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对动态缓变 Reissner-Nordström 黑洞量子辐射特征的研究*

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考虑到 Reissner-Nordström 黑洞的质量 M 和荷电量 Q 随时间 t 缓慢变化的情况,研究了此黑洞的量子辐射特征.结果表明,在动态缓变 Reissner-Nordström 时空中荷电 Dirac 粒子的量子热辐射谱与其黑洞蒸发率及随时间 t 缓变的 $M(t)$ 和 $Q(t)$ 等因素有关;动态缓变 Reissner-Nordström 黑洞量子非热辐射的最大能量与量子热辐射谱中的化学势相等.

关键词:黑洞,缓变时空,辐射

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1. 引 言

根据 William 等人对荷电黑洞演化的研究结果[1],在黑洞质量 M 和荷电量 Q 随时间 t 缓慢变化时,可以把具有球对称性的 Reissner-Nordströmr 黑洞的 M 和 Q 认为是随时间 t 缓变的函数,把 Reissner-Nordström 线元中的 M 和 Q 分别改写为 $M(t)$ 和 $Q(t)$ 后的动态缓变线元仍是爱因斯坦场方程的解,在此动态缓变 Reissner-Nordström 时空中荷电粒子的量子辐射特征是一个还未被深入研究的课题.近年来,天体物理学研究者已对用超前爱丁顿坐标表示的动态黑洞的量子热辐射和非热辐射进行了一系列有效的研究,这些研究促进了黑洞物理学的发展[2—19].本文从研究动态缓变 Reissner-Nordström 时空开始,在考虑 $M=M(t)$,$Q=Q(t)$ 前提下,研究荷电 Dirac 粒子的量子热辐射和量子非热辐射,并对采取的出发点、采用的研究方法和所得结论进行了讨论.

2. 动态缓变 Reissner-Nordström 时空中荷电 Dirac 粒子的量子热辐射

根据文献[1],对于球对称荷电缓变的 Reissner-Nordström 黑洞而言,在黑洞缓变过程中,$M=M(t)$,$Q=Q(t)$,用 Reissner-Nordström 度规描述此动态球对称缓变的时空线元为

$$\mathrm{d}s^2=-\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]\mathrm{d}t^2+\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]^{-1}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2). \tag{1}$$

由此可得出逆变度规张量 $g^{\mu\nu}$ 为

$$g^{\mu\nu}=\begin{pmatrix} g^{00} & 0 & 0 & 0\\ 0 & g^{11} & 0 & 0\\ 0 & 0 & g^{22} & 0\\ 0 & 0 & 0 & g^{33}\end{pmatrix}, \tag{2}$$

其中

$$g^{00}=-\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]^{-1},\quad g^{22}=\frac{1}{r^2},$$

$$g^{11}=1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2},\quad g^{33}=\frac{1}{r^2\sin^2\theta}. \tag{3}$$

由零超曲面方程[5—7]

$$g^{\mu\nu}\frac{\partial F}{\partial x^\mu}\frac{\partial F}{\partial x^\nu}=0, \tag{4}$$

可得出黑洞的视界 r_{H} 为

$$r_{\mathrm{H}}=\frac{M(t)+\sqrt{M^2(t)-Q^2(t)(1-\dot{r}_{\mathrm{H}}^2)}}{1-\dot{r}_{\mathrm{H}}^2}, \tag{5}$$

其中 $\dot{r}_{\mathrm{H}}=\frac{\mathrm{d}r_{\mathrm{H}}}{\mathrm{d}t}$ 描述黑洞的蒸发率.在(1)式表示的弯

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曲时空中,四分量 Dirac 方程可表示为[7]

$$\gamma^{\mu}(\partial_{\mu}\Psi - \Gamma_{\mu}\Psi) + \mathrm{i}m_0\Psi = 0, \tag{6}$$

其中

$$\Gamma_{\mu} = -\frac{1}{4}\gamma^{\alpha}(\gamma_{\alpha,\mu} - \gamma_{\lambda}\Gamma^{\lambda}_{\alpha\mu}). \tag{7}$$

如果引用 Penrose 的旋系数,(1)式表示的弯曲时空中的荷电粒子 Dirac 方程可以表述为[11—13]

$$\begin{aligned}
&(D + \varepsilon - \rho + \mathrm{i}eA_{\mu}l^{\mu})F_1\\
&+ (\bar{\sigma} + \pi - \alpha + \mathrm{i}eA_{\mu}\bar{m}^{\mu})F_2\\
&- \mathrm{i}\frac{1}{\sqrt{2}}\mu_0 G_1 = 0,\\
&(\Delta + \mu - \gamma + \mathrm{i}eA_{\mu}n^{\mu})F_2\\
&+ (\delta + \beta - \tau + \mathrm{i}eA_{\mu}m^{\mu})F_1\\
&- \mathrm{i}\frac{1}{\sqrt{2}}\mu_0 G_2 = 0,\\
&(D + \varepsilon^* - \rho^* + \mathrm{i}eA_{\mu}l^{\mu})G_2\\
&- (\delta + \pi^* - \alpha^* + \mathrm{i}eA_{\mu}m^{\mu})G_1\\
&- \mathrm{i}\frac{1}{\sqrt{2}}\mu_0 F_2 = 0,\\
&(\Delta + \mu^* - \gamma^* + \mathrm{i}eA_{\mu}n^{\mu})G_1\\
&- (\bar{\sigma} + \beta^* - \tau^* + \mathrm{i}eA_{\mu}\bar{m}^{\mu})G_2\\
&- \mathrm{i}\frac{1}{\sqrt{2}}\mu_0 F_1 = 0,
\end{aligned} \tag{8}$$

其中 μ_0, e 分别为荷电 Dirac 粒子的质量和电荷.

在动态缓变 Reissner-Nordström 时空中,可以构造零标架 $Z_{m\mu}$ 为

$$\begin{aligned}
Z_{m\mu} &= (Z_{1\mu}, Z_{2\mu}, Z_{3\mu}, Z_{4\mu})\\
&= (l_{\mu}, n_{\mu}, m_{\mu}, \bar{m}_{\mu}),
\end{aligned} \tag{9}$$

其中

$$\begin{aligned}
l_{\mu} &= \frac{1}{\sqrt{2B}}\left[\frac{B}{r}, r, 0, 0\right],\\
n_{\mu} &= \frac{1}{\sqrt{2B}}\left[-\frac{B}{r}, r, 0, 0\right],\\
m_{\mu} &= \frac{r}{\sqrt{2}}[0, 0, 1, \mathrm{i}\sin\theta],\\
\bar{m}_{\mu} &= \frac{r}{\sqrt{2}}[0, 0, -1, \mathrm{i}\sin\theta].
\end{aligned} \tag{10}$$

由(10)式可得出零标架的另一协变形式 Z^{μ}_{m} 为

$$\begin{aligned}
Z^{\mu}_{m} &= (Z^{\mu}_{1}, Z^{\mu}_{2}, Z^{\mu}_{3}, Z^{\mu}_{4})\\
&= (l^{\mu}, n^{\mu}, m^{\mu}, \bar{m}^{\mu}),
\end{aligned} \tag{11}$$

其中

$$\begin{aligned}
l^{\mu} &= \frac{1}{\sqrt{2B}}\left[-r, \frac{B}{r}, 0, 0\right],\\
n^{\mu} &= \frac{1}{\sqrt{2B}}\left[r, \frac{B}{r}, 0, 0\right],\\
m^{\mu} &= \frac{r^{-1}}{\sqrt{2}}\left[0, 0, 1, \frac{\mathrm{i}}{\sin\theta}\right],\\
\bar{m}^{\mu} &= \frac{r^{-1}}{\sqrt{2}}\left[0, 0, -1, \frac{\mathrm{i}}{\sin\theta}\right],
\end{aligned} \tag{12}$$

其中

$$B = [r^2 - 2M(t)r + Q^2(t)]. \tag{13}$$

由零标架的协变分量 $Z_{m\mu}$ 和 Z^{μ}_{m} 的关系式

$$Z_{m\mu}Z^{\mu}_{n} = \eta_{mn}, \tag{14}$$

容易得出

$$\begin{aligned}
&l_{\mu}l^{\mu} = n_{\mu}n^{\mu} = m_{\mu}m^{\mu} = \bar{m}_{\mu}\bar{m}^{\mu} = 0,\\
&l_{\mu}n^{\mu} = -m_{\mu}\bar{m}^{\mu} = 1,\\
&l_{\mu}m^{\mu} = l_{\mu}\bar{m}^{\mu} = n_{\mu}m^{\mu} = n_{\mu}\bar{m}^{\mu} = 0,\\
&g_{\mu\nu} = l_{\mu}n_{\nu} + n_{\mu}l_{\nu} - m_{\mu}\bar{m}_{\nu} - \bar{m}_{\mu}m_{\nu}.
\end{aligned} \tag{15}$$

根据(1)—(3)式和 $\Gamma^{\alpha}_{\mu\lambda} = \frac{1}{2}g^{\alpha\nu}(g_{\mu\nu,\lambda} + g_{\nu\lambda,\mu} - g_{\lambda\mu,\nu})$,通过计算得出非零的联络分量分别为

$$\begin{aligned}
\Gamma^{0}_{00} &= g^{00}\left(\frac{\dot{M}}{r} - \frac{Q\dot{Q}}{r^2}\right),\\
\Gamma^{0}_{11} &= -g^{00}\left(\frac{\dot{M}}{r} - \frac{Q\dot{Q}}{r^2}\right)\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-2},\\
\Gamma^{1}_{00} &= g^{11}\left(\frac{M}{r^2} - \frac{Q^2}{r^3}\right),\\
\Gamma^{1}_{11} &= -g^{11}\left(\frac{M}{r^2} - \frac{Q^2}{r^3}\right)\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-2},\\
\Gamma^{1}_{10} &= \Gamma^{1}_{01} = g^{11}\left(\frac{\dot{M}}{r} - \frac{Q\dot{Q}}{r^2}\right)\\
&\quad\times\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-2},\\
\Gamma^{0}_{01} &= \Gamma^{0}_{10} = -g^{00}\left(\frac{M}{r^2} - \frac{Q^2}{r^3}\right),\\
\Gamma^{1}_{22} &= -g^{11}r, \Gamma^{1}_{33} = -g^{11}r\sin^2\theta,\\
\Gamma^{3}_{13} &= \Gamma^{3}_{31} = g^{33}r\sin^2\theta,\\
\Gamma^{3}_{23} &= \Gamma^{3}_{32} = g^{33}r^2\sin\theta\cos\theta,\\
\Gamma^{2}_{33} &= g^{22}r^2\sin\theta\cos\theta.
\end{aligned} \tag{16}$$

(16)式及以下的数学表达式中的 $M = M(t)$, $Q = Q(t)$. 由(16),(10),(12)式以及 Ricci 旋度系数 $\gamma^{np}_{m} = Z_{m\mu;\nu}Z^{n\mu}Z^{p\nu}$,通过计算零标架的普通微分和协变微分,可得 Newman-Penrose 旋系数如下:

$$\begin{aligned}
\varepsilon &= \frac{1}{2}(\gamma_{121} - \gamma_{341}) = \frac{1}{2}(l_{\mu;\nu}n^{\mu}l^{\nu} - m_{\mu;\nu}\bar{m}^{\mu}l^{\nu})\\
&= \frac{Br^{-2}}{4\sqrt{2B}} + \frac{1}{2\sqrt{2B}}\left(\frac{M}{r} - \frac{Q^2}{r^2}\right)
\end{aligned}$$

$$-\frac{1}{2}\frac{r^2}{B\sqrt{2B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right),$$

$$\rho=\gamma_{134}=l_{\mu;\nu}m^{\mu}\bar{m}^{\nu}=-\frac{\sqrt{B}}{\sqrt{2}r^2},$$

$$\mu=-\gamma_{243}=-n_{\mu;\nu}\bar{m}^{\mu}m^{\nu}=\frac{\sqrt{B}}{\sqrt{2}r^2},$$

$$\beta=\frac{1}{2}(\gamma_{123}-\gamma_{343})$$

$$=\frac{1}{2}(l_{\mu;\nu}n^{\mu}m^{\nu}-m_{\mu;\nu}\bar{m}^{\mu}m^{\nu})$$

$$=\frac{r^{-1}}{4\sqrt{2}}\frac{\cos\theta}{\sin\theta},$$

$$\gamma=\frac{1}{2}(\gamma_{122}-\gamma_{342})=\frac{1}{2}(l_{\mu;\nu}n^{\mu}n^{\nu}-m_{\mu;\nu}\bar{m}^{\mu}n^{\nu})$$

$$=\frac{Br^{-2}}{4\sqrt{2B}}-\frac{1}{2\sqrt{2B}}\left(\frac{M}{r}-\frac{Q^2}{r^2}\right)$$

$$-\frac{1}{2}\frac{r^2}{B\sqrt{2B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right),$$

$$\kappa=\pi=\alpha=\lambda=\sigma=\tau=\nu=0. \tag{17}$$

由(12)式可得微分算子如下：

$$D=l^{\mu}\partial_{\mu}=\frac{1}{\sqrt{2B}}\left(-r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right),$$

$$\Delta=n^{m}\partial_{\mu}=\frac{1}{\sqrt{2B}}\left(r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right),$$

$$\delta=m^{\mu}\partial_{\mu}=\frac{r^{-1}}{\sqrt{2}}\left(\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right),$$

$$\bar{\delta}=\bar{m}^{\mu}\partial_{\mu}=\frac{r^{-1}}{\sqrt{2}}\left(-\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right). \tag{18}$$

将(10),(12),(17),(18)式代入方程(8)中,得出动态缓变 Reissner-Nordström 时空中荷电粒子的 Dirac 方程为

$$\left[\frac{1}{\sqrt{B}}\left(-r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right)\right.$$

$$+\frac{5\sqrt{B}}{4r^2}+\frac{1}{2\sqrt{B}}\left(\frac{M}{r}-\frac{Q^2}{r^2}\right)$$

$$\left.-\frac{r^2}{2\sqrt{B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)-\mathrm{i}eA_0\frac{r}{\sqrt{B}}\right]F_1$$

$$+r^{-1}\left(-\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right)F_2-\mathrm{i}\mu_0G_1=0,$$

$$\left[\frac{1}{\sqrt{B}}\left(r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right)\right.$$

$$+\frac{3\sqrt{B}}{r^2}+\frac{1}{2\sqrt{B}}\left(\frac{M}{r}-\frac{Q^2}{r^2}\right)$$

$$\left.+\frac{r^2}{2B\sqrt{B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)+\mathrm{i}eA_0\frac{r}{\sqrt{B}}\right]F_2$$

$$+\left[r^{-1}\left(\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right)+\frac{1}{4r}\frac{\cos\theta}{\sin\theta}\right]F_1$$

$$-\mathrm{i}\mu_0G_2=0,$$

$$\left[\frac{1}{\sqrt{B}}\left(-r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right)\right.$$

$$+\frac{5\sqrt{B}}{4r^2}+\frac{1}{2\sqrt{B}}\left(\frac{M}{r}-\frac{Q^2}{r^2}\right)$$

$$\left.-\frac{r^2}{2B\sqrt{B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)-\mathrm{i}eA_0\frac{r}{\sqrt{B}}\right]G_2$$

$$-r^{-1}\left(-\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right)G_1-\mathrm{i}\mu_0F_2=0,$$

$$\left[\frac{1}{\sqrt{B}}\left(r\frac{\partial}{\partial t}+Br^{-1}\frac{\partial}{\partial r}\right)\right.$$

$$+\frac{3\sqrt{B}}{r^2}+\frac{1}{2\sqrt{B}}\left(\frac{M}{r}-\frac{Q^2}{r^2}\right)$$

$$\left.+\frac{r^2}{2B\sqrt{B}}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)+\mathrm{i}eA_0\frac{r}{\sqrt{B}}\right]G_1$$

$$+\left[r^{-1}\left(-\frac{\partial}{\partial\theta}+\mathrm{i}\sin^{-1}\theta\frac{\partial}{\partial\phi}\right)\right.$$

$$\left.+\frac{1}{4r}\frac{\cos\theta}{\sin\theta}\right]G_2-\mathrm{i}\mu_0F_1=0. \tag{19}$$

对此方程作如下形式的变量分离：

$$F_1=\mathrm{e}^{\mathrm{i}m\phi}f_1(t,r,\theta)=\mathrm{e}^{\mathrm{i}m\phi}R_-(t,r)S_-(\theta),$$

$$F_2=\mathrm{e}^{\mathrm{i}m\phi}f_2(t,r,\theta)=\mathrm{e}^{\mathrm{i}m\phi}R_+(t,r)S_+(\theta),$$

$$G_1=\mathrm{e}^{\mathrm{i}m\phi}g_1(t,r,\theta)=\mathrm{e}^{\mathrm{i}m\phi}R_+(t,r)S_-(\theta),$$

$$G_2=\mathrm{e}^{\mathrm{i}m\phi}g_2(t,r,\theta)=\mathrm{e}^{\mathrm{i}m\phi}R_-(t,r)S_+(\theta), \tag{20}$$

并将(20)式代入方程(19),可得

$$\left[\left(-r^2\frac{\partial}{\partial t}+B\frac{\partial}{\partial r}\right)+\frac{5B}{4r}+\frac{1}{2}\left(M-\frac{Q^2}{r}\right)\right.$$

$$\left.-\frac{r^3}{2B}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)-\mathrm{i}eA_0r^2\right]R_-$$

$$-\sqrt{B}(\mathrm{i}\mu_0r+\lambda)R_+=0,$$

$$\left[\left(r^2\frac{\partial}{\partial t}+B\frac{\partial}{\partial r}\right)+\frac{3B}{4r}+\frac{1}{2}\left(M-\frac{Q^2}{r}\right)\right.$$

$$\left.+\frac{r^3}{2B}\left(\dot{M}-\frac{Q\dot{Q}}{r}\right)+\mathrm{i}eA_0r^2\right]R_+$$

$$-\sqrt{B}(\mathrm{i}\mu_0r+\lambda)R_-=0,$$

$$\left(\frac{\partial}{\partial\theta}+\frac{m}{\sin\theta}\right)S_+-\lambda S_-=0,$$

$$\left(\frac{\partial}{\partial\theta}-\frac{m}{\sin\theta}+\frac{1}{4}\frac{\cos\theta}{\sin\theta}\right)S_-+\lambda S_+=0, \tag{21}$$

其中 m,λ 为分离变量过程中引进的常数,m 的物理意义为 Dirac 粒子角动量的轴向分量.由(21)式可知,弯曲时空中 Dirac 方程完全退耦为用普通导数表示的偏微分方程,其中前两个是径向方程,后两个属于角向方程,在研究辐射问题时,只需考虑径向方程即可.由(21)式前两个方程可得关于 R_+ 的径向方程(R_- 也有类似结果)为

$$D_0 D_1 R_+ - \frac{D_0[\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)]}{\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)} D_1 R_+ - B(\mathrm{i}\mu_0 r + \lambda)^2 R_+ = 0, \tag{22}$$

其中

$$\begin{aligned} D_0 &= - r^2 \frac{\partial}{\partial t} + B \frac{\partial}{\partial r} + \frac{5B}{4r} + \frac{1}{2}\left(M - \frac{Q^2}{r}\right) \\ &\quad - \frac{r^2}{2B}\left(\dot{M} - \frac{Q\dot{Q}}{r}\right) - \mathrm{i}eA_0 r^2 \\ &= - r^2 \frac{\partial}{\partial t} + B \frac{\partial}{\partial r} + \widetilde{D}_0, \\ D_1 &= - r^2 \frac{\partial}{\partial t} + B \frac{\partial}{\partial r} + \frac{3B}{4r} + \frac{1}{2}\left(M - \frac{Q^2}{r}\right) \\ &\quad - \frac{r^2}{2B}\left(\dot{M} - \frac{Q\dot{Q}}{r}\right) - \mathrm{i}eA_0 r^2 \\ &= r^2 \frac{\partial}{\partial t} + B \frac{\partial}{\partial r} + \widetilde{D}_1 . \end{aligned} \tag{23}$$

采用 tortoise 坐标变换[9]

$$\begin{aligned} r_* &= r + \frac{1}{2k}\ln[r - r_{\mathrm{H}}(t)], \\ t_* &= t - t_0, \end{aligned} \tag{24}$$

可得

$$\begin{aligned} \frac{\partial}{\partial r} &= \frac{1 + 2k(r - r_{\mathrm{H}})}{2k(r - r_{\mathrm{H}})} \frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t_*} - \frac{\dot{r}_{\mathrm{H}}}{2k(r - r_{\mathrm{H}})} \frac{\partial}{\partial r_*}, \\ \frac{\partial^2}{\partial r^2} &= \left[\frac{1 + 2k(r - r_{\mathrm{H}})}{2k(r - r_{\mathrm{H}})}\right]^2 \frac{\partial^2}{\partial r_*^2} \\ &\quad - \frac{1}{2k(r - r_{\mathrm{H}})^2} \frac{\partial}{\partial r_*}, \\ \frac{\partial^2}{\partial t^2} &= \frac{\partial^2}{\partial t_*^2} - \frac{\dot{r}_{\mathrm{H}}}{2k(r - r_{\mathrm{H}})} \frac{\partial^2}{\partial r_* \partial t_*} \\ &\quad - \frac{\ddot{r}_{\mathrm{H}}(r - r_{\mathrm{H}}) + \dot{r}_{\mathrm{H}}^2}{2k(r - r_{\mathrm{H}})^2} \frac{\partial}{\partial r_*} \\ &\quad + \frac{\dot{r}_{\mathrm{H}}^2}{4k^2(r - r_{\mathrm{H}})^2} \frac{\partial^2}{\partial r_*^2}, \\ \frac{\partial^2}{\partial t \partial r} &= \left[\frac{1 + 2k(r - r_{\mathrm{H}})}{2k(r - r_{\mathrm{H}})}\right] \frac{\partial^2}{\partial t_* \partial r_*} \\ &\quad - \left[\frac{1 + 2k(r - r_{\mathrm{H}})}{2k(r - r_{\mathrm{H}})}\right] \frac{\dot{r}_{\mathrm{H}}}{2k(r - r_{\mathrm{H}})} \frac{\partial^2}{\partial r_*^2} \\ &\quad + \frac{\dot{r}_{\mathrm{H}}}{2k(r - r_{\mathrm{H}})^2} \frac{\partial}{\partial r_*} . \end{aligned} \tag{25}$$

将(23)—(25)式代入(22)式,可得

$$\begin{aligned} &\frac{(g^{11})^2[1 + 2k(r - r_{\mathrm{H}})]^2 - \dot{r}_{\mathrm{H}}^2}{4k\dot{r}_{\mathrm{H}}(r - r_{\mathrm{H}})} \frac{\partial^2 R_+}{\partial r_*^2} \\ &- \frac{2k(r - r_{\mathrm{H}})}{\dot{r}_{\mathrm{H}}} \frac{\partial^2 R_+}{\partial t_*^2} + 2 \frac{\partial^2 R_+}{\partial t_* \partial r_*} \\ &+ C \frac{\partial R_+}{\partial r_*} + D \frac{\partial R_+}{\partial t_*} + ER_+ = 0, \end{aligned} \tag{26}$$

其中

$$\begin{aligned} C &= \frac{[1 + 2k(r - r_{\mathrm{H}})]}{r^4 \dot{r}_{\mathrm{H}}}\Big\{2B(r - M) \\ &\quad + 2r^2(\dot{M}r - Q\dot{Q}) + B\widetilde{D}_1 + B\widetilde{D}_0 \\ &\quad - \frac{\sqrt{B}D_0[\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)]}{\mathrm{i}\mu_0 r + \lambda}\Big\} \\ &\quad + \frac{-(g^{11})^2 + \dot{r}_{\mathrm{H}}^2 + \ddot{r}_{\mathrm{H}}(r - r_{\mathrm{H}})}{\dot{r}_{\mathrm{H}}(r - r_{\mathrm{H}})} \\ &\quad - \frac{1}{r^4}\Big\{2Br + r^2\widetilde{D}_0 - r^2\widetilde{D}_1 \\ &\quad - \frac{r^2 D_0[\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)]}{\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)}\Big\}, \\ D &= \frac{2k(r - r_{\mathrm{H}})}{r^4 \dot{r}_{\mathrm{H}}}\Big\{2Br + r^2\widetilde{D}_0 - r^2\widetilde{D}_1 \\ &\quad - \frac{r^2 D_0[\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)]}{\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)}\Big\}, \\ E &= \frac{2k(r - r_{\mathrm{H}})}{r^4 \dot{r}_{\mathrm{H}}}\Big\{\widetilde{D}_0\widetilde{D}_1 - B(\mathrm{i}\mu_0 r + \lambda)^2 \\ &\quad - \frac{\widetilde{D}_1 D_0[\sqrt{B}(\mathrm{i}\mu_0 + \lambda)]}{\sqrt{B}(\mathrm{i}\mu_0 r + \lambda)} - r^2\Big[\frac{3}{2r}(\dot{M}r + Q\dot{Q}) \\ &\quad + \frac{1}{2}\left(\dot{M} - \frac{2Q\dot{Q}}{r}\right) + \frac{r^3}{B}(\dot{M}r - Q\dot{Q})\left(\dot{M} - \frac{Q\dot{Q}}{r}\right) \\ &\quad + \frac{r^3}{2B}\left(\ddot{M} - \frac{\dot{Q}^2 + Q\ddot{Q}}{r}\right) + \mathrm{i}e\dot{A}_0 r^2\Big] \\ &\quad + B\Big[\frac{3}{2r}(r - M) - \frac{3B}{4r^2} + \frac{Q^2}{2r^2} \\ &\quad + \frac{3Br^2 - r^3(r - M)}{B^2}\left(\dot{M} - \frac{Q\dot{Q}}{r}\right) \\ &\quad + \frac{rQ\dot{Q}}{2B} - \mathrm{i}eQ\Big]\Big\}. \end{aligned} \tag{27}$$

(26)式等号左端第一项的系数在 $r \to r_{\mathrm{H}}$ 时为$\frac{0}{0}$型.令

$$A = \lim_{r \to r_H} \frac{(g^{11})^2 [1 + 2k(r - r_H)]^2 - \dot{r}_H^2}{4k\dot{r}_H(r - r_H)} = 1, \tag{28}$$

则当 $r \to r_H$ 时,(26)式化为

$$\frac{\partial^2 R_+}{\partial r_*^2} + 2\frac{\partial^2 R_+}{\partial t_* \partial r_*} + C' \frac{\partial R_+}{\partial r_*} = 0. \tag{29}$$

由(27)式可得(29)式中的系数 C' 为

$$\begin{aligned} C' &= \lim_{r \to r_H} C \\ &= \frac{\ddot{r}_H}{\dot{r}_H} + \frac{4 - \dot{r}_H}{r_H} - \frac{1}{r_H^2}\left(M + \frac{Q^2}{r_H^2}\right) \\ &\quad + \frac{1}{r_H \dot{r}_H}\left(2 + \frac{1}{r_H}\right)\left(\dot{M} - \frac{Q\dot{Q}}{r_H}\right) + \mathrm{i}\,\frac{2eQ}{r_H} \\ &= \alpha_0 + \mathrm{i}2\omega_0, \end{aligned} \tag{30}$$

其中

$$\omega_0 = eA_0 = \frac{eQ}{r_H}, \tag{31}$$

$$\begin{aligned} \alpha_0 &= \frac{\ddot{r}_H}{\dot{r}_H} + \frac{4 - \dot{r}_H}{r_H} - \frac{1}{r_H^2}\left(M + \frac{Q^2}{r_H^2}\right) \\ &\quad + \frac{1}{r_H \dot{r}_H}\left(2 + \frac{1}{r_H}\right)\left(\dot{M} - \frac{Q\dot{Q}}{r_H}\right). \end{aligned} \tag{32}$$

求解方程(29)得到 Dirac 粒子进出动态缓变 Reissner-Nordström 黑洞视界的径向波函数为

$$\begin{aligned} R^{\mathrm{in}} &= R_+^{\mathrm{in}} = \mathrm{e}^{\mathrm{i}\omega t_*}, \\ R^{\mathrm{out}} &= R_+^{\mathrm{out}} = \mathrm{e}^{-2\mathrm{i}(\omega - \omega_0) r_*} \mathrm{e}^{-\alpha_0 r_*}. \end{aligned} \tag{33}$$

由此可知,R_+^{in} 在视界上解析,但是,R_+^{out} 在视界处具有对数奇异性,采用 Damour-Ruffini 的解析延拓法,可以将 R_+^{out} 延拓到视界内部,即

$$\begin{aligned} \widetilde{R}_+^{\mathrm{out}} &= \mathrm{e}^{-\mathrm{i}\omega t_*} \mathrm{e}^{2\mathrm{i}(\omega - \omega_0) r_*} \mathrm{e}^{-\alpha_0 r_*} \\ &\quad \times \mathrm{e}^{\pi(\omega - \omega_0)/k} \mathrm{e}^{\mathrm{i}\alpha_0 \pi / 2k}, \end{aligned} \tag{34}$$

所以出射波在视界处的散射概率为

$$\left| R_+^{\mathrm{out}} / \widetilde{R}_+^{\mathrm{out}} \right|^2 = \mathrm{e}^{-2\pi(\omega - \omega_0)/k}. \tag{35}$$

根据 Domour-Ruffini-Sannam 的方法,得到出射波谱为

$$N_\omega = \frac{1}{\mathrm{e}^{(\omega - \omega_0)/k_B T} + 1}, \tag{36}$$

其中 k_B 为 Boltzmann 常数,T 为辐射温度.由(28)式可得

$$k = \frac{r_H(1 - \dot{r}_H) - M}{2Mr_H - Q^2}, \tag{37}$$

所以辐射温度为

$$T = \frac{k}{2\pi k_B} = \frac{1}{2\pi k_B} \frac{r_H(1 - \dot{r}_H) - M}{2Mr_H - Q^2}. \tag{38}$$

显然,(36)式为费米子的黑体辐射谱,表征动态缓变 Reissner-Nordström 黑洞时空中荷电 Dirac 粒子的量子热辐射[16—21],这就说明 M 和 Q 随时间 t 缓变时,Reissner-Nordström 黑洞确实在辐射 Dirac 粒子.由(36),(37)和(38)式可见,这一系列结果都是具有一定意义的新结论,这一系列结论在 M 和 Q 随 t 缓变的前提条件下成立.当 $\dot{M} = \dot{Q} = 0$,$\dot{r}_H = 0$,$Q = 0$ 时,由(38)式可知 $T = 1/8\pi M k_B$,此即 Schwarzschild 黑洞 Hawking 热辐射温度.

3. 动态缓变 Reissner-Nordström 时空中 Hamilton-Jacobi 方程的解与量子非热辐射

在(1)式表示的弯曲时空中,荷电 e、质量为 μ_0 的粒子的运动方程(Hamilton-Jacobi 方程)为[22]

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu} + eA_\mu\right)\left(\frac{\partial S}{\partial x^\nu} + eA_\nu\right) + \mu_0^2 = 0, \tag{39}$$

其中 S 为 Hamilton 主函数,A_μ(或 A_ν)为与 $Q(t)$ 相应的电磁四矢,对动态缓变 Reissner-Nordström 黑洞而言,如(31)式所述,$A_\mu = \left(\frac{Q(t)}{r},0,0,0\right)$.把(3)式代入(39)式,得

$$\begin{aligned} &-\left[1 - \frac{2M(t)}{r} + \frac{Q^2(t)}{r^2}\right]^{-1}\left(\frac{\partial S}{\partial t} + \frac{eQ(t)}{r}\right)^2 \\ &+ \left[1 - \frac{2M(t)}{r} + \frac{Q^2(t)}{r^2}\right]\left(\frac{\partial S}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 \\ &+ \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial S}{\partial \phi}\right)^2 + \mu_0^2 = 0. \end{aligned} \tag{40}$$

把(25)式代入(40)式,令

$$S = R(r_*, t_*) + Y(\theta, \phi), \tag{41}$$

并用 λ' 表示分离变量过程中引进的常数,则在 tortoise 坐标下的径向方程为

$$\begin{aligned} &\left[1 - \frac{2M(t)}{r} + \frac{Q^2(t)}{r^2}\right]^{-1} \\ &\times \left[\frac{\partial R}{\partial t_*} - \frac{\dot{r}_H}{2k(r - r_H)} \frac{\partial R}{\partial r_*} + \frac{eQ(t)}{r}\right]^2 \\ &- \left[1 - \frac{2M(t)}{r} + \frac{Q^2(t)}{r^2}\right] \\ &\times \left[\frac{1 + 2k(r - r_H)}{2k(r - r_H)}\right]^2 \left(\frac{\partial R}{\partial r_*}\right)^2 - \frac{\lambda'^2}{r^2} - \mu_0^2 = 0. \end{aligned} \tag{42}$$

令

$$\frac{\partial R}{\partial t_*} = \frac{\partial S}{\partial t_*} = -\omega_N, \tag{43}$$

其中 ω_N 为粒子能量.把(43)式代入(42)式,得

$$\left(\frac{\partial R}{\partial r_*}\right)^2 \left\{ -\dot{r}_H^2 + [1 + 2k(r - r_H)]^2 \right.$$

$$\times\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]^2\Big\}\frac{1}{4k\dot{r}_{\rm H}(r-r_{\rm H})}$$

$$-\left(\omega_N-\frac{eQ(t)}{r}\right)\frac{\partial R}{\partial r_*}-\frac{k(r-r_{\rm H})}{\dot{r}_{\rm H}}$$

$$\times\left(\omega_N-\frac{eQ(t)}{t}\right)^2+\frac{k(r-r_{\rm H})}{\dot{r}_{\rm H}}\left(\frac{\lambda'^2}{r^2}+\mu_0^2\right)$$

$$\times\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]=0. \quad (44)$$

由此可得粒子能量 ω 所满足的关系式为

$$\left(\omega_N-\frac{eQ(t)}{r}\right)^2\dot{r}_{\rm H}^2-\Big\{-\dot{r}_{\rm H}^2+[1+2k(r-r_{\rm H})]^2$$

$$\times\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]^2\Big\}\Big\{-\left(\omega-\frac{eQ(t)}{r}\right)^2$$

$$+\left(\frac{\lambda'^2}{r^2}+\mu_0^2\right)\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]\Big\}\geqslant 0. \quad (45)$$

由此可得

$$\omega_N^{\pm}=\frac{eQ(t)}{r}\pm\frac{\Big\{\left(\mu_0^2+\frac{\lambda'^2}{r^2}\right)\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]\left[\left(1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right)^2[1+2k(r-r_{\rm H})]^2-\dot{r}_{\rm H}^2\right]\Big\}^{1/2}}{[2k(r-r_{\rm H})+1]\left[1-\frac{2M(t)}{r}+\frac{Q^2(t)}{r^2}\right]} \quad (46)$$

和

$$\omega\geqslant\omega_N^+,\quad \omega\leqslant\omega_N^-. \quad (47)$$

在视界附近,即在 $r\to r_{\rm H}$ 时,由(5)和(46)式可得

$$\lim_{r\to r_{\rm H}}\omega_N^{\pm}=\omega_N^0=\frac{eQ(t)}{r_{\rm H}}$$

$$=\frac{eQ(t)(1-\dot{r}_{\rm H})}{M(t)+\sqrt{M^2(t)-Q^2(t)(1-\dot{r}_{\rm H})}}. \quad (48)$$

(46)—(48)式表明了动态缓变 Reissner-Nordström 时空中荷电粒子的能级分布特征.(48)式表明,在黑洞视界处,出现了粒子能级交错现象,这就使得处于负能态而能量高于最低正能量的费米子将通过量子隧道效应穿过禁区而成为出射的正能粒子,这是与温度无关的量子非热辐射.由于黑洞视界附近存在的电势把该处一切粒子的基态能量(即真空态能级)升高或降低,当一种粒子的基态能级高于它在平直时空中的最低激发态能级时,就会产生这种粒子的自发辐射[23,24].对于玻色子,在自发辐射的同时,还会有受激辐射发生.

4. 讨 论

在动态缓变 Reissner-Nordström 时空中量子非热辐射与温度无关,是由于黑洞视界附近存在电势而导致的非热辐射,量子非热辐射的最大能量为 ω_N^0,由(48)式确定,辐射条件为 $\mu_0<\omega_N\leqslant\omega_N^0$.由于 M,Q 随时间缓慢变化,因此辐射条件也随时间缓慢变化.实际上,如果处于负能态的费米子的能量满足量子非热辐射条件,则处于负能态的费米子将从负能级跃迁到正能级,与此同时,将会出现一个正能粒子和一个负能态的空穴,即有粒子对通过隧道效应而实化,这就产生了量子非热效应.因此,量子非热辐射要在一定的条件下才能发生.

动态缓变 Reissner-Nordström 黑洞的量子热辐射是与温度有关的辐射,辐射谱与辐射温度有关,辐射温度不仅与黑洞的质量 $M(t)$和电荷 $Q(t)$有关,而且还与黑洞的蒸发率 $\dot{r}_{\rm H}$ 有关.当 $Q=0$,$\dot{r}_{\rm H}=0$,$\dot{M}=\dot{Q}=0$ 时,黑洞演变为 Schwarzschild 黑洞.由(36)和(38)式可知,此时的黑洞仍有温度为 $1/8\pi Mk_{\rm B}$ 的量子热辐射,而此时的黑洞没有量子非热辐射发生[25—31].

需要进一步说明的是,(36)式中 ω_0 起化学势的作用,因此,由(31)和(48)式可知,动态缓变 Reissner-Nordström 黑洞的量子热辐射谱中的 ω_0 与量子非热辐射的最大能量 ω_N^0 相等,关于这一点,可以说是量子热辐射与量子非热辐射的联系.值得说明的是,在(33)和(34)式中 α_0 如(32)式所示,由(32)式可见,α_0 的表达式中有一项与 $\ddot{r}_{\rm H}$ 有关,$\ddot{r}_{\rm H}=\frac{d^2r_{\rm H}}{dt^2}$,在此之前的文献中未曾见过此 $\ddot{r}_{\rm H}$,由(10),(12),(18),(22),(23),(27)和(30)式可见,本文中出现 $\ddot{r}_{\rm H}$ 是必然的.由于 $\dot{r}_{\rm H}=\frac{{\rm d}r_{\rm H}}{{\rm d}t}$为描述黑洞蒸发率的物理量,因此,$\ddot{r}_{\rm H}=\frac{{\rm d}\dot{r}_{\rm H}}{{\rm d}t}=\frac{{\rm d}^2r_{\rm H}}{{\rm d}t^2}$为描述黑洞蒸发率随时间变化的特征的一个物理量.

由 Dirac 粒子的径向运动方程(26)和非热辐射粒子的径向运动方程(44)可见,Dirac 粒子径向运动方程中$\frac{\partial^2R_+}{\partial r_*^2}$项的系数与非热辐射粒子径向运动方

程中$\left(\frac{\partial R}{\partial r_*}\right)^2$项的系数相同，在 $r \to r_H$ 时，此系数属于$\frac{0}{0}$型，当对此系数求 $r \to r_H$ 的极限时，令此极限值为 1，都可以求出 k 值，由(37)和(38)式可知，实际上 k 为黑洞视界的表面重力，$k/2\pi k_B$ 即为黑洞量子热辐射温度，因此，在研究黑洞量子非辐射过程中，对粒子径向运动方程中$\left(\frac{\partial R}{\partial r_*}\right)^2$项的系数求 $r \to r_H$ 的极限，并令此极限值为 1，也可以求出黑洞视界表面重力 k，从而知道黑洞的热辐射温度，这是有意义的结论.

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Discussion on the characteristics of the quantum radiation of unstationary and slowly-changing Reissner-Nordström black hole*

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Abstract

Taking into account the case that the mass M and charge Q of Reissner-Nordström black hole varying slowly with time t, the discussion on the quantum radiation characteristics of the black hole shows that the thermal radiation spectrum of charged Dirac particles is connected with the black hole's evaporation rate together with the factors $M(t)$, $Q(t)$ etc. changing with time, and that the maximum energy of the quantum nonthermal radiation of nonstationary and slowly-changing Reissner-Nordström black hole is equal to the chemical potential in the quantum thermal radiation spectrum.

Keywords: black hole, slowly-changing space-time, radiation

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