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Vaidya-Bonner-de Sitter 黑洞对狄拉克粒子的热辐射*

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分析讨论了 Vaidya-Bonner-de Sitter 黑洞视界附近的狄拉克方程,准确地定出了 Vaidya-Bonner-de Sitter 黑洞的 Hawking 温度和辐射谱,同时计算出事件视界方程,所得结果与用零曲面方程得到的结果一致.

关键词:黑洞,视界,狄拉克方程,热辐射
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1. 引 言

1974 年,Hawking 发现黑洞存在温度为 $T = K/2\pi$ 的量子辐射[1],其中 K 为黑洞视界面上的表面重力,T 为 Hawking 温度.此后人们在黑洞热力学的研究上做了大量工作,特别是在计算黑洞的 Hawking 温度与其辐射谱方面,取得了许多有价值的成果.继 Hawking 之后,人们又采用各种各样的方法来研究 Hawking 温度和黑洞辐射谱.主要的方法有路径积分法[2]、温度格林函数法[3]、Damour-Ruffini 法[4]、Unruh 法[5]和 Zhao 等人在 1991 年提出的决定动态黑洞热效应的方法[6,7]等等.Zhao 等人的方法在计算黑洞的温度方面,具有简单、精确和适应性强等特点.此方法同时可用于研究稳态和动态黑洞,包括球对称黑洞、非球对称黑洞以及非渐进平直时空中的黑洞等等.本文从视界附近的狄拉克方程出发,采用此方法计算 Vaidya-Bonner-de Sitter 黑洞的事件视界和温度,并采用文献[8,9]中方法,计算黑洞的辐射谱,得到的结果可以返回原来已知的 Vaidya-Bonner-de Sitter 黑洞的结果.

2. 度规与联络

Vaidya-Bonner-de Sitter 黑洞的时空线元[7]为

$$\mathrm{d}s^2 = -\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{\lambda r^2}{3}\right)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2). \quad (1)$$

将(1)式给出的线元用号差(+,-,-,-)重新表示为

$$\mathrm{d}s^2 = \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{\lambda r^2}{3}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \quad (2)$$

式中 $m=m(v)$ 和 $Q=Q(v)$ 分别为黑洞的质量和电荷,λ 为宇宙常数,v 为超前爱丁顿坐标.

由(2)式容易求出度规的行列式和不为零的逆变度规分量为

$$g = -r^4\sin^2\theta, \quad (3)$$

$$g^{01} = g^{10} = -1,$$
$$g^{11} = -\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}-\frac{\lambda r^2}{3}\right),$$
$$g^{22} = -\frac{1}{r^2}, \quad g^{33} = -\frac{1}{r^2\sin^2\theta}, \quad (4)$$

可算出不为零的联络分量为

$$\Gamma^0_{00} = \frac{rm-Q^2}{r^3} - \frac{1}{3}\lambda r,$$
$$\Gamma^0_{22} = -r, \quad \Gamma^0_{33} = -r\sin^2\theta,$$
$$\Gamma^1_{00} = \frac{\dot{m}r - Q\dot{Q}}{r^2} + \frac{\Delta}{r^2}\left(\frac{rm-Q^2}{r^3} - \frac{1}{3}\lambda r\right),$$
$$\Gamma^1_{10} = \Gamma^1_{01} = -\left(\frac{rm-Q^2}{r^3} - \frac{1}{3}\lambda r\right),$$
$$\Gamma^1_{22} = -\frac{\Delta}{r}, \quad \Gamma^1_{33} = -\frac{\Delta}{r}\sin^2\theta,$$

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$$\Gamma^2_{21} = \Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta,$$

$$\Gamma^3_{31} = \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta, \qquad (5)$$

式中 $\dot{m} = \frac{\mathrm{d}m}{\mathrm{d}v}$，$\dot{Q} = \frac{\mathrm{d}Q}{\mathrm{d}v}$，$\Delta = r^2 - 2rm + Q^2 - \frac{1}{3}\lambda r^4$.

3. 零标架的选取

选取如下零标架形式:

$$l_\mu = \left(\frac{1}{2\Delta}\right)^{\frac{1}{2}}\left(\frac{\Delta}{r}, 0, 0, 0\right),$$

$$n_\mu = \left(\frac{1}{2\Delta}\right)^{\frac{1}{2}}\left(\frac{\Delta}{r}, -2r, 0, 0\right),$$

$$m_\mu = \frac{r}{\sqrt{2}}(0, 0, 1, \mathrm{i}\sin\theta),$$

$$\bar{m}_\mu = \frac{r}{\sqrt{2}}(0, 0, 1, -\mathrm{i}\sin\theta), \qquad (6)$$

由零标架的协变分量和度规可得它的逆变分量为

$$l^\mu = \left(\frac{1}{2\Delta}\right)^{\frac{1}{2}}\left(0, -\frac{\Delta}{r}, 0, 0\right),$$

$$n^\mu = \left(\frac{1}{2\Delta}\right)^{\frac{1}{2}}\left(2r, \frac{\Delta}{r}, 0, 0\right),$$

$$m^\mu = \frac{1}{\sqrt{2}r}\left(0, 0, -1, \frac{-\mathrm{i}}{\sin\theta}\right),$$

$$\bar{m}^\mu = \frac{1}{\sqrt{2}r}\left(0, 0, -1, \frac{\mathrm{i}}{\sin\theta}\right). \qquad (7)$$

不难验证,所选取的零标架满足以下条件:

$$g_{\mu\nu} = l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu,$$

$$l_\mu l^\mu = n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0, \qquad (8)$$

$$l^\mu n_\mu = -m_\mu \bar{m}^\mu = 1,$$

$$l_\mu m^\mu = l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0.$$

4. 旋系数的计算

根据文献[11]及(6),(7)式,得不为零的零标架下的旋系数为

$$\varepsilon = \frac{1}{2}(l_{\mu;\nu} n^\mu l^\nu - m_{\mu;\nu}\bar{m}^\mu l^\nu)$$

$$= -\frac{rm - Q^2 - \frac{1}{3}\lambda r^4}{2r^2\sqrt{2\Delta}},$$

$$\rho = l_{\mu;\nu} m^\mu \bar{m}^\nu = \frac{\Delta}{r^2\sqrt{2\Delta}},$$

$$\alpha = \frac{1}{2}(l_{\mu;\nu} n^\mu \bar{m}^\nu - m_{\mu;\nu}\bar{m}^\mu \bar{m}^\nu) = \frac{1}{2\sqrt{2}r}\cot\theta,$$

$$\beta = \frac{1}{2}(l_{\mu;\nu} n^\mu m^\nu - m_{\mu;\nu}\bar{m}^\mu m^\nu)$$

$$= -\frac{1}{2\sqrt{2}r}\cot\theta,$$

$$\mu = -n_{\mu;\nu}\bar{m}^\mu m^\nu = \left(\frac{1}{2\Delta}\right)^{\frac{1}{2}}\frac{\Delta}{r^2},$$

$$\gamma = \frac{1}{2}(l_{\mu;\nu} n^\mu n^\nu - m_{\mu;\nu}\bar{m}^\mu n^\nu)$$

$$= \frac{1}{\sqrt{2\Delta}}\left[\frac{r}{\Delta}(Q\dot{Q} - \dot{m}\, r) - \frac{rm - Q^2 - \frac{1}{3}\lambda r^4}{2r^2}\right]. \qquad (9)$$

5. 弯曲时空的狄拉克方程

在存在电磁场的情况下,带电粒子的狄拉克方程可写为

$$(\nabla_{ab} + \mathrm{i}eA_{ab})P^a + \frac{\mathrm{i}}{\sqrt{2}}\mu_0 \bar{Q}_b = 0,$$

$$(\nabla_{ab} - \mathrm{i}eA_{ab})Q^a + \frac{\mathrm{i}}{\sqrt{2}}\mu_0 \bar{P}_b = 0, \qquad (10)$$

式中 μ_0,e 分别为狄拉克粒子的静止质量和电荷,P^a,Q^a 为两个二分量旋量,∇_{ab}为旋协变微分,A_{ab} 为电磁四矢的旋分量. 此方程可用零标架和旋系数表出[10]

$$(D + \varepsilon - \rho + \mathrm{i}eA_\mu l^\mu)F_1 + (\bar{\delta} + \pi - \alpha$$

$$+ \mathrm{i}eA_\mu \bar{m}^\mu)F_2 = \frac{\mathrm{i}}{\sqrt{2}}\mu_0 G_1,$$

$$(\Delta' + \mu - \gamma + \mathrm{i}eA_\mu n^\mu)F_2 + (\delta + \beta$$

$$- \tau + \mathrm{i}eA_\mu m^\mu)F_1 = \frac{\mathrm{i}}{\sqrt{2}}\mu_0 G_2,$$

$$(D + \varepsilon^* - \rho^* + \mathrm{i}eA_\mu l^\mu)G_2 - (\delta + \pi^*$$

$$- \alpha^* + \mathrm{i}eA_\mu m^\mu)G_1 = \frac{\mathrm{i}}{\sqrt{2}}\mu_0 F_2,$$

$$(\Delta' + \mu^* - \gamma^* + \mathrm{i}eA_\mu n^\mu)G_1 - (\bar{\delta} + \beta^*$$

$$- \tau^* + \mathrm{i}eA_\mu \bar{m}^\mu)G_2 = \frac{\mathrm{i}}{\sqrt{2}}\mu_0 F_1, \qquad (11)$$

式中 $F_1 = P^0$,$F_2 = P^1$,$G_1 = \bar{Q}^{\dot{1}}$,$G_2 = -\bar{Q}^{\dot{0}}$,旋系数 π,τ 为零,电磁四矢为

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0\right),$$

$$A_{0\dot{0}} = A_\mu l^\mu, \quad A_{0\dot{1}} = A_\mu m^\mu,$$

$$A_{1\hat{0}} = A_{\mu}\bar{m}^{\mu}, \quad A_{1\hat{1}} = A_{\mu}n^{\mu}, \tag{12}$$

4 个微分算子为

$$D = l^{\mu}\partial_{\mu} = -\frac{1}{\sqrt{2\Delta}}\frac{\Delta}{r}\frac{\partial}{\partial r},$$

$$\Delta' = n^{\mu}\partial_{\mu} = \frac{1}{\sqrt{2\Delta}}\left(2r\frac{\partial}{\partial v} + \frac{\Delta}{r}\frac{\partial}{\partial r}\right),$$

$$\delta = m^{\mu}\partial_{\mu} = -\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right),$$

$$\bar{\delta} = \bar{m}^{\mu}\partial_{\mu} = -\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right). \tag{13}$$

把(9),(13)式代入(11)式,通过化简,可得

$$\sqrt{\Delta}\left[\frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3rm + Q^2 - \lambda r^4}{2r}\right]F_1 + \left[\frac{\partial}{\partial r} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi} + \frac{1}{2}\cot\theta\right]F_2 + \mathrm{i}\mu_0 rG_1 = 0, \tag{14}$$

$$\sqrt{\Delta}\left[\frac{2r^2}{\Delta}\frac{\partial}{\partial v} + \frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r} - \frac{r^2}{\Delta^2}(\dot{Q}Q - \dot{m}r) + \frac{2\mathrm{i}erQ}{\Delta}\right]F_2 - \left(\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi} + \frac{1}{2}\cot\theta\right)F_1 - \mathrm{i}\mu_0 rG_2 = 0, \tag{15}$$

$$\sqrt{\Delta}\left[\frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r}\right]G_2 - \left[\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi} + \frac{1}{2}\cot\theta\right]G_1 + \mathrm{i}e\mu_0 rF_2 = 0, \tag{16}$$

$$\sqrt{\Delta}\left[\frac{2r^2}{\Delta}\frac{\partial}{\partial v} + \frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r} - \frac{r^2}{\Delta^2}(\dot{Q}Q - \dot{m}r) + \frac{2\mathrm{i}erQ}{\Delta}\right]G_1 - \left(\frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi} + \frac{1}{2}\cot\theta\right)G_2 - \mathrm{i}\mu_0 rF_1 = 0. \tag{17}$$

分离变量为

$$G_1 = R_+(v,r)S_-(\theta)\mathrm{e}^{\mathrm{i}p\varphi},$$

$$G_2 = R_-(v,r)S_+(\theta)\mathrm{e}^{\mathrm{i}p\varphi},$$

$$F_1 = R_-(v,r)S_-(\theta)\mathrm{e}^{\mathrm{i}p\varphi},$$

$$F_2 = R_+(v,r)S_+(\theta)\mathrm{e}^{\mathrm{i}p\varphi}. \tag{18}$$

将(18)式代入(14)—(17)式,可得

$$\tilde{D}_0\tilde{D}_1 R_+ + \frac{\lambda - \mathrm{i}\mu_0 r}{\sqrt{\Delta}}\left[\tilde{D}_0\frac{\sqrt{\Delta}}{\lambda - \mathrm{i}\mu_0 r}\right] \times \tilde{D}_1 R_+ - \frac{\lambda^2 + \mu_0^2 r^2}{\Delta}R_+ = 0, \tag{19}$$

$$\tilde{D}_1\tilde{D}_0 R_- + \frac{\lambda + \mathrm{i}\mu_0 r}{\sqrt{\Delta}}\left[\tilde{D}_1\frac{\sqrt{\Delta}}{\lambda + \mathrm{i}\mu_0 r}\right] \times \tilde{D}_0 R_- - \frac{\lambda^2 + \mu_0^2 r^2}{\Delta}R_- = 0, \tag{20}$$

$$L_- L_+ S_+ + \lambda^2 S_+ = 0, \tag{21}$$

$$L_+ L_- S_- + \lambda^2 S_- = 0, \tag{22}$$

式中

$$\tilde{D}_0 = \frac{\partial}{\partial r} + \frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r\Delta},$$

$$\tilde{D}_1 = \frac{2r^2}{\Delta}\frac{\partial}{\partial v} + \frac{\partial}{\partial r} + \frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r} - \frac{r^2}{\Delta^2}(\dot{Q}Q - \dot{m}r) + \frac{2\mathrm{i}erQ}{\Delta},$$

$$L_{\pm} = \frac{\partial}{\partial\theta} \pm \frac{K}{\sin\theta}\frac{\partial}{\partial\varphi} + \frac{1}{2}\cot\theta, \tag{23}$$

式中 λ 都是分离变常数.本文研究 Hawking 辐射,仅对径向方程有兴趣.下面研究 R_+ 的方程,R_- 也有类似结果,下面用 R 代替 R_+.

6. 乌龟坐标变换、解析延拓及辐射谱

(19)式可进一步表示为

$$\frac{\partial^2 R}{\partial r^2} + \frac{2r^2}{\Delta}\frac{\partial^2 R}{\partial v\partial r} + \left(\frac{1}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r} + A + B\right)\frac{\partial R}{\partial r} + \left[\frac{r}{\Delta^2}\left(2r^2 - 7mr + 5Q^2 + \frac{4}{3}\lambda r^4\right) + \frac{2r^2}{\Delta}A\right]\frac{\partial R}{\partial v} + \left[AB + \frac{B}{\Delta}\frac{2r^2 - 3mr + Q^2 - \lambda r^4}{2r} + \frac{\partial B}{\partial r} - \frac{1}{\Delta}(\lambda^2 + \mu_0^2 r^2)\right]R = 0, \tag{24}$$

式中

$$A = \frac{4r^2 - 5mr + Q^2 - \frac{7}{3}\lambda r^4}{2r\Delta} - \frac{\mathrm{i}\mu_0}{\lambda - \mathrm{i}\mu_0 r},$$

$$B = \frac{2r^2 - 4mr + Q^2 - \lambda r^4}{2r\Delta} - \frac{r^2}{\Delta^2}(\dot{Q}Q - \dot{m}r) + \frac{2\mathrm{i}erQ}{\Delta},$$

$$\frac{\partial B}{\partial r} = \frac{4r^2(\dot{Q}Q - mr)\left(r - m - \frac{2}{3}\lambda r^3\right)}{\Delta^3} + \frac{1}{\Delta^2}\left[\dot{m}r^2 - 2r(\dot{Q}Q - \bar{m}r) - 4\mathrm{i}eQr\left(r - m - \frac{2}{3}\lambda r^3\right) - 2\left(r - m - \frac{2}{3}\lambda r^3\right)\right.$$

$$\times\left(Q^2-3mr+2r^2-\lambda r^4\right)\Big]+\frac{1}{\Delta}\frac{2r^2-Q^2+3\lambda r^4+2\mathrm{i}eQr^2}{2r^2}. \quad (25)$$

作乌龟坐标变换

$$\begin{cases} r_* = r+\dfrac{1}{2K}\ln(r-r_j), \\ v_* = v-v_0, \end{cases} \quad (26)$$

式中 j 的取值是 H 和 C，分别表示有关的值，对应于 Vaidya-Bonner-de Sitter 时空中黑洞的事件视界（外视界）和时空的宇宙视界，K 为可调常数，在乌龟坐标变换下，K 与 v_0 均为常数. 微分算子用乌龟坐标表示为[9]

$$\frac{\partial}{\partial r}=\left[1+\frac{1}{2K(r-r_j)}\right]\frac{\partial}{\partial r_*},$$

$$\frac{\partial}{\partial v}=\frac{\partial}{\partial v_*}-\frac{\dot r_j}{2K(r-r_j)}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial r^2}=\left[1+\frac{1}{2K(r-r_j)}\right]^2\frac{\partial^2}{\partial r_*^2}-\frac{1}{2K(r-r_j)^2}\frac{\partial}{\partial r_*},$$

$$\frac{\partial^2}{\partial v\partial r}=\left[1+\frac{1}{2K(r-r_j)}\right]\frac{\partial^2}{\partial v_*\partial r_*}-\left[1+\frac{1}{2K(r-r_j)}\right]\frac{\dot r_j}{2K(r-r_j)}\frac{\partial^2}{\partial r_*^2}+\frac{\dot r_j}{2K(r-r_j)}\frac{\partial}{\partial r_*}, \quad (27)$$

式中 $\dot r_j=\dfrac{\mathrm{d}r_j}{\mathrm{d}v}$. 将(27)式代入(19)式，可得

$$\frac{\Delta[2K(r-r_j)+1]-2r^2\dot r_j}{2K(r-r_j)r^2}\frac{\partial^2 R}{\partial r_*^2}+2\frac{\partial^2 R}{\partial r_*\partial v_*}+\frac{2K(r-r_j)}{1+2K(r-r_j)}\Big[\frac{1}{r\Delta}\Big(2r^2-7mr+5Q^2+\frac{4}{3}\lambda r^4\Big)+2A\Big]\frac{\partial R}{\partial v_*}+\Big\{\frac{\Delta-2\dot r_j r^2}{[1+2K(r-r_j)](r-r_j)r^2}+\frac{\dot r_j}{\Delta[1+2K(r-r_j)]r}\Big(2r^2-7mr+5Q^2+\frac{4}{3}\lambda r^4+A+B\Big)\Big\}\frac{\partial R}{\partial v_*}+\frac{2\Delta K(r-r_j)}{[1+2K(r-r_j)]r^2}\times\Big[AB+\frac{B}{\Delta}\frac{2r^2-3mr+Q^2-\lambda r^4}{2r}+\frac{\partial B}{\partial r}-\frac{1}{\Delta}(\lambda^2+\mu_0^2r^2)\Big]R=0. \quad (28)$$

令 $\dfrac{\partial^2 R}{\partial r_*^2}$ 项的系数为 C，则

$$C=\frac{\Delta[2K(r-r_j)+1]-2r^2\dot r_j}{2K(r-r_j)r^2}. \quad (29)$$

由于当 $r\to r_j$ 时，C 的分母为零，因此要求当 $r\to r_j$ 时，C 的分子也为零，即

$$\lim_{\substack{r\to r_j\\ v\to v_0}}\{\Delta[2K(r-r_j)+1]-2r^2\dot r_j\}=0. \quad (30)$$

得到视界满足的方程[11]为

$$r_j^2-2mr_j+Q^2-\frac{1}{3}\lambda r_j^4-2r_j^2\dot r_j=0. \quad (31)$$

从(29)式可以看出，当 $r\to r_j$ 时，C 的极限为 $\dfrac{0}{0}$ 型，可用罗必塔法则求极限并调节参数 K，使 C 的极限为 1，即

$$\lim_{\substack{r\to r_j\\ v\to v_0}}\frac{\Delta[2K(r-r_j)+1]-2r^2\dot r_j}{2K(r-r_j)r^2}=1.$$

解之，得

$$K_j=\frac{r_j-m-\dfrac{2}{3}\lambda r_j^2-2r_j\dot r_j}{2mr_j-Q^2+\dfrac{1}{3}\lambda r_j^4}. \quad (32)$$

于是，在视界附近，也即在 $r\to r_j$ 情况下，(28)式可化成

$$\frac{\partial^2 R}{\partial r_*^2}+2\frac{\partial^2 R}{\partial v_*\partial r_*}+(\alpha_0+2\mathrm{i}\lambda_0)\frac{\partial R}{\partial r_*}=0, \quad (33)$$

式中

$$\alpha_0=\frac{2r_j^2-3mr_j+Q^2-\lambda r_j^4}{2r_j^3}-\frac{\dot QQ-\dot m r_j}{2r_j^2\dot r_j},$$

$$\lambda_0=\frac{eQ}{r_j}. \quad (34)$$

解(33)式，得狄拉克粒子进出视界的径向波函数为

$$R^{\mathrm{in}}=R_+^{\mathrm{in}}\approx \mathrm{e}^{-\mathrm{i}\omega v_*},$$

$$R^{\mathrm{out}}=R_+^{\mathrm{out}}\approx \mathrm{e}^{-\mathrm{i}\omega v_*}\mathrm{e}^{-2\mathrm{i}(\omega-\omega_0)r_*}\mathrm{e}^{-\alpha_0 r_*}. \quad (35)$$

容易看出 R^{out} 在视界处不解析. 利用发展的 Damour-Ruffini 方法进行解析延拓[12]，最后可得辐射谱为

$$N_\omega^2=\frac{1}{\exp\left(\dfrac{\omega-\omega_0}{k_{\mathrm B}T}\right)+1}, \quad (36)$$

$$T_j=\frac{K_j}{2\pi k_{\mathrm B}}=\frac{r_j-m-\dfrac{2}{3}\lambda r_j^2-2r_j\dot r_j}{2\pi k_{\mathrm B}\left(2mr_j-Q^2+\dfrac{1}{3}\lambda r_j^4\right)}, \quad (37)$$

式中 $k_{\mathrm B}$ 为玻尔兹曼常数，T 为辐射温度（在自然坐标中，若把 $k_{\mathrm B}$ 看作 1，则 $T_j=\dfrac{K_j}{2\pi}$），(36)式为费米子

的黑体辐射谱.可以看到,Vaidya-Bonner-de Sitter 黑洞确实在辐射狄拉克粒子.

7.结 论

从求解视界附近的狄拉克方程出发,准确地定出了 Vaidya-Bonner-de Sitter 黑洞的 Hawking 温度和辐射谱.同时计算出视界方程,所得结果与零曲面方程得到的结果一致.

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Dirac radiation of Vaidya-Bonner-de Sitter black hole*

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Abstract

Dirac equation near horizons of Vaidya-Bonner-de Sitter black hole is discussed. Its Hawking temperature and radiation spectrum are calculated accurately. At the same time, the equation of event horizon is given, which is consistent with the result calculated by zero-curve equation.

Keywords: black hole, horizon, Dirac equation, radiation

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