\mu}(t, \boldsymbol{a})$满足结构方程(26),则

$$I_s(t, \boldsymbol{a}, \varepsilon) = \varepsilon^{k}(\tilde{R}^{\cdot}_{\mu}\tilde{F}^{k}_{\mu} - \tilde{B}^{\cdot}\tilde{f}^{k} + \tilde{G}^{k}) \tag{27}$$

为该系统的 s 阶绝热不变量.

证明

$$\begin{aligned}\frac{\mathrm{d}I_s}{\mathrm{d}t} &= \varepsilon^{k}\Big\{\left(\frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial a^{\nu}}\dot{a}^{\nu}\right)\tilde{F}^{k}_{\mu} + \tilde{R}^{\cdot}_{\mu}\dot{\tilde{F}}^{k}_{\mu} \\ &\quad - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial t} + \frac{\partial \tilde{B}^{\cdot}}{\partial a^{\nu}}\dot{a}^{\nu}\right)\tilde{f}^{k} - \tilde{B}^{\cdot}\dot{\tilde{f}}^{k} \\ &\quad - (\tilde{R}^{\cdot}_{\mu}\dot{a}^{\mu} - \tilde{B}^{\cdot})\dot{\tilde{f}}^{k} - Q_{\mu}(\tilde{F}^{k-1}_{\mu} - \dot{a}^{\mu}\tilde{f}^{k-1}) \\ &\quad - \left(\frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t}\tilde{f}^{k} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial a^{\nu}}\tilde{F}^{k}_{\nu}\right)\dot{a}^{\nu} + \frac{\partial \tilde{B}^{\cdot}}{\partial t}\tilde{f}^{k} + \frac{\partial \tilde{B}^{\cdot}}{\partial a^{\mu}}\tilde{F}^{k}_{\mu}\Big\} \\ &= \varepsilon^{k}\Big\{\Big[\frac{\partial \tilde{B}^{\cdot}}{\partial a^{\mu}} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t} - \left(\frac{\partial \tilde{R}^{\cdot}_{\nu}}{\partial a^{\mu}} - \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial a^{\nu}}\right)\dot{a}^{\nu}\Big]\tilde{F}^{k}_{\mu} \\ &\quad - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^{\mu}} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t}\right)\dot{a}^{\mu}\tilde{f}^{k} - Q_{\mu}(\tilde{F}^{k-1}_{\mu} - \dot{a}^{\mu}\tilde{f}^{k-1})\Big\} \\ &\times \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^{\mu}} + \frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial t}\right)\dot{a}^{\mu} = \varepsilon\dot{a}^{\mu}Q_{\mu} \\ &\quad - \left(\frac{\partial \tilde{R}^{\cdot}_{\mu}}{\partial a^{\nu}} - \frac{\partial \tilde{R}^{\cdot}_{\nu}}{\partial a^{\mu}}\right)\dot{a}^{\nu}\dot{a}^{\mu} = \varepsilon\dot{a}^{\mu}Q_{\mu},\end{aligned} \tag{28}$$

那么有

$$\frac{\mathrm{d}I_s}{\mathrm{d}t} = \varepsilon^{k}[\varepsilon Q_{\mu}(\tilde{F}^{k}_{\mu} - \dot{a}^{\mu}\tilde{f}^{k}) - Q_{\mu}(\tilde{F}^{k-1}_{\mu} - \dot{a}^{\mu}\tilde{f}^{k-1})],$$

展开后可得

$$\frac{\mathrm{d}I_s}{\mathrm{d}t} = \varepsilon^{s+1} Q_\mu (\tilde{F}^s_\mu - \dot{a}^\mu \tilde{f}^s)$$
$$(\mu = 1,\cdots,2n ; s = 1,2,\cdots). \tag{29}$$

(27)式含有相对论性质量,不同于非相对论性的相应公式.

5. 相对论性 Birkhoff 系统对称性摄动的逆问题

假设相对论性 Birkhoff 系统存在 s 阶绝热不变量

$$I_s = \varepsilon^k \lambda_k (t, a^\mu)$$
$$(k = 0,1,2,\cdots ; \mu = 1,\cdots,2n), \tag{30}$$

由绝热不变量的定义,得

$$\frac{\mathrm{d}I_s}{\mathrm{d}t} = \varepsilon^k \left(\frac{\partial \lambda_k}{\partial t} + \frac{\partial \lambda_k}{\partial a^\mu} \dot{a}^\mu \right) = \varepsilon^{s+1} Q_\mu (\tilde{F}^s_\mu - \dot{a}^\mu \tilde{f}^s). \tag{31}$$

将方程(18)等号两端乘以($\tilde{F}_\mu - \dot{a}^\mu \tilde{f}$),并对 μ 求和,得

$$\left(\tilde{\omega}^{\cdot}_{\mu\nu} \dot{a}^\nu - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) + \varepsilon Q_\mu \right) (\tilde{F}_\mu - \dot{a}^\mu \tilde{f}) = 0. \tag{32}$$

(31)和(32)式等号两端分别相加,得

$$\varepsilon^k \left(\frac{\partial \lambda_k}{\partial t} + \frac{\partial \lambda_k}{\partial a^\mu} \dot{a}^\mu \right) + \left(\tilde{\omega}^{\cdot}_{\mu\nu} \dot{a}^\nu - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) + \varepsilon Q_\mu \right) (\tilde{F}_\mu - \dot{a}^\mu \tilde{f}) = \varepsilon^{s+1} Q_\mu (\tilde{F}_\mu - \dot{a}^\mu \tilde{f})$$
$$(k = 0,1,2,\cdots ; \mu, \nu = 1,\cdots,2n), \tag{33}$$

式中

$$\tilde{F}_\mu = \tilde{F}^0_\mu + \varepsilon \tilde{F}^1_\mu + \cdots + \varepsilon^k \tilde{F}^k_\mu + \cdots ;$$
$$\tilde{f} = \tilde{f}^0 + \varepsilon \tilde{f}^1 + \cdots + \varepsilon^k \tilde{f}^k + \cdots \tag{34}$$

将(33)式展开并整理后,得

$$\frac{\partial \lambda_0}{\partial t} + \frac{\partial \lambda_0}{\partial a^\mu} \dot{a}^\mu + \left(\tilde{\omega}^{\cdot}_{\mu\nu} \dot{a}^\nu - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \right) \times (\tilde{F}^0_\mu - \dot{a}^\mu \tilde{f}^0) + \varepsilon \left[\frac{\partial \lambda_1}{\partial t} + \frac{\partial \lambda_1}{\partial a^\mu} \dot{a}^\mu + \left(\tilde{\omega}^{\cdot}_{\mu\nu} \dot{a}^\nu - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) + \varepsilon Q_\mu \right) (\tilde{F}^1_\mu - \dot{a}^\mu \tilde{f}^1) \right] + \cdots + \varepsilon^k \left[\frac{\partial \lambda_k}{\partial t} + \frac{\partial \lambda_k}{\partial a^\mu} \dot{a}^\mu + \left(\tilde{\omega}^{\cdot}_{\mu\nu} \dot{a}^\nu - \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) + \varepsilon Q_\mu \right) \times (\tilde{F}^k_\mu - \dot{a}^\mu \tilde{f}^k) \right] + \cdots$$
$$= \varepsilon^{s+1} (\tilde{F}^s_\mu - \dot{a}^\mu \tilde{f}^s). \tag{35}$$

由(35)式中同次 ε^k 中 $\dot{a}^\nu$ 的系数为零,得

$$\frac{\partial \lambda_0}{\partial a^\nu} + \tilde{\omega}^{\cdot}_{\mu\nu} \tilde{F}^0_\mu + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^0 = 0$$
$$(\mu, \nu = 1,\cdots,2n), \tag{36}$$

$$\frac{\partial \lambda_1}{\partial a^\nu} + \tilde{\omega}^{\cdot}_{\mu\nu} \tilde{F}^1_\mu + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^1 = 0$$
$$(\mu, \nu = 1,\cdots,2n), \tag{37}$$

$$\frac{\partial \lambda_2}{\partial a^\nu} + \tilde{\omega}^{\cdot}_{\mu\nu} \tilde{F}^2_\mu + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^2 + Q_\mu \tilde{f}^1 = 0$$
$$(\mu, \nu = 1,\cdots,2n), \tag{38}$$

$$\cdots,\cdots$$

$$\frac{\partial \lambda_k}{\partial a^\nu} + \tilde{\omega}^{\cdot}_{\mu\nu} \tilde{F}^k_\mu + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^k + Q_\mu \tilde{f}^{k-1} = 0$$
$$(\mu, \nu = 1,\cdots,2n). \tag{39}$$

因 $\tilde{\omega}^{\cdot}_{\mu\nu}$ 非退化,从方程(36)解得

$$\tilde{F}^0_\mu = \tilde{\omega}^{\cdot\mu\nu} \left(\frac{\partial \lambda_0}{\partial a^\nu} + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^0 \right)$$
$$(\mu, \nu = 1,\cdots,2n). \tag{40}$$

令

$$\lambda_0 = \tilde{R}^{\cdot}_\mu \tilde{F}^0_\mu - \tilde{B}^{\cdot} \tilde{f}^0 + \tilde{G}^0, \tag{41}$$

由此解得

$$\tilde{f}^0 = \frac{1}{\tilde{B}^{\cdot}} (\tilde{R}^{\cdot}_\mu \tilde{F}^0_\mu - \lambda_0 + \tilde{G}^0). \tag{42}$$

(40)和(42)式确定了无扰动部分对应的时间和空间的生成元.同理可得小扰动作用下生成元的各阶摄动项的结果为

$$\tilde{F}^1_\mu = \tilde{\omega}^{\cdot}_{\mu\nu} \left(\frac{\partial \lambda_1}{\partial a^\nu} + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^1 \right)$$
$$(\mu, \nu = 1,\cdots,2n), \tag{43}$$

$$\tilde{f}^1 = \frac{1}{\tilde{B}^{\cdot}} (\tilde{R}^{\cdot}_\mu \tilde{F}^1_\mu - \lambda_1 + \tilde{G}^1), \tag{44}$$

$$\tilde{F}^2_\mu = \tilde{\omega}^{\cdot}_{\mu\nu} \left(\frac{\partial \lambda_2}{\partial a^\nu} + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^2 + Q_\mu \tilde{f}^1 \right)$$
$$(\mu, \nu = 1,\cdots,2n), \tag{45}$$

$$\tilde{f}^2 = \frac{1}{\tilde{B}^{\cdot}} (\tilde{R}^{\cdot}_\mu \tilde{F}^2_\mu - \lambda_2 + \tilde{G}^2), \tag{46}$$

$$\tilde{F}^k_\mu = \tilde{\omega}^{\cdot}_{\mu\nu} \left(\frac{\partial \lambda_k}{\partial a^\nu} + \left(\frac{\partial \tilde{B}^{\cdot}}{\partial a^\mu} + \frac{\partial \tilde{R}^{\cdot}_\mu}{\partial t} \right) \tilde{f}^k + Q_\mu \tilde{f}^{k-1} \right)$$
$$(\mu, \nu = 1,\cdots,2n), \tag{47}$$

$$\tilde{f}^k = \frac{1}{\tilde{B}^{\cdot}} (\tilde{R}^{\cdot}_\mu \tilde{F}^k_\mu - \lambda_k + \tilde{G}^k). \tag{48}$$

由(43)—(48)式知,生成元的一阶摄动项 F^1_μ, f^1 与 F^0_μ,f^0 之间不迭代,由(43)和(44)式可直接求得 ;而生成元的高阶摄动项之间互相迭代,且具有形式相同的迭代关系,可以逐次求得.

定理 3　对于受到小干扰力 εQ_μ 作用的相对论性 Birkhoff 系统,若存在 s 阶绝热不变量

$$I_s = \varepsilon^k \lambda_k(t, a^\mu)$$
$$(\mu = 1, \cdots, 2n; k = 0, 1, 2, \cdots),$$

则存在相应的无限小对称变换(40)—(48)式.

6. 相对论性 Birkhoff 系统与经典 Birkhoff 系统的对称性摄动之间的关系

在粒子的速度 $|\dot{r}| \ll c$ 的经典近似下,取 $\sqrt{1-(\dot{r}_i)^2/c^2}$ 关于幂级数展开式的前两项,则相对论性质量为

$$m_i = m_{0i}(1-(\dot{r}_i)^2 c^{-2})^{-1/2} \approx m_{0i}.$$

相对论性 Birkhoff 函数和相对论性 Birkhoff 函数组为

$$\tilde{B}^\cdot = B^\cdot(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx B(t, \boldsymbol{a}),$$
$$\tilde{R}^\cdot_\nu = R^\cdot_\nu(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx R^\cdot_\nu(t, \boldsymbol{a}),$$

无限小变换生成元及其摄动项为

$$\tilde{f}^0(t, \boldsymbol{a}) = f^0(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx f^0(t, \boldsymbol{a}),$$
$$\tilde{F}^0_\mu = F^0_\mu(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx F^0_\mu(t, \boldsymbol{a}),$$
$$\tilde{f}^1(t, \boldsymbol{a}) = f^1(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx f^1(t, \boldsymbol{a}),$$
$$\tilde{F}^1_\mu = F^1_\mu(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx F^1_\mu(t, \boldsymbol{a}),$$
$$\tilde{f}^2(t, \boldsymbol{a}) = f^2(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx f^2(t, \boldsymbol{a}),$$
$$\tilde{F}^2_\mu = F^2_\mu(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx F^2_\mu(t, \boldsymbol{a}),$$
$$\cdots, \cdots, \cdots,$$
$$\tilde{f}^k(t, \boldsymbol{a}) = f^k(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx f^k(t, \boldsymbol{a}),$$
$$\tilde{F}^k_\mu = F^k_\mu(m_i(t, \boldsymbol{a}), t, \boldsymbol{a}) \approx F^k_\mu(t, \boldsymbol{a}).$$

本文导出经典 Birkhoff 系统的对称性摄动及其逆问题.

7. 例 子

系统的 Birkhoff 函数和 Birkhoff 函数组分别为

$$B^\cdot = m((a^1)^2 + (a^2)^2); \quad R^\cdot_1 = R^\cdot_2 = 0,$$
$$R^\cdot_3 = ma^1, \quad R^\cdot_4 = ma^2, \tag{49}$$

式中

$$m = m_0 / \sqrt{1-((a^1)^2+(a^2)^2)/c^2}, \tag{50}$$

m 为相对论性质量,m_0 为静止质量,c 为光速.若系统受到小扰动作用

$$\varepsilon Q_1 = -\frac{\varepsilon}{1+b^2t^2}, \quad \varepsilon Q_2 = -\frac{\varepsilon bt}{1+b^2t^2}, \tag{51}$$

试研究该相对论系统的对称性摄动和绝热不变量.

首先研究系统的精确不变量.结构方程(16)给出为

$$\begin{aligned}
&\tilde{F}^0_1 m \frac{c^2-(a^2)^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^3 \\
&+ \tilde{F}^0_2 m \frac{a^1a^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^3 \\
&+ \tilde{F}^0_2 m \frac{c^2-(a^1)^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^4 \\
&+ \tilde{F}^0_1 m \frac{a^1a^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^4 \\
&+ (\dot{\tilde{F}}^0_3 - \dot{a}^3\dot{\tilde{f}}^0)ma^1 + (\dot{\tilde{F}}^0_4 - \dot{a}^4\dot{\tilde{f}}^0)ma^2 \\
&- 2\tilde{F}^0_1 ma^1 - \tilde{F}^0_1 ma^1\left(\frac{((a^1)^2+(a^2)^2)}{c^2-(a^1)^2-(a^2)^2}\right) \\
&- 2\tilde{F}^0_2 ma^2 - \tilde{F}^0_2 ma^2\left(\frac{((a^1)^2+(a^2)^2)}{c^2-(a^1)^2-(a^2)^2}\right) \\
&+ m(a^1\dot{a}^3 + a^2\dot{a}^4 - (a^1)^2 - (a^2)^2)\dot{\tilde{f}}^0 + \dot{\tilde{G}}^0 = 0,
\end{aligned} \tag{52}$$

方程(52)有解

$$\tilde{f}^0 = d_1, \quad \tilde{F}^0_3 = d_2, \quad \tilde{F}^0_4 = d_3,$$
$$\tilde{F}^0_1 = \tilde{F}^0_2 = 0 \qquad (d_1, d_2, d_2 \text{ 为常数});$$
$$\tilde{G}^0 = \text{const}. \tag{53}$$

由方程(17),可得精确不变量

$$I_0 = d_2 ma^1 + d_3 ma^2 - d_1 m((a^1)^2 + (a^2)^2) + \tilde{G}^0 = \text{const}. \tag{54}$$

可见,该精确不变量(54)式含有相对论性质量,不同于非相对论形式下的精确不变量.

其次研究该系统的绝热不变量.在小干扰力作用下该系统的结构方程为

$$\begin{aligned}
&\tilde{F}^k_1 m \frac{c^2-(a^2)^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^3 \\
&+ \tilde{F}^k_2 m \frac{a^1a^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^3 \\
&+ \tilde{F}^k_2 m \frac{c^2-(a^1)^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^4 \\
&+ \tilde{F}^k_1 m \frac{a^1a^2}{c^2-(a^1)^2-(a^2)^2}\dot{a}^4 \\
&+ (\dot{\tilde{F}}^k_3 - \dot{a}^3\dot{\tilde{f}}^k)ma^1 + (\dot{\tilde{F}}^k_4 - \dot{a}^4\dot{\tilde{f}}^k)ma^2 \\
&- 2\tilde{F}^k_1 ma^1 - \tilde{F}^k_1 ma^1\left(\frac{((a^1)^2+(a^2)^2)}{c^2-(a^1)^2-(a^2)^2}\right) \\
&- 2\tilde{F}^k_2 ma^2 - \tilde{F}^k_2 ma^2\left(\frac{((a^1)^2+(a^2)^2)}{c^2-(a^1)^2-(a^2)^2}\right) \\
&+ m(a^1\dot{a}^3 + a^2\dot{a}^4 - (a^1)^2 - (a^2)^2)\dot{\tilde{f}}^k \\
&- \frac{1}{1+b^2t^2}(\tilde{F}^{k-1}_1 - \dot{a}^1\tilde{f}^{k-1}) \\
&- \frac{bt}{1+b^2t^2}(\tilde{F}^{k-1}_2 - \dot{a}^1\tilde{f}^{k-1}) + \dot{\tilde{G}}^0 = 0.
\end{aligned} \tag{55}$$

当 $k=1$ 时,方程(55)有解

$$\tilde{f}^1 = d_4,\quad \tilde{F}_3^1 = d_5,\quad \tilde{F}_4^1 = d_6,$$
$$\tilde{F}_1^1 = \tilde{F}_2^1 = 0 \qquad (d_4, d_5, d_6 \text{ 为常数});$$
$$\tilde{G}^1 = \text{const}. \tag{56}$$

在小干扰力作用下,与该系统相应的一阶绝热不变量为

$$I_1 = I_0 + \varepsilon\left(d_5 ma^1 + d_6 ma^2 - d_4 m((a^1)^2 + (a^2)^2)\right) + \tilde{G}^1 = \text{const}. \tag{57}$$

当 $k = 2$ 时,方程(55)有解

$$\tilde{f}^2 = d_7,\quad \tilde{F}_3^2 = d_8,\quad \tilde{F}_4^2 = d_9,$$
$$\tilde{F}_1^2 = \tilde{F}_2^2 = 0 \qquad (d_7, d_8, d_9 \text{ 为常数});$$
$$\dot{\tilde{G}}^2 = -\frac{d_4}{1 + b^2 t^2}\dot{a}^1 - d_4\frac{bt}{1 + b^2 t^2}\dot{a}^1;$$
$$\tilde{G}^2 = -d_4\int(1 + bt)\dot{a}^1 \mathrm{d}t. \tag{58}$$

在小干扰力作用下,与该系统相应的二阶绝热不变量为

$$I_2 = I_1 + \varepsilon^2\left[d_8 ma^1 + d_8 ma^2 - d_7 m((a^1)^2 + (a^2)^2)\right] - d_4\int(1 + bt)\dot{a}^1 \mathrm{d}t. \tag{59}$$

同理可以求得在小干扰力作用下,与该系统相应的 s 阶绝热不变量,它们都含有相对论性质量,不同于非相对论形式的 s 阶绝热不变量.

[1] Djukic D S and Vujanovic B 1975 *Acta Mech*. **23** 13

[2] Vujanovic B 1978 *Int*. *Non-Linear Mech*. **13** 185

[3] Vujanovic B 1986 *Acta Mech*. **65** 63

[4] Mei F X 1999 *Applications of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing:Science Press) (in Chinese) [梅凤翔 1999 李群和李代数对约束力学系统的应用(北京:科学出版社)]

[5] Mei F X 2000 *Int*. *J*. *Non-Linear Mech*. **35** 229

[6] Mei F X 2000 *Appl*. *Mech*. *Rev*. **53** 283

[7] Li Z P 1981 *Acta Phys*. *Sin*. **30** 1599 and 1699 (in Chinese) [李子平 1981 物理学报 **30** 1599 和 1699]

[8] Li Z P 1985 *Acta Mech*. *Sin*. **5** 379

[9] Li Z P and Li X 1990 *Int*. *J*. *Theor*. *Phys*. **29** 765

[10] Zhao Y Y and Mei F X 1999 *Symmetries and Invariants of Mechanical Systems* (Beijing:Science Press) p 164 (in Chinese) [赵跃宇、梅凤翔 1999 力学系统的对称性与不变量(北京:科学出版社)第 164 页]

[11] Mei F X, Liu D and Luo Y 1991 *Advanced Analytical Mechanics* (Beijing:Beijing Institute of Technology Press) p 728 (in Chinese) [梅凤翔、刘 端、罗 勇 1991 高等分析力学(北京:北京理工大学出版社)第 728 页]

[12] Burgers J M 1917 *Ann*. *Phys*. **52** 195

[13] Kruskal M 1962 *J*. *Math*. *Phys*. **3** 806

[14] Djukic D S 1981 *Int*. *J*. *Nov-Linear Mech*. **16** 489

[15] Bulanov S V and Shaharina S G 1992 *Nucl*. *Fusion* **32** 1531

[16] Nemov V V 1999 *Phys*. *Plasmas* **6** 122

[17] Notte J, FaJans J, Chu R and Wurtele J S 1993 *Phys*. *Rev*. *Lett*. **70** 3900

[18] Muller J, Burgdorfer J and Noid D W 1995 *J*. *Chem*. *Phys*. **103** 4985

[19] Zhao Y Y and Mei F X 1996 *Acta Mech*. *Sin*. **28** 207 (in Chinese) [赵跃宇、梅凤翔 1996 力学学报 **28** 207]

[20] Chen X W, Zhang R C and Mei F X 2000 *Acta Mech*. *Sin*. **16** 282

[21] Chen X W and Mei F X 2000 *Chin*. *Phys*. **9** 721

[22] Chen X W and Mei F X 2001 *Chin*. *Quart*. *Mech*. **22** 204 (in Chinese) [陈向炜、梅凤翔 2001 力学季刊 **22** 204]

[23] Zhang Y 2002 *Acta Phys*. *Sin*. **51** 2417 (in Chinese) [张 毅 2002 物理学报 **51** 2417]

[24] Zhang Y 2002 *Acta Phys*. *Sin*. **51** 1666 (in Chinese) [张 毅 2002 物理学报 **51** 1666]

[25] Birkhoff G D 1927 *Dynamical Systems* (New York:A M S College Publ., Providence, RI)

[26] Santilli R M 1983 *Foundations of Theoretical Mechanics* Ⅱ (New York:Springer-Verlag)

[27] Mei F X, Shi R C, Zhang Y F and Wu H B 1996 *Dynamical of Birkhoffian Systems* (Beijing:Beijing Institute of Technology Press) (in Chiese) [梅凤翔、史荣昌、张永发、吴惠彬 1996 Birkhoff 系统动力学(北京:北京理工大学出版社)]

[28] Mei F X 2001 *Int*. *J*. *Non-Linear Mech*. **36** 817

[29] Mei F X 1993 *China Sci*. A **23** 709

[30] Fu J L *et al* 2001 *Acta Phys*. *Sin*. **50** 2289 (in Chinese) [傅景礼等 2001 物理学报 **50** 2289]

[31] Fu J L and Wang X M 2000 *Acta Phys*. *Sin*. **49** 1023 (in Chinese) [傅景礼、王新民 2000 物理学报 **49** 1023]

[32] Fu J L 2001 *Acta Math*. *Phys*. *Sci*. **21** 70 (in Chinese) [傅景礼 2001 数学物理学报 **21** 70]

[33] Fu J L, Chen L Q, Xue Y and Luo S K 2002 *Acta Phys*. *Sin*. **51** 2683 (in Chinese) [傅景礼、陈立群、薛 纭、罗绍凯 2002 物理学报 **51** 2683]

[34] Fu J L *et al* 2003 *Chin*. *Phys*. **12** 695

[35] Luo S K 1991 *Shanghai J*. *Mech*. **12** 61 (in Chinese) [罗绍凯 1991 上海力学 **12** 61]

[36] Luo S K 1996 *Appl*. *Math*. *Mech*. **17** 645

Perturbation to the symmetries of relativistic Birkhoffian systems and the inverse problems

Fu Jing-Li[1)2)] Chen Li-Qun[2)] Xie Feng-Ping[1)]

[1)] (*Insititute of Mathematical Mechanics and Mathematical Physics*, *Shangqiu Teachers College*, *Shangqiu* 476000, *China*)

[2)] (*Shanghai Institute of Applied Mathematics and Mechanics*, *Shanghai University*, *Shanghai* 200072, *China*)

(Received 10 December 2002; revised manuscript received 10 February 2003)

Abstract

In this paper, we have studied the positive problems of perturbation to the symmetries for relativistic Birkhoffian systems under the action of a small force of the disturbance. The fundamental theory, equations of motion and equation of small disturbance of relativistic Birkhoffian systems were established. Lie symmetries and conserved quantity of this systems were given. Perturbation to the symmetries of the systems under infinitesimal transformations, s-order adiabatic invariants, existence conditions and the form of adiabatic invariants were studied. We have also studied the inverse problems of perturbation to the symmetries for relativistic Birkhoffian systems. We obtain the perturbation to the symmetries of the systems under infinitesimal transformations, when the systems possess s-order adiabatic invariants. The relationship between relativistic Birkhoffian systems and classical Birkhoffian systems were studied. Finally an example was presented to illustrate the result.

Keywords: Lie symmetry, perturbation, adiabatic invariants, relativity

PACC: 0320, 0414