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高阶微商系统中正则 Ward 恒等式和 Abel 规范理论中动力学质量的产生*

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基于含复合场的正则 Ward 恒等式, 研究了含高阶微商的 Abel 理论中动力学规范对称破缺. 得到了包括费米子和束缚态的质量谱. 讨论了高阶微商项的影响.

关键词: 正则 Ward 恒等式, 约束, 动力学对称破缺, Abel 规范理论

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1. 引　言

动力学对称破缺在粒子物理中起着重要的作用, 它提出一种不同于 Higgs 机理的产生破缺的可能机理, 在动力学自发破缺机理中不需要引入 Higgs 标量场, 拉氏函数只包含所要研究的场, 自发破缺解存在于自己所满足的(非微扰的)方程之中. 它包含的基本场及参数更少, 因而一直引起人们的研究兴趣[1—7]. 它的缺点是计算困难, 因为动力学自发破缺是一个非微扰效应, 不能作微扰展开. 最近, 开展了包含复合场的 Ward-Takahashi(W-T)恒等式的研究. 在一些模型的研究中, 这种方法能够得到包括费米子和束缚态的质量谱. 但在存在明显破缺的情况下, 用这种方法能更方便地讨论相结构、质量谱、PCAC 等[3,8], 文献[1]曾将它推广到规范对称动力学破缺的情况, 讨论规范玻色子获得质量的机理[9], 当规范对称动力学破缺时, 矢量介子获得的质量与 Schwinger 机理[10]结果一致. 由高阶微商描述的动力系统一直被广泛研究[11—15], 基于相空间的路径积分, 建立了高阶微商奇异拉氏量系统在相空间中的正则 Ward 恒等式[15]. 拉氏量中含高阶微商项, 在量子理论中它能改变费曼图的收敛性. 本文对含高阶微商项的 Cornwall-Norton 模型[16]和含高阶微商项的 Jackiw-Johnson 模型[17], 基于高阶微商约束理论的正则 Ward 恒等式[15], 讨论高阶微商项对 Abel 规范对称理论中, 动力学质量产生的影响.

2. 含高阶微商项的 Cornwall-Norton 模型

在 Cornwall-Norton 模型中引入高阶微商项, 其拉氏密度为

$$\mathscr{L} = \bar{\psi}(\mathrm{i}\gamma\partial - m_0)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{c}{4}\partial_\rho G_{\mu\nu}\partial^\rho G^{\mu\nu} + g\bar{\psi}\gamma^\mu A_\mu + g'\bar{\psi}\gamma^\mu\tau_2\psi B^\mu, \tag{1}$$

其中 $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, c 为任意常数. 由 Ostrogradsky 变换引入正则动量[11], 设场变量 $\bar{\psi}, \psi, A_\mu, B_\mu, C_\mu = \dot{B}_\mu$ 的正则动量分别为 $\pi, \bar{\pi}, \pi^\mu, P^\mu, Q^\mu$, 则

$$\begin{aligned} P^0 &= -c\partial_0\partial_i G^{0i}, \\ P^i &= G^{i0} + c(\nabla^2 G^{0i} + \partial_0\partial_j G^{ji}) - \partial_0 Q^i, \\ Q^0 &= 0, \\ Q^i &= -c\partial_0 G^{0i}, \end{aligned} \tag{2}$$

$$\begin{aligned} \pi &= \frac{\partial\mathscr{L}}{\partial\dot{\bar{\psi}}} = 0, \\ \bar{\pi} &= \frac{\partial\mathscr{L}}{\partial\dot{\psi}} = -\bar{\psi}\mathrm{i}\gamma^0, \\ \pi^\mu &= \frac{\partial\mathscr{L}}{\partial\dot{A}_\mu} = -F^{0\mu}. \end{aligned} \tag{3}$$

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初级约束为

$$\Phi_1 = \pi \approx 0, \quad \Phi_2 = \bar{\pi} + \bar{\psi} \mathrm{i}\gamma^0 \approx 0,$$
$$\Phi_3 = \pi^0 \approx 0, \quad \Phi_4 = Q^0 \approx 0. \tag{4}$$

正则哈密顿密度为

$$\begin{aligned}
\mathscr{H}_c =& \dot{\psi}\bar{\pi} + \dot{\bar{\psi}}\pi + \dot{A}_\mu \pi^\mu + C_\mu P^\mu + \dot{C}_\mu Q^\mu - \mathscr{L} \\
=& \dot{\psi}\bar{\pi} + (F_{0i} + \partial_i A_0)\pi^i + C_\mu P^\mu \\
&+ \left(-\frac{1}{c}Q_i + \partial_i C^0\right) Q^i - \bar{\psi}(\mathrm{i}\gamma\partial - m_0)\psi \\
&+ \frac{1}{2}F_{0i}F^{0i} + \frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}G_{0i}G^{0i} + \frac{1}{4}G_{ij}G^{ij} \\
&+ \frac{c}{4}\partial_i G_{jk}\partial^i G^{jk} + \frac{c}{4}\partial_0 G_{ij}\partial^0 G^{ij} \\
&+ \frac{c}{2}\partial_k G_{0j}\partial^k G^{0j} + \frac{c}{2}\partial_0 F_{0j}\partial^0 F^{0j} \\
&- g\bar{\psi}\gamma^\mu\psi A_\mu - g'\bar{\psi}\gamma^\mu\tau_2\psi B_\mu \\
=& C_\mu P^\mu - \bar{\psi}(\mathrm{i}\gamma^i\partial_i - m_0)\psi + \frac{1}{2}\pi_i^2 \\
&- A_0\partial_i\pi^i + \frac{1}{4}F_{ij}F^{ij} - \frac{1}{2c}Q_iQ^i + \partial_i C^0 Q^i \\
&+ \frac{1}{4}G_{ij}G^{ij} + \frac{1}{2}C_iC^i + \frac{1}{2}\partial_i B_0\partial^i B_0 \\
&- C_i\partial^i B^0 + \frac{c}{4}\partial_i G_{jk}\partial^i G^{jk} + \frac{c}{4}\partial_0 G_{ij}\partial^0 G^{ij} \\
&+ \frac{c}{2}\partial_k G_{0j}\partial^k G^{0j} - g\bar{\psi}\gamma^\mu\psi A_\mu - g'\bar{\psi}\gamma^\mu\tau_2\psi B_\mu .
\end{aligned} \tag{5}$$

总哈密顿量为

$$H_T = H_c + \int \mathrm{d}^3 x \lambda^i \Phi_i, \tag{6}$$

$\lambda^i(x)$为约束乘子.初级约束的自洽性条件给出次级约束

$$\begin{aligned}
\{\Phi_1, H_T\} = \{\pi, H_T\} =& (\mathrm{i}\gamma^i\partial_i - m_0)\psi \\
&+ g\gamma^\mu\psi A^\mu + g'\gamma^\mu\tau_2\psi B_\mu \\
&- \lambda_2 \mathrm{i}\gamma^0 \approx 0,
\end{aligned} \tag{7a}$$

$$\begin{aligned}
\{\Phi_2, H_T\} = \{\bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0, H_T\} =& (\mathrm{i}\gamma^i\partial_i - m_0)\bar{\psi} \\
&- g\bar{\psi}\gamma^\mu A_\mu - g'\bar{\psi}\gamma^\mu\tau_2 B_\mu \\
&+ \lambda_1 \mathrm{i}\gamma^0 \approx 0,
\end{aligned} \tag{7b}$$

可确定乘子 λ_1, λ_2

$$\begin{aligned}
\lambda_1 =& -\mathrm{i}\gamma^0(\mathrm{i}\gamma^i\partial_i - m_0)\bar{\psi} + \mathrm{i}\gamma^0 g\gamma^\mu\bar{\psi}A^\mu \\
&+ \mathrm{i}\gamma^0 g'\gamma^\mu\tau_2\psi B_\mu,
\end{aligned} \tag{8a}$$

$$\begin{aligned}
\lambda_2 =& -\mathrm{i}\gamma^0(\mathrm{i}\gamma^i\partial_i - m_0)\psi - \mathrm{i}\gamma^0 g\psi\gamma^\mu A^\mu \\
&- \mathrm{i}\gamma^0 g'\bar{\psi}\gamma^\mu\tau_2 B_\mu .
\end{aligned} \tag{8b}$$

次级约束为

$$\chi_1 = \{\Phi_3, H_T\} = \{\pi^0, H_T\} = \partial_i\pi^i + g\bar{\psi}\gamma^0\psi \approx 0, \tag{9a}$$

$$\chi_1' = \{\Phi_4, H_T\} = \{Q^0, H_T\} = -P^0 + \partial_i Q^i \approx 0, \tag{9b}$$

$$\begin{aligned}
\chi_2' =& \{\chi_1', H_T\} = \{-P^0 + \partial_i Q^i, H_T\} \\
=& -\partial_i P^i - g'\bar{\psi}\gamma^0\tau_2\psi \approx 0.
\end{aligned} \tag{9c}$$

此外,再无约束.第一类约束为 Φ_3, Φ_4, χ_1',第二类约束为 $\Phi_1, \Phi_2, \chi_1, \chi_2'$. 它们不构成第二类约束的最小数目,它们的线性组合

$$\Phi_6 = \partial_i\pi^i + \mathrm{i}g(\bar{\pi}\psi + \bar{\psi}\pi) \approx 0, \tag{10a}$$

$$\Phi_7 = -\partial_i P^i - \mathrm{i}g(\bar{\pi}\tau_2\psi + \bar{\psi}\tau_2\pi) \approx 0 \tag{10b}$$

是第一类约束,因而第一类约束是 $R_1 \equiv \Phi_3 \approx 0, R_2 \equiv \Phi_6 \approx 0, R_3 \equiv \Phi_4 \approx 0, R_4 \equiv \Phi_5 \approx 0, R_5 \equiv \Phi_7 \approx 0$ 第二类约束是 $R_6 \equiv \Phi_1 \approx 0, R_7 \equiv \Phi_2 \approx 0$. 按 Faddeev-Senjanovic(F-S)路径积分量子化方案[18],相应于每一个第一类约束需取一规范条件,该条件取为

$$\begin{aligned}
&\Omega_1 = \partial_i\pi_i + \partial_i\partial_i A_0 \approx 0, \\
&\Omega_2 = \partial_i A_i \approx 0, \\
&\Omega_3 = C_0 \approx 0, \\
&\Omega_4 = \partial_i C^i \approx 0, \\
&\Omega_5 = \partial_i B^i \approx 0.
\end{aligned} \tag{11}$$

对复合场引入外源,由 F-S 量子化方案[18],略去与场量无关的项,则 Green 函数的相空间生成泛函为

$$\begin{aligned}
Z[J] =& \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi^\mu\mathrm{D}B_\mu \\
&\times \mathrm{D}P^\mu\mathrm{D}C_\mu\mathrm{D}Q^\mu\delta(R_1)\delta(R_2)\delta(R_3) \\
&\times \delta(R_4)\delta(R_5)\delta(\Omega_1)\delta(\Omega_2)\delta(\Omega_3) \\
&\times \delta(\Omega_4)\delta(\Omega_5)\delta(R_6)\delta(R_7) \\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}^3 x[\mathscr{L}^p + J_\mu B_\mu + J'_\mu C_\mu + I_\mu A_\mu \right. \\
&\left. + \bar{\eta}\psi + \bar{\psi}\eta + \bar{\psi}\tau_a\psi K_a]\right\},
\end{aligned} \tag{12}$$

其中

$$\mathscr{L}^p = \dot{\psi}\bar{\pi} + \dot{\bar{\psi}}\pi + \dot{A}_\mu\pi^\mu + C_\mu P^\mu + \dot{C}_\mu Q^\mu - \mathscr{H}_c .$$

利用 δ 函数的性质,(12)式可写为

$$\begin{aligned}
Z[J] =& \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}\pi^\mu\mathrm{D}B_\mu \\
&\times \mathrm{D}P^\mu\mathrm{D}C_\mu\mathrm{D}Q^\mu\mathrm{D}\mu_k\mathrm{D}\omega_l \\
&\times \exp\left\{\mathrm{i}\int \mathrm{d}^3 x[\mathscr{L}^p_{\mathrm{eff}} + J_\mu B_\mu + J'_\mu C_\mu + I_\mu A_\mu \right. \\
&\left. + \bar{\eta}\psi + \bar{\psi}\eta + U_k\mu_k + V_l\omega_l + \bar{\psi}\tau_a\psi K_a]\right\} \\
\equiv& \mathrm{e}^{\mathrm{i}W[J]},
\end{aligned} \tag{13}$$

其中 $\mathscr{L}^p_{\mathrm{eff}} = \mathscr{L}^p + \mu_k R_k + \omega_l\Omega_l$.

在下列变换下

$$\delta\psi(x) = \mathrm{i}[\alpha(x) + \tau_2\beta(x)]\psi(x),$$

$$\delta\bar{\pi}(x) = -[\alpha(x) + \tau_2\beta(x)]\bar{\psi}(x)\gamma^0,$$
$$\delta\pi(x) = 0,$$
$$\delta A_\mu(x) = \frac{1}{g}\partial_\mu\alpha(x),$$
$$\delta B_\mu(x) = \frac{1}{g'}\partial_\mu\beta(x),$$
$$\delta\pi^\mu(x) = 0,$$
$$\delta P^\mu(x) = 0,$$
$$\delta Q^\mu(x) = 0,$$
$$\delta C_\mu(x) = \delta(\partial_0 B_\mu(x)) = \frac{1}{g'}\partial_\mu\partial_0\beta(x), \quad (14)$$

$\mathscr{L}^p$ 不变,此变换的 Jacobi 行列式为 1. 又

$$\delta(\mu_k R_k + \omega_l \Omega_l) = \frac{1}{g}\omega_1 \nabla^2\partial_0\alpha(x) + \frac{1}{g}\omega_2\nabla^2\alpha(x) + \frac{1}{g'}\omega_3\partial_0\partial_0\beta(x) + \frac{1}{g'}\omega_4\nabla^2\partial_0\beta(x) + \frac{1}{g'}\omega_5\nabla^2\beta(x). \quad (15)$$

作泛函 Legendre 变换,引入正规顶角生成泛函 $\Gamma[\phi]$,得 $\Gamma[\phi]$ 满足的 Ward 恒等式为

$$-\frac{1}{g'}\partial_0\partial_0\omega_3(x) - \frac{1}{g'}\partial_0\nabla^2\omega_4(x) + \frac{1}{g'}\nabla^2\omega_5(x) + \frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)}\mathrm{i}\tau_2\psi_c(x) + \bar{\psi}_c(x)\mathrm{i}\tau_2\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)} + \frac{1}{g'}\partial^\mu\frac{\delta\Gamma[\phi]}{\delta B_c^\mu(x)} - \frac{1}{g'}\partial_0\partial^\mu\frac{\delta\Gamma[\phi]}{\delta C_c^\mu(x)} - 2\varepsilon_{2ab}\sigma_c^a(x)\frac{\delta\Gamma[\phi]}{\delta\sigma_c^b(x)} = 0, \quad (16)$$

其中引进 $\sigma^a(x) = aG^a(x)$ 来描述束缚态 $G^a(x)$,从(16)式可得关于两点顶角的 Ward 恒等式. 对(16)式中 $\psi_c(y)$ 和 $\bar{\psi}_c(z)$ 求导,无外源时 $\psi_c(x) = \bar{\psi}_c(x) = 0$,(16)式为

$$\delta(x-z)\mathrm{i}\tau_2\frac{\delta^2\Gamma[\phi]}{\delta\psi_c(y)\delta\bar{\psi}_c(x)} + \delta(x-y)\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(x)}\mathrm{i}\tau_2 + 2\varepsilon_{2ab}\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\sigma_c^a(x)} \times\sigma_c^b(x) - \frac{1}{g'}\partial_\mu\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta B_c^\mu(x)} - \frac{1}{g'}\partial_0\partial_\mu\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta C_c^\mu(x)} = 0. \quad (17)$$

作 Fourier 变换,(17)式变为

$$\Gamma^{(2)}_{\psi,\bar{\psi}}(p)\mathrm{i}\tau_2 - \mathrm{i}\tau_2\Gamma^{(2)}_{\psi,\bar{\psi}}(p+k) = 2\varepsilon_{2ab}\Gamma^{(3)}_{\psi,\bar{\psi};\sigma_a}(p+k,-p;-k)\sigma_c^b - \frac{\mathrm{i}}{g'}k_\mu\Gamma^{(3)}_{\psi,\bar{\psi};B_\mu}(p+k,-p;-k) + \frac{1}{g'}k_\mu k_0\Gamma^{(3)}_{\psi,\bar{\psi};C_\mu}(p+k,-p;-k). \quad (18)$$

当 $k_\mu\to 0$ 时,上式则变为

$$-\mathrm{i}[\tau_2,\Gamma^{(2)}_{\psi,\bar{\psi}}(p)] = 2\varepsilon_{2ab}\Gamma^{(3)}_{\psi,\bar{\psi};\sigma_a}(p,-p;0)\sigma_c^b. \quad (19)$$

可见高阶微商项对费米子的质量无贡献. 对(16)式的 $B_c^\nu(y)$ 求导,得

$$\frac{\delta^2\Gamma[\phi]}{\delta B_c^\nu(y)\delta\psi_c(x)}\mathrm{i}\tau_2\psi_c(x) + \bar{\psi}_c(x)\mathrm{i}\tau_2\frac{\delta^2\Gamma[\phi]}{\delta B_c^\nu(y)\delta\bar{\psi}_c(x)} + \frac{1}{g'}\partial^\mu\frac{\delta^2\Gamma[\phi]}{\delta B_c^\nu(y)\delta B_c^\mu(x)} - 2\varepsilon_{2ab}\sigma_c^a(x)\frac{\delta^2\Gamma[\phi]}{\delta B_c^\nu(y)\delta\sigma_c^b(x)} - \frac{1}{g'}\partial^0\partial^\mu\frac{\delta^2\Gamma[\phi]}{\delta B_c^\nu(y)\delta C_c^\mu(x)} = 0. \quad (20)$$

作 Fourier 变换,类似于(17)式的讨论,则上式变为

$$\frac{\mathrm{i}}{g'}p_\mu\Gamma^{(2)}_{B_\nu,B_\mu}(p) + \frac{1}{g'}p_\mu p_0\Gamma^{(2)}_{B_\nu,C_\mu}(p) = -2\sigma_c^3\Gamma^{(2)}_{B_\nu,\sigma_1}(p). \quad (21)$$

应用关系式

$$\lim_{p\to 0}\Gamma^{(2)}_{B_\nu,B_\mu}(p) = -z_B\delta_{\mu,\nu}m_B^2, \quad (22)$$

得到规范玻色子的质量为

$$m_B^2 = -\lim_{p\to 0}z_B^{-1}g'\mathrm{i}\left[\frac{p_\mu p_\nu p_0}{p^2}\frac{1}{g'}\Gamma^{(2)}_{B_\nu,C_\mu}(p) + \frac{p_\nu}{p^2}\Gamma^{(2)}_{B_\nu,\sigma_1}(p)\sigma_c^3\right]. \quad (23)$$

此时,高阶微商项对规范玻色子的质量有贡献.

3. 含高阶微商项的 Jackiw-Johnson 模型

在 Jackiw-Johnson 模型中引入高阶微商项,其拉氏密度为

$$\mathscr{L} = \bar{\psi}\mathrm{i}\gamma\cdot\partial\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{c}{4}\partial_\rho F_{\mu\nu}\partial^\rho F^{\mu\nu} + gJ_{5\mu}A_\mu, \quad (24)$$

其中 $J_{5\mu} = \mathrm{i}\bar{\psi}\gamma_\mu\gamma_5\psi$,$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\mu$,$C$ 为常数.

设场变量 $\bar{\psi}$,ψ,A_μ,$B_\mu = \dot{A}_\mu$ 的正则共轭动量分别为 π,$\bar{\pi}$,P^μ,Q^μ,由 Ostrorgradsky 变换,引入正则动量,有

$$P^0 = -c\partial_0\partial_i F^{0i},$$
$$P^i = F^{0i} + c(\nabla^2 F^{0i} + \partial_0\partial_j F^{ji}) - \partial_0 Q^i,$$
$$Q^0 = 0,$$
$$Q^i = -c\partial_0 F^{0i},$$
$$\pi = \frac{\partial \mathscr{L}}{\partial \dot{\psi}} = -\bar{\psi}\mathrm{i}\gamma^0. \tag{25}$$

初级约束为

$$\Phi_1 = \pi \approx 0, \quad \Phi_2 = \bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0 \approx 0,$$
$$\Phi_3 = Q^0 \approx 0.$$

$$\mathscr{H}_c = B_\mu P^\mu - \frac{1}{2c}Q^i Q_i + \partial_i B_0 Q^i + \frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}B_i B^i + \frac{1}{2}\partial_i A_0 \partial^i A^0 - B_i \partial^i A_0 + \frac{c}{4}\partial_i F_{jk}\partial^i F^{jk} + \frac{c}{4}\partial_0 F_{ij}\partial^0 F^{ij} + \frac{c}{2}\partial_k F_{0j}\partial^k F^{0j} - \bar{\psi}\mathrm{i}\gamma^i\partial_i\psi - gJ_{5\mu}A_\mu. \tag{26}$$

总哈密顿量为

$$H_T = H_c + \int \mathrm{d}^3 x \lambda_i \Phi_i, \tag{27}$$

$\lambda_i(x)$为约束乘子.初级约束 Φ_1,Φ_2 的自洽性条件给出乘子 λ_1,λ_2 满足的方程.次级约束为

$$\chi_1 = \{\Phi_3, H_T\} = \{Q^0, H_T\} = -P^0 + \partial_i Q^i \approx 0, \tag{28a}$$

$$\chi_2 = \{\chi_1, H_T\} = \{-P^0 + \partial_i Q^i, H_T\} = -\partial_i P^i - gJ_{50} \approx 0. \tag{28b}$$

此外,再无约束.第一类约束为 $\Phi_3 = Q^0$,$\Phi_4 = \chi_1$;第二类约束为 Φ_1,Φ_2,χ_2.它们不构成第二类约束的最小数目,其线性组合

$$\Phi_5 = -\partial_i P^i + g(\bar{\pi}\gamma_5\psi + \bar{\psi}\gamma_5\pi) \tag{29}$$

为第一类约束.因此,第一类约束的最大数目为

$$\Phi_3 = Q^0,$$
$$\Phi_4 = -P^0 + \partial_i Q^i,$$
$$\Phi_5 = -\partial_i P^i + g(\bar{\pi}\gamma_5\psi + \bar{\psi}\gamma_5\pi). \tag{30}$$

第二类约束为

$$\Phi_1 = \pi, \quad \Phi_2 = \bar{\pi} + \bar{\psi}\mathrm{i}\gamma^0. \tag{31}$$

对第一类约束取相应的规范条件

$$f_1 = B_0 \approx 0, \quad f_2 = \partial_i B^i \approx 0,$$
$$f_3 = \partial_i A^i \approx 0. \tag{32}$$

对复合场引入外源,由 F-S 量子化方案,略去与场量无关的项,则 Green 函数的生成泛涵为

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}P^\mu\mathrm{D}B_\mu\mathrm{D}Q^\mu \times \delta(f_1)\delta(f_2)\delta(f_3)\delta(\Phi_3)\delta(\Phi_4)\delta(\Phi_5) \times \delta(\Phi_1)\delta(\Phi_2)\exp\left\{\mathrm{i}\int \mathrm{d}^4 x[J_\mu A_\mu + J'_\mu B_\mu + \bar{\eta}\psi + \bar{\psi}\eta + \bar{\psi}\psi K + \bar{\psi}\gamma_5\psi K_5 + \mathscr{L}^p]\right\}. \tag{33}$$

利用 δ 函数的性质,(33)式可写为

$$Z[J] = \int \mathrm{D}\bar{\psi}\mathrm{D}\pi\mathrm{D}\psi\mathrm{D}\bar{\pi}\mathrm{D}A_\mu\mathrm{D}P^\mu\mathrm{D}B_\mu \times \mathrm{D}Q^\mu\mathrm{D}\mu_k\mathrm{D}\omega_l\exp\left\{\mathrm{i}\int \mathrm{d}^4 x[\mathscr{L}^p_{\mathrm{eff}} + J_\mu A_\mu + J'_\mu B_\mu + \bar{\eta}\psi + \bar{\psi}\eta + U_k\mu_k + V_l\omega_l \bar{\psi}\psi K + \bar{\psi}\gamma_5\psi K_5]\right\}, \tag{34}$$

其中 $\mathscr{L}^p_{\mathrm{eff}} = \mathscr{L}^p + \mathscr{L}_m = \mathscr{L}^p + \omega_l f_l + \mu_k\Phi_k$.

在下列变换下

$$\delta\psi(x) = \mathrm{i}\alpha(x)\gamma_5\psi(x),$$
$$\delta\pi(x) = 0,$$
$$\delta\bar{\pi}(x) = \bar{\psi}\alpha(x)\gamma_5\gamma^0,$$
$$\delta A_\mu(x) = -\frac{\mathrm{i}}{g}\partial_\mu\alpha(x),$$
$$\delta P^\mu(x) = 0,$$
$$\delta Q^\mu(x) = 0,$$
$$\delta B_\mu(x) = \delta(\partial_0 A_\mu) = -\frac{\mathrm{i}}{g}\partial_\mu\partial_0\alpha(x), \tag{35}$$

$\mathscr{L}^p$ 是不变的.又

$$\delta(\omega_l f_l + \mu_k\Phi_k) = -\frac{\mathrm{i}}{g}\omega_l\partial_0\partial_0\alpha(x) - \frac{\mathrm{i}}{g}\omega_2\nabla^2\partial_0\alpha(x) - \frac{\mathrm{i}}{g}\omega_3\nabla^2\alpha(x). \tag{36}$$

作泛涵 Legendre 变换,引入顶角生成泛涵 $\Gamma[\phi]$,最后得 Ward 恒等式为

$$\frac{\mathrm{i}}{g}\partial_0\partial_0\omega_l + \frac{\mathrm{i}}{g}\partial_0\nabla^2\omega_2 - \frac{\mathrm{i}}{g}\nabla^2\omega_3 + \frac{\Gamma[\phi]}{\delta\psi_c(x)}\frac{\mathrm{i}}{2}\gamma_5\psi_c(x) - \bar{\psi}_c(x)\frac{\mathrm{i}}{2}\gamma_5\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)} + \frac{1}{2g}\partial^\mu\partial^0\frac{\delta\Gamma[\phi]}{\delta B^\mu_c(x)} - \frac{\mathrm{i}}{2g}\partial^\mu\frac{\delta\Gamma[\phi]}{\delta A^\mu_c(x)} + G(x)\frac{\delta\Gamma[\phi]}{\delta G_5(x)} - G_5(x)\frac{\delta\Gamma[\phi]}{\delta G(x)} = 0. \tag{37}$$

为了描述凝聚的量子涨落,引入标量和赝标量场 $\sigma(x)$,$\pi(x)$,即

$$\sigma(x) = aG(x), \quad \pi(x) = aG_5(x), \tag{38}$$

则(37)式变为

$$\frac{\mathrm{i}}{g}\partial_0\partial_0\omega_l + \frac{\mathrm{i}}{g}\partial_0\nabla^2\omega_2 - \frac{\mathrm{i}}{g}\nabla^2\omega_3$$

$$+\frac{\delta\Gamma[\phi]}{\delta\psi_c(x)}\frac{\mathrm{i}}{2}\gamma_5\psi_c(x)-\bar{\psi}_c(x)\frac{\mathrm{i}}{2}\gamma_5\frac{\delta\Gamma[\phi]}{\delta\bar{\psi}_c(x)}$$
$$+\frac{1}{2g}\partial^\mu\partial^0\frac{\delta\Gamma[\phi]}{\delta B_c^\mu(x)}-\frac{\mathrm{i}}{2g}\partial^\mu\frac{\delta\Gamma[\phi]}{\delta A_c^\mu(x)}$$
$$+\sigma_c(x)\frac{\delta\Gamma[\phi]}{\delta\pi_c(x)}-\pi_c(x)\frac{\delta\Gamma[\phi]}{\delta\sigma_c(x)}=0. \qquad (39)$$

对(39)式的 $\psi_c(y)$，$\bar{\psi}_c(z)$ 求导，得

$$\delta(x-z)\frac{\mathrm{i}}{2}\gamma_5\frac{\delta^2\Gamma[\phi]}{\delta\psi_c(y)\delta\bar{\psi}_c(x)}$$
$$-\delta(x-z)\frac{\delta^2\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(x)}\frac{\mathrm{i}}{2}\gamma_5$$
$$-\bar{\psi}_c(x)\frac{\mathrm{i}}{2}\gamma_5\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)}$$
$$+\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\bar{\psi}_c(x)}\frac{\mathrm{i}}{2}\gamma_5\psi_c(x)$$
$$-\frac{\mathrm{i}}{2g}\partial_\mu\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta A\mu_c(x)}$$
$$+\frac{1}{2g}\partial^\mu\partial^0\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta B_c^\mu(x)}$$
$$-\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\sigma_c(x)}\pi_c(x)$$
$$+\frac{\delta^3\Gamma[\phi]}{\delta\bar{\psi}_c(z)\delta\psi_c(y)\delta\pi_c(x)}\sigma_c(x)=0. \qquad (40)$$

现在选择如下破缺：

$$\langle\bar{\psi}(x)\psi(x)\rangle\neq 0, \qquad (41a)$$
$$\langle\bar{\psi}(x)\mathrm{i}\gamma_5\psi(x)\rangle\neq 0, \qquad (41b)$$

对(40)式作 Fourier 变换，有

$$\frac{\mathrm{i}}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(p+k)+\Gamma^{(2)}_{\psi,\bar{\psi}}(p)\frac{\mathrm{i}}{2}\gamma_5$$
$$=\frac{1}{2g}k_\mu\Gamma^{(3)}_{\psi,\bar{\psi};A_\mu}(p+k,-p;-k)$$
$$+\Gamma^{(3)}_{\psi,\bar{\psi},\pi}(p+k,-p;-k)\sigma_c$$
$$+\frac{1}{2g}k_\mu k_0\Gamma^{(3)}_{\psi,\bar{\psi};B_\mu}(p+k,-p;-k). \qquad (42)$$

当 $k_\mu\to 0$ 时，(42)式变为

$$\frac{\mathrm{i}}{2}\gamma_5\Gamma^{(2)}_{\psi,\bar{\psi}}(p)+\Gamma^{(2)}_{\psi,\bar{\psi}}(p)\frac{\mathrm{i}}{2}\gamma_5=\Gamma^{(3)}_{\psi,\bar{\psi};\pi}(p,-p;0). \qquad (43)$$

高阶微商项对费米子的质量无贡献.

对 $A_c^\mu(x)$ 求导，类似计算可得

$$\sigma_c\Gamma^{(2)}_{A_\nu,\pi}(p)-\frac{1}{2g}p_\mu\Gamma^{(2)}_{A_\mu,B_\mu}(p)+\frac{1}{2g}p_\mu p_0\Gamma^{(2)}_{A_\mu,B_\mu}=0. \qquad (44)$$

于是，有

$$m_A^2=\lim_{p\to 0}z_A^{-1}2g\mathrm{i}\left[\frac{p_\nu}{p^2}\Gamma^{(2)}_{A_\nu,\pi}(p)\sigma_c+\frac{1}{2g}\frac{p_\mu p_\nu p_0}{p^2}\Gamma^{(2)}_{A_\nu,B_\mu}(p)\right]. \qquad (45)$$

此时，高阶微商项对规范玻色子质量有贡献.上面发展的方法可以进一步推广到非 Abel 规范理论的动力学对称破缺研究中.

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Canonical Ward identity and dynamical mass generation for Abelian gauge theory in higher-order derivative systems*

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Abstract

Based on the canonical Ward-Takahashi identities to include composite fields to study gauge symmetry dynamical breaking for Abelian gauge theory in higher-order derivative systems, the mass spectra of both fermion and bound states are obtained. The effect of higher-order derivative terms on mass generation is discussed.

Keywords: canonical Ward identity, constraints, dynamical symmetry breaking, Abelian gauge theory

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