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起源于引力场的 Vaidya-Bonner-de Sitter 黑洞的量子熵

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利用 brick-wall 模型研究了引力场对 Vaidya-Bonner-de Sitter 黑洞熵的量子修正. 当黑洞事件视界不随超前时间变化时,结果与 Reissner-Nordström-de Sitter 黑洞的量子熵完全相同.

关键词:引力场, brick-wall 模型, Vaidya-Bonner-de Sitter 黑洞, 熵
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1. 引 言

黑洞熵的起源是理论物理探讨的重要课题之一. 自 Bekenstein 提出黑洞有一个正比于事件视界面积的内禀熵以来,人们利用各种途径来理解这个熵的统计性质,特别是 1985 年, 't Hooft[1] 利用 WKB 近似研究了 Schwarzschild 黑洞的量子熵,为了解决物态密度在事件视界附近的发散问题,引进了 brick-wall 模型. 该模型已经用于各种稳态黑洞[2—7] 情况. 最近,人们又将 brick-wall 模型推广到动态黑洞[8—14] 情况,研究了标量场或 Dirac 场对黑洞熵的量子修正. 但是 WKB 近似的正确性要求场方程必须有一个定态解,而在 (v, r) 坐标系中,场方程不可能存在这样的解. 不过许多工作[15—19] 表明,对于动态黑洞仅在 Tortoise 坐标系中,场方程在事件视界附近有一个渐近定态解. 所以,对于动态黑洞必须在 Tortoise 坐标系中才能利用 WKB 近似. 本文利用 brick-wall 模型,研究引力场对 Vaidya-Bonner-de Sitter 黑洞的量子修正.

2. 场方程

Vaidya-Bonner-de Sitter 黑洞的时空线元[20] 为

$$\mathrm{d}s^2 = \left[1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} - \frac{1}{3}\Lambda r^2\right]\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \qquad (1)$$

在坐标系 $(x^0 = v, x^1 = r, x^2 = \theta, x^3 = \varphi)$ 中选择零标架如下:

$$l^\mu = \delta_1^\mu, \quad n^\mu = -\delta_0^\mu - \frac{1}{2}\left[1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} - \frac{1}{3}\Lambda r^2\right]\delta_1^\mu,$$
$$m^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu + \frac{\mathrm{i}}{\sin\theta}\delta_3^\mu\right),$$
$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(\delta_2^\mu - \frac{\mathrm{i}}{\sin\theta}\delta_3^\mu\right). \qquad (2)$$

由此得到所有非零旋系数

$$\rho = -\frac{1}{r}, \quad \alpha = -\frac{1}{2\sqrt{2}r}\cot\theta = -\omega,$$
$$\mu = -\frac{1}{2r}\left(1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} - \frac{1}{3}\Lambda r^2\right),$$
$$\gamma = \frac{1}{2}\left(\frac{M(v)}{r^2} - \frac{Q^2(v)}{r^3} - \frac{1}{3}\Lambda r\right) \qquad (3)$$

和 Weyl 张量的非零独立分量

$$\Psi_2 = -\frac{M(v)}{r^3} + \frac{Q^2(v)}{r^4}. \qquad (4)$$

(3) 和 (4) 式告诉我们 Vaidya-Bonner-de Sitter 度规是 Petrov D 类的,所以无源引力方程可用微扰方法简化为[21]

$$[(\mathrm{D} - 3\varepsilon + \bar{\varepsilon} - 4\rho - \bar{\rho})(\Delta - 4\gamma + \mu) - (\delta + \bar{\pi} - \bar{\alpha} - 3\omega - 4\tau)(\bar{\delta} + \pi - 4\alpha) - 3\Psi_2]\Psi_0^B = 0,$$
$$[(\Delta + 3\gamma - \bar{\gamma} + 4\mu + \bar{\mu})(\mathrm{D} + 4\varepsilon - \rho) - (\bar{\delta} - \bar{\tau} + \bar{\omega} + 3\alpha + 4\pi)(\delta - \tau + 4\omega) - 3\Psi_2]\Psi_4^B = 0, \qquad (5)$$

其中 $\mathrm{D} = l^\mu\partial_\mu$, $\Delta = n^\mu\partial_\mu$, $\delta = m^\mu\partial_\mu$. 将 (2), (3), (4)

式代入(5)式，并令 $\Psi_0^B,\Psi_4^B={}_p\rho_l(\upsilon,r)\,{}_pY_l^m(\theta,\varphi)$，其中${}_pY_l^m(\theta,\varphi)$是自旋-加权球谐函数[22]．$l$ 为角量子数，m 是磁量子数，且满足 $l\geqslant|p|$，$-l\leqslant m\leqslant l$．自旋态 $p=\pm2$．则由(5)式，径向函数${}_p\rho_l(\upsilon,r)$满足方程

$$\begin{aligned}&\left\{A\frac{\partial^2}{\partial r^2}+\frac{2\partial^2}{\partial r\partial\upsilon}+\frac{2}{r}(p+3)\frac{\partial}{\partial\upsilon}\right.\\&+\left[\frac{2A}{r}(p+3)+(p+1)A'-\frac{2Ap}{r}\right]\frac{\partial}{\partial r}\\&+\frac{A}{r^2}(p+3)(p+2)+\frac{1}{r}(p+3)\\&\times\left[(p+1)A'-\frac{2Ap}{r}\right]+\frac{1}{6}(2p+1)\\&\left.\times(p+1)r\left(\frac{A}{r}\right)''-\frac{1+\lambda^2}{r^2}\right\}{}_p\rho_l=0,\end{aligned}\tag{6}$$

其中 $A=1-\dfrac{2M(\upsilon)}{r}+\dfrac{Q^2(\upsilon)}{r^2}-\dfrac{1}{3}\Lambda r^2$，$A'=\dfrac{\partial A}{\partial r}$，$\lambda^2=(l-p)(l+p+1)$．

引进新坐标[15]　$\hat{r}=r-r_H(\upsilon)$，$\hat{\upsilon}=\upsilon$，

$$\begin{aligned}&\frac{\partial^n}{\partial r^n}=\frac{\partial^n}{\partial\hat{r}^n},\ \frac{\partial}{\partial\upsilon}=\frac{\partial}{\partial\hat{\upsilon}}-\dot{r}_H\frac{\partial}{\partial\hat{r}},\\&\frac{\partial^2}{\partial r\partial\upsilon}=\frac{\partial^2}{\partial\hat{r}\partial\hat{\upsilon}}-\dot{r}_H\frac{\partial^2}{\partial\hat{r}^2},\end{aligned}\tag{7}$$

其中 r_H 为时空事件视界，$\dot{r}_H=\dfrac{\mathrm{d}r_H}{\mathrm{d}\upsilon}$．

将(7)式代入(6)式，方程变为

$$\begin{aligned}&\left\{(A-2\dot{r}_H)\frac{\partial^2}{\partial\hat{r}^2}+\frac{2\partial^2}{\partial\hat{r}\partial\hat{\upsilon}}+\frac{2}{r}(p+3)\frac{\partial}{\partial\hat{\upsilon}}\right.\\&+\left[\frac{2A}{r}(p+3)+(p+1)A'-\frac{2Ap}{r}\right.\\&\left.-\frac{2\dot{r}_H}{r}(p+3)\right]\frac{\partial}{\partial\hat{r}}+\frac{A}{r^2}(p+3)(2-p)\\&+\frac{A'}{r}(p+3)(p+1)+\frac{1}{6}(2p+1)(p+1)\\&\left.\times r\left(\frac{A}{r}\right)''-\frac{1+\lambda^2}{r^2}\right\}{}_p\rho_l=0.\end{aligned}\tag{8}$$

定义 Tortoise 坐标变换[15]

$$\mathrm{d}r_*=\frac{1-2\eta}{A-2\dot{r}_H}\mathrm{d}\hat{r}+\eta\mathrm{d}\hat{\upsilon},\ \upsilon_*=\hat{\upsilon},\tag{9}$$

其中 η 是保证 $\mathrm{d}r_*$ 为全微分的调节因子，在事件视界附近 $\eta\approx\dot{r}_H$．则有

$$\begin{aligned}&\frac{\partial}{\partial\hat{r}}=\frac{1-2\eta}{A-2\dot{r}_H}\frac{\partial}{\partial r_*},\ \frac{\partial}{\partial\hat{\upsilon}}=\frac{\partial}{\partial\upsilon_*}+\eta\frac{\partial}{\partial r_*},\\&\frac{\partial^2}{\partial\hat{r}^2}=\left(\frac{\partial}{\partial\hat{r}}\frac{1-2\eta}{A-2\dot{r}_H}\right)\frac{\partial}{\partial r_*}+\left(\frac{1-2\eta}{A-2\dot{r}_H}\right)^2\frac{\partial^2}{\partial r_*^2},\\&\frac{\partial^2}{\partial\hat{r}\partial\hat{\upsilon}}=\left(\frac{\partial}{\partial\hat{\upsilon}}\frac{1-2\eta}{A-2\dot{r}_H}\right)\frac{\partial}{\partial r_*}+\eta\frac{1-2\eta}{A-2\dot{r}_H}\frac{\partial^2}{\partial r_*^2}\\&\qquad+\frac{1-2\eta}{A-2\dot{r}_H}\frac{\partial^2}{\partial r_*\partial\upsilon_*}.\end{aligned}\tag{10}$$

将(10)式代入(8)式，化简后得到引力场方程

$$\begin{aligned}&\left\{\frac{1-2\eta}{A-2\dot{r}_H}\frac{\partial^2}{\partial r_*^2}+\frac{2(1-2\eta)}{A-2\dot{r}_H}\frac{\partial^2}{\partial r_*\partial\upsilon_*}\right.\\&+\frac{2(p+3)}{r}\frac{\partial}{\partial\upsilon_*}+\left[(A-2\dot{r}_H)\left(\frac{\partial}{\partial\hat{r}}\frac{1-2\eta}{A-2\dot{r}_H}\right)\right.\\&+2\left(\frac{\partial}{\partial\hat{\upsilon}}\frac{1-2\eta}{A-2\dot{r}_H}\right)+2(p+3)\frac{\eta}{r}\\&+\left(\frac{6A}{r}+(p+1)A'-2\dot{r}_H(p+3)\frac{1}{r}\right)\\&\left.\left.\times\frac{1-2\eta}{A-2\dot{r}_H}\right]\frac{\partial}{\partial r_*}+V-\frac{\lambda^2}{r^2}\right\}{}_p\rho_l=0,\end{aligned}\tag{11}$$

其中

$$\begin{aligned}V&=\frac{A}{r^2}(p+3)(2-p)+\frac{A'}{r}(p+3)(p+1)\\&\quad+\frac{1}{6}(2p+1)(p+1)r\left(\frac{A}{r}\right)''-\frac{1}{r^2}\\&=\frac{4}{r^2}+\frac{4M(\upsilon)}{r^3}(p-2)+\frac{3Q^2(\upsilon)}{r^4}(2-p)\\&\quad-\frac{1}{3}\Lambda(7p+16).\end{aligned}$$

3. 黑洞的量子熵

如引言中所述，在 WKB 近似下，${}_p\rho_l(\upsilon_*,r_*)$在黑洞附近的薄区域内可以写成如下形式：

$${}_p\rho_l(\upsilon_*,r_*)=\exp[-\mathrm{i}E\upsilon_*+\mathrm{i}W(r_*,p,l,E)],\tag{12}$$

其中 E 为引力子的能量．将(12)式代入(11)式，得

$$\begin{aligned}&\left\{-\frac{1-2\eta}{A-2\dot{r}_H}\left(\frac{\partial W}{\partial r_*}\right)^2+\frac{\mathrm{i}(1-2\eta)}{A-2\dot{r}_H}\frac{\partial^2W}{\partial r_*^2}\right.\\&+\frac{2E(1-2\eta)}{A-2\dot{r}_H}\frac{\partial W}{\partial r_*}-2\mathrm{i}E(p+3)\frac{1}{r}\\&+\mathrm{i}\left[(A-2\dot{r}_H)\left(\frac{\partial}{\partial\hat{r}}\frac{1-2\eta}{A-2\dot{r}_H}\right)+2\left(\frac{\partial}{\partial\hat{\upsilon}}\frac{1-2\eta}{A-2\dot{r}_H}\right)\right.\\&+2(p+3)\frac{\eta}{r}+\left(\frac{6A}{r}+(p+1)A'-2\dot{r}_H(p+3)\right.\\&\left.\left.\left.\times\frac{1}{r}\right)\frac{1-2\eta}{A-2\dot{r}_H}\right]\frac{\partial W}{\partial r_*}+V-\frac{\lambda^2}{r^2}\right\}{}_p\rho_l=0.\end{aligned}\tag{13}$$

将(13)式中实部与虚部分离

$$-\frac{1-2\eta}{A-2\dot{r}_H}\left(\frac{\partial W}{\partial r_*}\right)^2+\frac{2E(1-2\eta)}{A-2\dot{r}_H}\frac{\partial W}{\partial r_*}$$

$$+V-\frac{\lambda^2}{r^2}=0, \tag{14}$$

$$\frac{1-2\eta}{A-2\dot{r}_H}\frac{\partial^2 W}{\partial r_*^2}-2E(p+3)\frac{1}{r}+\Big[(A$$

$$-2\dot{r}_H)\left(\frac{\partial}{\partial\hat{r}}\frac{1-2\eta}{A-2\dot{r}_H}\right)+2\left(\frac{\partial}{\partial\hat{v}}\frac{1-2\eta}{A-2\dot{r}_H}\right)$$

$$+2(p+3)\frac{\eta}{r}+\left(\frac{6A}{r}+(p+1)A'\right.$$

$$\left.-2\dot{r}_H(p+3)\frac{1}{r}\right)\frac{1-2\eta}{A-2\dot{r}_H}\Big]\frac{\partial W}{\partial r_*}=0, \tag{15}$$

则径向波数为[23]

$$K(r_*,p,l,E)=-E+\frac{\partial W}{\partial r_*}=\pm\Big[E^2$$

$$+\frac{A-2\dot{r}_H}{1-2\eta}\left(V-\frac{\lambda^2}{r^2}\right)\Big]^{\frac{1}{2}}. \tag{16}$$

根据半经典的量子化规则,能谱由下式给出:

$$\oint K(r_*,p,l,E)\,\mathrm{d}r_*=2n\pi, \tag{17}$$

其中 n 为主量子数. 采用薄膜 brick-wall 模型[24,25],在事件视界 r_H 附近的一个薄区域 a 内,则有

$$\mathrm{d}r_*\approx\frac{1-2\dot{r}_H}{A-2\dot{r}_H}\mathrm{d}\hat{r}, \tag{18}$$

$$\int_a\Big[E^2+\frac{A-2\dot{r}_H}{1-2\dot{r}_H}\left(V-\frac{\lambda^2}{r^2}\right)\Big]^{\frac{1}{2}}\frac{1-2\dot{r}_H}{A-2\dot{r}_H}\mathrm{d}\hat{r}=n\pi. \tag{19}$$

能量不超过 E 的本征态数目由下式给出:

$$g(E)=\sum_p\sum_l(2l+1)n$$

$$\approx\frac{1}{\pi}\sum_p\int_{|p|}^{l_{\max}}(2l+1)\,\mathrm{d}l\int_a\Big[E^2+\frac{A-2\dot{r}_H}{1-2\dot{r}_H}$$

$$\times\left(V-\frac{\lambda^2}{r^2}\right)\Big]^{\frac{1}{2}}\frac{1-2\dot{r}_H}{A-2\dot{r}_H}\mathrm{d}\hat{r}. \tag{20}$$

在对 l 的积分中,上下限的确定满足角动量 $l\geqslant|p|$ 和波数是实数的要求,则(20)式对 l 积分后变为

$$g(E)=\frac{2}{3\pi}\sum_p\int_a\frac{(1-2\dot{r}_H)^2r^2}{(A-2\dot{r}_H)^2}$$

$$\times\Big[E^2+\frac{A-2\dot{r}_H}{1-2\dot{r}_H}\left(V-\frac{4}{r^2}\right)\Big]^{\frac{3}{2}}\mathrm{d}\hat{r}. \tag{21}$$

由量子统计理论,系统的自由能可表示为

$$F=\frac{1}{\beta}\int_0^\infty\mathrm{d}E\,\frac{\mathrm{d}g(E)}{\mathrm{d}E}\ln(1-\mathrm{e}^{-\beta E})$$

$$=-\int_0^\infty\frac{g(E)}{\mathrm{e}^{\beta E}-1}\mathrm{d}E. \tag{22}$$

将(21)式代入(22)式,对能量 E 积分后,得

$$F=-\frac{2}{3\pi}\sum_p\Big[\frac{\Gamma(4)\zeta(4)}{\beta^4}\int_a\frac{(1-2\dot{r}_H)^2r^2}{(A-2\dot{r}_H)^2}\mathrm{d}\hat{r}$$

$$-\frac{3\zeta(2)}{2\beta^2}\int_a\frac{1-2\dot{r}_H}{A-2\dot{r}_H}(4-Vr^2)\,\mathrm{d}\hat{r}\Big], \tag{23}$$

其中 $\frac{1}{\beta}$ 为温度, $\Gamma(n)$ 为伽玛函数, $\zeta(n)$ 为 Riemann-Zeta 函数. 由量子统计理论知熵 $S=\beta^2\frac{\partial F}{\partial\beta}$,所以(23)式对应的熵为

$$S=\frac{16\Gamma(4)\zeta(4)}{3\pi\beta^3}\int_a\frac{(1-2\dot{r}_H)^2r^2}{(A-2\dot{r}_H)^2}\mathrm{d}\hat{r}$$

$$-\frac{2\zeta(2)}{\pi\beta}\sum_p\int_a\frac{1-2\dot{r}_H}{A-2\dot{r}_H}(4-Vr^2)\,\mathrm{d}\hat{r}. \tag{24}$$

在(24)式积分中,视界方程

$$A-2\dot{r}_H=-\frac{\Lambda}{3r^2}\Big[r^4-\frac{3(1-2\dot{r}_H)r^2}{\Lambda}+\frac{6Mr}{\Lambda}-\frac{3Q^2}{\Lambda}\Big]$$

$$=-\frac{\Lambda}{3r^2}(r-r_1)(r-r_2)(r-r_3)(r-r_4), \tag{25}$$

其中 $r_1<r_2<r_3$ 为 $(A-2\dot{r}_H)=0$ 的三个根[26],分别解释为内、外事件视界和宇宙视界, r_4 是一个负实根. 考虑外视界($r_2=r_H$)附近的薄区域,即 $a=\{(r,\theta,\varphi)\,|\,r_H+\xi\leqslant r\leqslant r_H+c\xi\}$, ξ 为薄区域与视界面的距离,是无穷小量, c 为大于 1 的小正数. 利用(25)式和(7)式可计算出(24)式中的前一项积分,将 V 代入(24)式,可计算出(24)式中的后一项积分,但此项积分对黑洞的熵无贡献.

最后,计算出 Vaidya-Bonner-de Sitter 黑洞的熵为

$$S=\frac{48\Gamma(4)\zeta(4)(1-2\dot{r}_H)^2}{\pi\Lambda^2\beta^3}\frac{(c-1)B}{c\xi}, \tag{26}$$

其中 $B=\frac{r_H^6}{(r_H-r_1)^2(r_H-r_3)^2(r_H-r_4)^2}$.

4. 结　　论

我们把 brick-wall 模型推广到了 Vaidya-Bonner-de Sitter 黑洞,得到了黑洞的量子熵(26)式. 将(26)式与已知的 Reissner-Nordström-de Sitter 黑洞熵的结果(文献[5]的(33)式)比较,形式上多了一个描述动

态黑洞的因子$(1-2\dot{r}_H)^2$. 当 $\dot{r}_H=0$ 时，与相应静态黑洞熵的结果[5]完全相同，即(26)式可回到静态黑洞情况.

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Quantum entropy of the Vaidya-Bonner-de Sitter black hole arising from gravitational fields

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Abstract

The quantum corrections to the entropy of the Vaidya-Bonner-de Sitter black hole due to gravitational fields are investigated by using the brick-wall model. When the event horizon of the black hole does not depend upon the advanced-time, the results coincide with that of the Reissner-Nordström-de Sitter black hole.

Keywords: gravitational fields, brick-wall model, Vaidya-Bonner-de Sitter black hole, entropy

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