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电子在周期驱动耦合量子阱中的振荡

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通过精确求解含时的量子体系,研究了在周期耦合驱动下电子在两量子阱中的受迫振荡.电子在双量子阱间的隧穿可由周期性外场控制.得到了电子被囚禁在单一量子阱中的条件.

关键词:量子阱,受迫振荡,周期驱动
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1.引 言

近年来,由于在输运过程中研究单电子效应已成为可能,电子在量子阱中的输运问题引起了广泛关注.但研究的重点仍集中在单量子阱,而微加工工艺的快速发展已可以制造耦合的量子阱系统,如一个耦合的量子阱分别与两个电极相连[1—6].与单量子阱相比,电子将会在耦合的量子阱中振荡.这种量子关联必然会影响整个系统中的电流,因而研究单电子在周期驱动耦合量子阱中的动力学是十分有意义的.

2.模型和精确解

如图 1 所示,当远离发射极和集电极时,电子在周期驱动的耦合量子阱中的 Hamiltonian 为

$$\hat{H} = E_+ \hat{a}_+^+ \hat{a}_+ + E_- \hat{a}_-^+ \hat{a}_- + \Omega_0(\hat{a}_+^+ \hat{a}_- e^{i\omega t} + \hat{a}_-^+ \hat{a}_+ e^{-i\omega t}), \quad (1)$$

式中,$\hat{a}_i^+$,$\hat{a}_i$ 表示在电子能级 E_i 上的产生和湮没算符,满足通常的反对易关系:$\{\hat{a}_i, \hat{a}_j^+\} = \delta_{ij}$,$\Omega_0$ 为周期耦合驱动振幅,ω 为驱动频率.

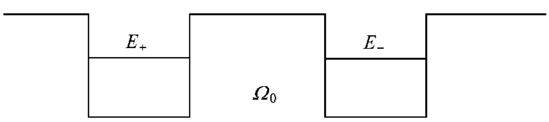

图 1 电子在耦合量子阱中的振荡

取 E_+ 与 E_- 的平均值为零点能,系统的 Hamiltonian 变为

$$\begin{aligned}\hat{H} &= \frac{E_+ - E_-}{2}(\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-) \\ &\quad + \Omega_0(\hat{a}_+^+ \hat{a}_- e^{i\omega t} + \hat{a}_-^+ \hat{a}_+ e^{-i\omega t}) \\ &= \varepsilon \hat{K}_0 + \Omega_0(\hat{K}_+ e^{i\omega t} + \hat{K}_- e^{-i\omega t}),\end{aligned} \quad (2)$$

式中 $\varepsilon = E_+ - E_-$,不失一般性,令 $\varepsilon \geqslant 0$.

$$\hat{K}_0 = \frac{1}{2}(\hat{a}_+^+ \hat{a}_+ - \hat{a}_-^+ \hat{a}_-),$$

$$\hat{K}_+ = \hat{a}_+^+ \hat{a}_-,$$

$$\hat{K}_- = \hat{a}_-^+ \hat{a}_+,$$

$$\hat{K}_+ = (\hat{K}_-)^+.$$

可以证明,$\hat{K}_0$,$\hat{K}_+$ 与 $\hat{K}_-$ 对易关系为

$$[\hat{K}_0, \hat{K}_\pm] = \pm \hat{K}_\pm,$$
$$[\hat{K}_+, \hat{K}_-] = 2\hat{K}_0. \quad (3)$$

含时 Schrödinger 方程为

$$i\frac{d}{dt} | \Psi(t)\rangle = \hat{H}(t) | \Psi(t)\rangle. \quad (4)$$

取 $\hbar = 1$.

为求解此含时体系,作如下幺正变换:

$$| \Psi(t)\rangle = \hat{R}(t) | \Psi'(t)\rangle, \quad (5)$$

式中,$\hat{R}(t)$为幺正算符,

$$\hat{R}(t) = \exp\left[\frac{\gamma(t)}{2}(\hat{K}_+ e^{-i\beta(t)} - \hat{K}_- e^{i\beta(t)})\right]. \quad (6)$$

代入 Schrödinger 方程(4),得到$|\Psi'(t)\rangle$满足的含时方程

$$i\frac{d}{dt} | \Psi'\rangle = (\hat{R}^+(t)\hat{H}(t)\hat{R}(t)$$

$$- \hat{R}^{+}(t)\mathrm{i}\dot{\hat{R}}(t))\mid \Psi'\rangle . \tag{7}$$

利用(6)式,不难得到下列关系式:

$$\hat{R}^{+}(t)\hat{K}_0\hat{R}(t) = \hat{K}_0\cos\gamma + \frac{1}{2}(\hat{K}_{+}\mathrm{e}^{-\mathrm{i}\beta} + \hat{K}_{-}\mathrm{e}^{\mathrm{i}\beta})\sin\gamma, \tag{8}$$

$$\hat{R}^{+}(t)\hat{K}_{+}\hat{R}(t) = \hat{K}_{+}\cos^2\frac{\gamma}{2} - \hat{K}_{-}\mathrm{e}^{\mathrm{i}2\beta}\sin^2\frac{\gamma}{2} - \hat{K}_0\mathrm{e}^{\mathrm{i}\beta}\sin\gamma, \tag{9}$$

$$\hat{R}^{+}(t)\hat{K}_{-}\hat{R}(t) = \hat{K}_{-}\cos^2\frac{\gamma}{2} - \hat{K}_{+}\mathrm{e}^{-\mathrm{i}2\beta}\sin^2\frac{\gamma}{2} - \hat{K}_0\mathrm{e}^{-\mathrm{i}\beta}\sin\gamma, \tag{10}$$

$$\begin{aligned}\hat{R}^{+}(t)\mathrm{i}\dot{\hat{R}}(t) = &- 2\hat{K}_0\dot{\beta}\sin^2\frac{\gamma}{2} \\ &+ \hat{K}_{+}\mathrm{e}^{-\mathrm{i}\beta}\left(\mathrm{i}\frac{\dot{\gamma}}{2} + \frac{\dot{\beta}}{2}\sin\gamma\right) \\ &+ \hat{K}_{-}\mathrm{e}^{\mathrm{i}\beta}\left(-\mathrm{i}\frac{\dot{\gamma}}{2} + \frac{\dot{\beta}}{2}\sin\gamma\right),\end{aligned} \tag{11}$$

取

$$\begin{aligned}\dot{\gamma} &= 2\Omega_0\sin(\omega t + \beta), \\ \frac{1}{2}(\dot{\beta} - \varepsilon)\sin\gamma &= \Omega_0\cos\gamma\cos(\omega t + \beta).\end{aligned} \tag{12}$$

将(8—12)式代入 Schrödinger 方程,得

$$\begin{aligned}\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\mid \Psi'(t)\rangle = &\Big(\varepsilon\cos\gamma - ^{\mathrm{i}(\beta+\omega t)}\Omega_0\sin\gamma \\ &- \mathrm{e}^{-\mathrm{i}(\beta+\omega t)}\Omega_0\sin\gamma \\ &+ 2\dot{\beta}\sin^2\frac{\gamma}{2}\Big)\hat{K}_0\mid \Psi'(t)\rangle .\end{aligned} \tag{13}$$

考虑绝热近似,$\dot{\gamma}=0$,$\dot{\beta}=0$,由(13)式得

$$\beta = 2n\pi - \omega t$$

$$\frac{1}{2}\varepsilon\sin\gamma + \Omega_0\cos\gamma = 0.$$

(13)式简化为

$$\begin{aligned}\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\mid \Psi'(t)\rangle = &\Big(\varepsilon - 2\varepsilon\sin^2\frac{\gamma}{2} \\ &- 2\Omega_0\sin\gamma\Big)\hat{K}_0\mid \Psi'(t)\rangle .\end{aligned} \tag{14}$$

对于远离发射极和集电极的耦合量子阱,$\hat{K}_0$ 的本征态为 $|+\rangle$,$|-\rangle$.

$$\hat{K}_0\mid +\rangle = \frac{1}{2}\mid +\rangle ,$$

$$\hat{K}_0\mid -\rangle = -\frac{1}{2}\mid -\rangle .$$

根据文献[7—10],方程(14)的通解为

$$\begin{aligned}\mid \Psi(t)\rangle = &C_{+}\mathrm{e}^{\mathrm{i}\alpha_{+}(t)}\hat{R}(t)\mid +\rangle \\ &+ C_{-}\mathrm{e}^{\mathrm{i}\alpha_{-}(t)}\hat{R}(t)\mid -\rangle ,\end{aligned} \tag{15}$$

式中

$$\begin{aligned}\alpha_{\pm}(t) = &\int_0^t \mathrm{d}t'\langle \pm \mid \hat{R}(t')\mathrm{i}\dot{\hat{R}}(t') \\ &- \hat{R}^{+}(t')\hat{H}(t')\hat{R}(t')\mid \pm\rangle .\end{aligned} \tag{16}$$

在绝热近似下,

$$\alpha_n(t) = -K_n\left(\varepsilon - 2\varepsilon\sin^2\frac{\gamma}{2} - 2\Omega_0\sin\gamma\right)t, \tag{17}$$

式中

$$K_{\pm} = \pm\frac{1}{2}.$$

$|\Psi(t)\rangle$也可写为

$$\mid \Psi(t)\rangle = \hat{U}(t)\mid \Psi(0)\rangle .$$

时间演化算符

$$\hat{U}(t) = \hat{R}(t)\mathrm{e}^{-\mathrm{i}\chi(t)\hat{K}_0}\hat{R}^{+}(0), \tag{18}$$

式中

$$\chi(t) = \left(\varepsilon - 2\varepsilon\sin^2\frac{\gamma}{2} - 2\Omega_0\sin\gamma\right)t.$$

3. 电子布居数的时间演化

1)设初始时刻,电子处于高能级量子阱中,

$$\mid \Psi(0)\rangle = |+\rangle ,$$

则在 t 时刻,量子阱中粒子数之差的平均值为

$$\begin{aligned}\bar{N}_{+-} &= 2\langle \Psi(t)\mid \hat{K}_0\mid \Psi(t)\rangle \\ &= 2\langle + \mid \hat{U}^{+}(t)\hat{K}_0\hat{U}(t)\mid +\rangle \\ &= 2\langle + \mid \hat{R}(0)\mathrm{e}^{\mathrm{i}\chi(t)\hat{K}_0}\Big[\hat{K}_0\cos\gamma + \frac{1}{2}(\hat{K}_{+}\mathrm{e}^{\mathrm{i}\beta} \\ &\quad + \hat{K}_{-}\mathrm{e}^{-\mathrm{i}\beta})\sin\gamma\Big]\mathrm{e}^{-\mathrm{i}\chi(t)\hat{K}_0}\hat{R}^{+}(0)\mid +\rangle \\ &= 2\cos\gamma\langle + \mid \hat{R}(0)\hat{K}_0\hat{R}^{+}(0)\mid +\rangle \\ &\quad + \mathrm{e}^{\mathrm{i}\chi(t)}\mathrm{e}^{-\mathrm{i}\beta}\sin\gamma\langle + \mid \hat{R}(0)\hat{K}_{+}\hat{R}^{+}(0)\mid +\rangle \\ &\quad + \mathrm{e}^{-\mathrm{i}\chi(t)}\mathrm{e}^{\mathrm{i}\beta}\sin\gamma\langle + \mid \hat{R}(0)\hat{K}_{-}\hat{R}^{+}(0)\mid +\rangle \\ &= \cos^2\gamma + \frac{1}{2}\mathrm{e}^{\mathrm{i}(\chi(t)-\beta)}\sin^2\gamma + \frac{1}{2}\mathrm{e}^{-\mathrm{i}(\chi(t)-\beta)}\sin^2\gamma \\ &= \frac{\varepsilon^2}{4\Omega_0^2 + \varepsilon^2} + \frac{4\Omega_0^2}{4\Omega_0^2 + \varepsilon^2}\cos(\chi(t) - \beta) \\ &= \frac{\varepsilon^2}{4\Omega_0^2 + \varepsilon^2} + \frac{4\Omega_0^2}{4\Omega_0^2 + \varepsilon^2}\cos\left[\left(\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2}\right)t\right].\end{aligned} \tag{19}$$

显然,当远离电极时,两量子阱中电子数之和为守恒量.

$$\bar{N} = \langle \Psi(t)\mid \hat{N}\mid \Psi(t)\rangle \equiv 1, \tag{20}$$

式中

$$\hat{N} = \hat{a}_1^+ \hat{a}_1 + \hat{a}_2^+ \hat{a}_2 .$$

故电子在高能级量子阱中的布居数随时间变化为

$$\begin{aligned}\bar{N}_+ &= \langle \Psi(t) \mid \hat{a}_+^+ \hat{a}_+ \mid \Psi(t) \rangle \\ &= \frac{1}{2} + \frac{\varepsilon^2}{8\Omega_0^2 + 2\varepsilon^2} \\ &\quad + \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2} \cos[(\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2})t] \\ &= \frac{2\Omega_0^2 + \varepsilon^2}{4\Omega_0^2 + \varepsilon^2} + \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2} \\ &\quad \times \cos[(\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2})t] . \end{aligned} \tag{21}$$

2)设初始时刻,电子处于叠加态,

$$| \Psi(0) \rangle_e = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle),$$

$$| \Psi(0) \rangle_o = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle),$$

在 t 时刻,电子在高能级量子阱中的布居数随时间变化为

$$\begin{aligned}\bar{N}_+^e &= \frac{4\Omega_0^2 + \varepsilon^2 + 2\Omega_0\varepsilon}{8\Omega_0^2 + 2\varepsilon^2} \\ &\quad - \frac{\Omega_0\varepsilon}{4\Omega_0^2 + \varepsilon^2}\cos[(\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2})t], \end{aligned} \tag{22}$$

$$\begin{aligned}\bar{N}_+^o &= \frac{4\Omega_0^2 + \varepsilon^2 - 2\Omega_0\varepsilon}{8\Omega_0^2 + 2\varepsilon^2} \\ &\quad + \frac{\Omega_0\varepsilon}{4\Omega_0^2 + \varepsilon^2}\cos[(\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2})t]. \end{aligned} \tag{23}$$

3)设初始时刻,电子处于低能级量子阱中

$$| \Psi(0) \rangle = |-\rangle,$$

在 t 时刻,电子在高能级量子阱中的布居数随时间变化为

$$\begin{aligned}\bar{N}_+ &= \langle \Psi(t) \mid \hat{a}_+^+ \hat{a}_+ \mid \Psi(t) \rangle \\ &= \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2} - \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2} \\ &\quad \times \cos[\omega - \sqrt{\varepsilon^2 + 4\Omega_0^2})t] . \end{aligned} \tag{24}$$

4. 讨 论

1) $\omega = 0$,由于隧穿耦合,电子仍在量子阱中作周期振荡.

设电子初始处于高能级量子阱中,则

$$\bar{N}_+ = \frac{2\Omega_0^2 + \varepsilon^2}{4\Omega_0^2 + \varepsilon^2} + \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2}\cos\sqrt{\varepsilon^2 + 4\Omega_0^2}\, t . \tag{25}$$

此计算结果给出与文献[5]相同的结果.若此时两量子阱能级差 $\varepsilon = 0$,电子在量子阱中振荡振幅最大,即发生振幅共振,布居数变化为

$$\bar{N}_+ = \frac{1}{2}(1 + \cos 2\Omega_0 t) . \tag{26}$$

若电子初始处于叠加态

$$| \Psi(0) \rangle_e = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle),$$

则

$$\begin{aligned}\bar{N}_+^e &= \frac{4\Omega_0^2 + \varepsilon^2 + 2\Omega_0\varepsilon}{8\Omega_0^2 + 2\varepsilon^2} \\ &\quad - \frac{\Omega^0\varepsilon}{4\Omega_0^2 + \varepsilon^2}\cos\sqrt{\varepsilon^2 + 4\Omega_0^2}\, t . \end{aligned} \tag{27}$$

当 $\varepsilon = 0$,布居数变化为 $\bar{N}_+^e = \frac{1}{2}$,即此时电子不会在量子阱中振荡.

若电子初始处于叠加态

$$| \Psi(0) \rangle_o = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle),$$

则

$$\begin{aligned}\bar{N}_+^o &= \frac{4\Omega_0^2 + \varepsilon^2 - 2\Omega_0\varepsilon}{8\Omega_0^2 + 2\varepsilon^2} \\ &\quad + \frac{\Omega_0\varepsilon}{4\Omega_0^2 + \varepsilon^2}\cos\sqrt{\varepsilon^2 + 4\Omega_0^2}\, t . \end{aligned} \tag{28}$$

当 $\varepsilon = 0$,布居数变化为 $\bar{N}_+^o = \frac{1}{2}$,此时电子也不会在量子阱中振荡.

若电子初始处于低能级中,则

$$\bar{N}_+ = \frac{2\Omega_0^2}{4\Omega_0^2 + \varepsilon^2}[1 - \cos(\sqrt{\varepsilon^2 + 4\Omega_0^2}\, t)] . \tag{29}$$

当 $\varepsilon = 0$,布居数变化为

$$\bar{N}_+ = \frac{1}{2}(1 - \cos 2\Omega_0 t) . \tag{30}$$

2) $\omega \neq 0$,若驱动频率

$$\omega = \sqrt{\varepsilon^2 + 4\Omega_0^2},$$

此时无论初始状态如何,电子始终被囚禁在量子阱中,布居数不再随时间变化.

若电子初始处于高能级量子阱中,则

$$\bar{N}_+ = 1,$$

电子被囚禁在高能级量子阱中.

若电子初始处于叠加态 $|\Psi(0)\rangle_e$ 或 $|\Psi(0)\rangle_o$,则

$$\bar{N}_+ = \frac{1}{2},$$

电子始终以相同概率处于两量子阱中.

若电子初始处于低能级量子阱中,则

$$\bar{N}_+ = 0,$$

电子被囚禁在低能级量子阱中.

3) $\omega \neq 0$, $\omega \neq \sqrt{\varepsilon^2 + 4\Omega_0^2}$, $\varepsilon = 0$.

若电子初始处于高能级量子阱中,则

$$\bar{N}_+ = \frac{1}{2}[1 + \cos(\omega - 2\Omega_0)t],$$

电子将在量子阱作振幅最大的振动.

若电子初始处于叠加态 $|\Psi(0)\rangle_o$ 或 $|\Psi(0)\rangle_e$,则

$$\bar{N}_+ = \frac{1}{2},$$

电子将被局域在两量子阱中,不再振动.

若电子初始处于低能级量子阱中,则

$$\bar{N}_+ = \frac{1}{2}[1 - \cos(\omega - 2\Omega_0)t],$$

电子将作振幅最大的振动.

5. 结　　论

本文分析了在含时耦合驱动下,由不同状态开始,电子在两个量子阱之间的振荡行为.得到振荡振幅和频率与耦合驱动振幅、频率及两量子阱间能级差的依赖关系.若电子初始处于某一量子阱中,当驱动频率 $\omega = \sqrt{\varepsilon^2 + 4\Omega_0^2}$ 时,电子在量子阱中的布居数不再随时间变化.若电子初始处于叠加态,则在另外一种情况下即两量子阱能级差为零时,电子在量子阱中的布居数也不随时间改变.此时电子被稳定地局域在量子阱中,故可以通过周期性外场来控制电子在耦合量子阱中的隧穿,以控制整个体系电流.

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Electron oscillation between coupled quantum wells with periodic driving

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Abstract

The behaviour of an electron between two quantum wells, is studied by exact solution of a time-dependent Schrödinger equation. The time-dependent electron occupation-probability is obtained analytically as a function of the energy difference between the two wells, coupling constant and driving frequency. The condition that electron can be trapped in a single quantum well is obtained.

Keywords: quantum wells, drived coupling, oscillation

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