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新 Tortoise 坐标变换与任意加速带电动态黑洞熵*

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采用一种新的 Tortoise 坐标变换,约化视界附近 Klein-Gordon 方程,得到了黑洞的 Hawking 温度;并用薄膜模型计算了黑洞熵,得到了熵与视界面积成正比的 Bekenstein 关系.用这种 Tortoise 坐标变换,还可以使计算动态黑洞熵时所用的截断因子变得与静态和稳态情况相同.

关键词:Tortoise 坐标变换,黑洞,Hawking 温度,熵

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1. 引 言

自从 Bekenstein 和 Hawking 提出了黑洞熵与其视界面积成正比以来[1,2],相关研究工作取得了很大进展.1985 年,'t Hooft 提出的 brick-wall 模型对黑洞熵的起源给出了一个统计解释[3].此后,为了黑洞熵统计起源的问题更加明晰,人们用相关的办法计算了各种黑洞的熵[4—6].最近一些工作把 brick-wall 模型发展成为薄膜模型[7].在用此模型求黑洞熵的过程中,用通常的 Tortoise 坐标变换,静态或稳态情况下的截断因子很简单,但动态情况下,若仍想得到黑洞熵与其面积成正比的结论,截断因子会变得很复杂,而且依赖于时空度规[8].

本文采用了一种新的 Tortoise 坐标变换[9],以任意加速带电动态黑洞作为例子,讨论了黑洞的 Hawking 温度和熵,得到了与通常的 Tortoise 坐标变换下不同的结果.特别是得到了和静态一样简单的截断因子,同时保证了熵与面积成正比的结论.这种新的坐标变换不失一般性,运用于其他动态黑洞可得到同样的效果.

2. 任意加速带电动态黑洞的时空线元

任意加速带电动态黑洞的时空度规用超前爱丁顿坐标表示为

$$\mathrm{d}s^2 = g_{00}\mathrm{d}v^2 + 2g_{01}\mathrm{d}v\mathrm{d}r + 2g_{02}\mathrm{d}v\mathrm{d}\theta + 2g_{03}\mathrm{d}v\mathrm{d}\varphi + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2, \quad (1)$$

其中

$$g_{00} = -\left[1 - \frac{2m}{r} - 2ar\cos\theta + \frac{Q^2}{r^2} - 4a\frac{Q^2}{r}\cos\theta - r^2\left(f^2 + h^2\sin^2\theta\right)\right],$$

$$g_{01} = g_{10} = 1,$$

$$g_{02} = g_{20} = r^2 f,$$

$$g_{03} = g_{30} = r^2 h\sin^2\theta,$$

$$g_{22} = r^2,$$

$$g_{33} = r^2\sin^2\theta, \quad (2)$$

而

$$f = -a\sin\theta + b\sin\varphi + c\cos\varphi,$$

$$h = \cot\theta\left(b\cos\varphi - c\sin\varphi\right),$$

参量 $m = m(v)$,$Q = Q(v)$分别为源质量和所带电荷,$a = a(v)$,$b = b(v)$,$c = c(v)$是加速度参量,a 为加速度的大小,b,c 描述了方向的改变.

容易算出度规行列式和逆变分量

$$g = -r^4\sin^2\theta, \quad (3)$$

$$g^{01} = g^{10} = 1,$$

$$g^{11} = 1 - \frac{2m}{r} - 2ar\cos\theta + \frac{Q^2}{r^2} - 4a\frac{Q^2}{r}\cos\theta,$$

$$g^{12} = g^{21} = -f, g^{13} = g^{31} = -h,$$

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$$g^{22}=\frac{1}{r^2},g^{33}=\frac{1}{r^2\sin^2\theta}. \tag{4}$$

令黑洞的视界面方程为

$$H=H(\upsilon,r,\theta,\varphi)=0, \tag{5}$$

或

$$r_h=r_h(\upsilon,\theta,\varphi), \tag{6}$$

则 $H(\upsilon,r,\theta,\varphi)$满足零曲面方程条件

$$g^{\mu\nu}\frac{\partial H}{\partial x^\mu}\frac{\partial H}{\partial x^\nu}=0. \tag{7}$$

从(5),(6)式可得

$$\frac{\partial H}{\partial r}\frac{\partial r}{\partial\theta}+\frac{\partial H}{\partial\theta}=0,$$
$$\frac{\partial H}{\partial r}\frac{\partial r}{\partial\varphi}+\frac{\partial H}{\partial\varphi}=0,$$
$$\frac{\partial H}{\partial r}\frac{\partial r}{\partial\upsilon}+\frac{\partial H}{\partial\upsilon}=0. \tag{8}$$

由(4),(7),(8)式可得事件视界 $r_h=r_h(\upsilon,\theta,\varphi)$满足

$$1-\frac{2m}{r_h}-2ar_h\cos\theta+\frac{Q^2}{r_h^2}-4a\frac{Q^2}{r_h}\cos\theta-2r_{h\upsilon}+2fr_{h\theta}+2hr_{h\varphi}+\frac{r_{h\theta}^2}{r_h^2}+\frac{r_{h\varphi}^2}{r_h^2\sin^2\theta}=0, \tag{9}$$

其中

$$r_{h\theta}=\left(\frac{\partial r}{\partial\theta}\right)_{r=r_h},$$
$$r_{h\upsilon}=\left(\frac{\partial r}{\partial\upsilon}\right)_{r=r_h},$$
$$r_{h\varphi}=\left(\frac{\partial r}{\partial\varphi}\right)_{r=r_h}. \tag{10}$$

定义一函数

$$F(r_h)=1-\frac{2m}{r_h}-2ar_h\cos\theta+\frac{Q^2}{r_h^2}-4a\frac{Q^2}{r_h}\cos\theta-2r_{h\upsilon}+2fr_{h\theta}+2hr_{h\varphi}+\frac{r_{h\theta}^2}{r_h^2}+\frac{r_{h\varphi}^2}{r_h^2\sin^2\theta}, \tag{11}$$

则 $F(r_h)=0$ 就是视界面方程式(9).

3. 黑洞的 Hawking 辐射

根据文献[10]提供的方法,我们研究任意加速带电黑洞的 Hawking 辐射,得到 Hawking 温度.

在弯曲时空中,标量粒子(Klein-Gorden 粒子)的动力学行为由 K-G 方程描述,即

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^\mu}-\mathrm{i}eA_\mu\right)\left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu}-\mathrm{i}eA_\nu\right)\Phi\right]-\mu^2\Phi=0,$$

式中 e 为 K-G 粒子所带电荷,A_μ 为黑洞所带电荷产生的电磁四矢,μ 为 K-G 粒子的质量. 利用 Lorentz 条件$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}A_\nu)=0$,可得

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\Phi\right)-2\mathrm{i}eA_\mu g^{\mu\nu}\frac{\partial}{\partial x^\nu}\Phi-e^2g^{\mu\nu}A_\mu A_\nu\Phi-\mu^2\Phi=0,$$

进而展开为

$$g^{11}\frac{\partial^2\Phi}{\partial r^2}+2\frac{\partial^2\Phi}{\partial\upsilon\partial r}+2g^{12}\frac{\partial^2\Phi}{\partial r\partial\theta}+2g^{13}\frac{\partial^2\Phi}{\partial r\partial\varphi}+g^{22}\frac{\partial^2\Phi}{\partial\theta^2}+g^{33}\frac{\partial^2\Phi}{\partial\varphi^2}+f_\upsilon\frac{\partial\Phi}{\partial\upsilon}+f_r\frac{\partial\Phi}{\partial r}+f_\theta\frac{\partial\Phi}{\partial\theta}+f_\varphi\frac{\partial\Phi}{\partial\varphi}+f_0\Phi=0, \tag{12}$$

其中

$$f_\upsilon=\frac{2}{r}-2\mathrm{i}eA_1,$$
$$f_r=\frac{2}{r}g^{11}+\frac{\partial g^{11}}{\partial r}+g^{12}\cot\theta+\frac{\partial g^{12}}{\partial\theta}+\frac{\partial g^{13}}{\partial\varphi}-2\mathrm{i}e(A_0+A_1g^{11}+A_2g^{12}+A_3g^{13}),$$
$$g_\theta=\frac{2}{r}g^{12}+g^{22}\cot\theta-2\mathrm{i}e(A_1g^{12}+A_2g^{22}),$$
$$f_\varphi=\frac{2}{r}g^{13}-2\mathrm{i}e(A_1g^{13}+A_3g^{33}),$$
$$f_0=-e^2g^{\mu\nu}A_\mu A_\nu-\mu^2. \tag{13}$$

我们给出的新 Tortoise 坐标变换为

$$r_*=\frac{1}{2\kappa(\upsilon_0,\theta_0,\varphi_0)}\ln[r-r_h(\upsilon,\theta,\varphi)],$$
$$\upsilon_*=\upsilon-\upsilon_0,$$
$$\theta_*=\theta-\theta_0,$$
$$\varphi_*=\varphi-\varphi_0, \tag{14}$$

式中 r_h 为黑洞的事件视界,κ 为调节参数(后面将看到 κ 即为表征黑洞 Hawking 辐射的温度函数)且在 Tortoise 变换下不变,$\upsilon_0,\theta_0,\varphi_0$ 也为与变换无关的任意常数.

在(14)式的坐标变换下,方程(12)变为

$$\frac{\hat{g}^{11}}{2\kappa(r-r_h)}\frac{\partial^2\Phi}{\partial r_*^2}+2\frac{\partial^2\Phi}{\partial\upsilon_*\partial r_*}+(2g^{12}-2g^{22}r_{h\theta})\frac{\partial^2\Phi}{\partial\theta_*\partial r_*}+(2g^{13}-2g^{33}r_{h\varphi})\frac{\partial^2\Phi}{\partial\varphi_*\partial r_*}+\left[-\frac{\hat{g}^{11}}{r-r_h}-(g^{22}r_{h\theta\theta}+g^{33}r_{h\varphi\varphi})-f_\upsilon r_{h\upsilon}\right.$$

$$+ f_r - f_\theta r_{h\theta} - f_\varphi r_{h\varphi}\Big] \frac{\partial \Phi}{\partial r_*}$$
$$+ 2\kappa(r - r_h)\left(g^{22} \frac{\partial^2}{\partial \theta_*} + g^{33} \frac{\partial^2}{\partial \varphi_*} + f_\upsilon \frac{\partial}{\partial \upsilon_*}\right.$$
$$\left. + f_\theta \frac{\partial}{\partial \theta_*} + f_\varphi \frac{\partial}{\partial \varphi_*} + f_0\right) \Phi = 0, \qquad (15)$$

其中

$$\hat{g}^{11} = g^{11} - 2r_{h\upsilon} - 2g^{12} r_{h\theta} - 2g^{13} r_{h\varphi} + g^{22} r_{h\theta}^2 + g^{33} r_{h\varphi}^2. \qquad (16)$$

比较(16)和(11)式可得

$$\lim_{r\to r_h} \hat{g}^{11} = F(r_h) = 0. \qquad (17)$$

当 $r \to r_h$ 时,$\frac{\partial^2 \Phi}{\partial r_*^2}$的系数设为 A,则有

$$A = \lim_{r\to r_h} \frac{\hat{g}^{11}}{2\kappa(r - r_h)} = \lim_{r\to r_h} \frac{\partial \hat{g}^{11}/\partial r}{2\kappa}$$
$$= \frac{1}{2\kappa} \lim_{r\to r_h} \frac{\partial \hat{g}^{11}}{\partial r}, \qquad (18)$$

可通过选择调节参数 κ 使 $A = 1$,则

$$\kappa = \frac{1}{2} \lim_{r\to r_h} \frac{\partial \hat{g}^{11}}{\partial r} = \frac{1}{r_h^2}(m + 2aQ^2 \cos\theta)$$
$$- \frac{1}{r_h^3}\left(Q^2 + r_{h\theta}^2 + \frac{r_{h\varphi}^2}{\sin^2\theta}\right) - a\cos\theta. \qquad (19)$$

用 Tortoise 坐标表示的 K-G 方程在视界附近应该具有典型的波动方程形式. 对于方程(15),当 $r \to r_h$(表示 $r \to r_h(\upsilon_0, \theta_0, \varphi_0)$), $\upsilon \to \upsilon_0$, $\theta \to \theta_0$, $\varphi \to \varphi_0$)时,显然可以约化成

$$\frac{\partial^2 \Phi}{\partial r_*^2} + 2\frac{\partial^2 \Phi}{\partial \upsilon_* \partial r_*} + B\frac{\partial^2 \Phi}{\partial \theta_* \partial r_*} + C\frac{\partial^2 \Phi}{\partial \varphi_* \partial r_*}$$
$$+ (D + \mathrm{i}2\omega_0)\frac{\partial \Phi}{\partial r_*} = 0, \qquad (20)$$

其中

$$B = \lim_{r\to r_h}(2g^{12} - 2g^{22} r_{h\theta}), \qquad (21)$$

$$C = \lim_{r\to r_h}(2g^{13} - 2g^{33} r_{h\varphi}), \qquad (22)$$

$$D = \lim_{r\to r_h}\Big[-\frac{\hat{g}^{11}}{r - r_h} - \left(g^{22} r_{h\theta\theta} + g^{33} r_{h\varphi\varphi} + \frac{2r_{h\upsilon}}{r}\right)$$
$$+ \left(\frac{2}{r}g^{11} + \frac{\partial g^{11}}{\partial r} + g^{12}\cot\theta + \frac{\partial g^{12}}{\partial \theta} + \frac{\partial g^{13}}{\partial \varphi}\right)$$
$$- \left(\frac{2}{r}g^{12} + g^{22}\cot\theta\right) r_{h\theta} - \frac{2}{r}g^{13} r_{h\varphi}\Big], \qquad (23)$$

$$\omega_0 = \lim_{r\to r_h}\big[eA_1 r_{h\upsilon} + e(A_1 g^{12} + A_2 g^{22}) r_{h\theta}$$
$$+ e(A_1 g^{13} + A_3 g^{33}) r_{h\varphi} - e(A_0 + A_1 g^{11} + A_2 g^{12} + A_3 g^{13})\big], \qquad (24)$$

显然 B, C, D, ω_0 均为有限值.

在视界附近,方程(20)的解可写为[11]

$$\Phi = R(r_*)\mathrm{e}^{-\mathrm{i}\omega\upsilon_* + \mathrm{i}k_\theta \theta_* + \mathrm{i}k_\varphi \varphi_*}. \qquad (25)$$

把(25)式代入(20)式,可得

$$\frac{\partial^2 R(r_*)}{\partial r_*^2} + \{D - \mathrm{i}[2(\omega - \omega_0) - Bk_\theta$$
$$- Ck_\varphi]\}\frac{\partial R(r_*)}{\partial r_*} = 0, \qquad (26)$$

则(26)式的解为

$$R_\omega^{\mathrm{in}} = \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}, R_\omega^{\mathrm{out}} = \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}\mathrm{e}^{-Dr_* + \mathrm{i}[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]r_*}, \qquad (27)$$

进而在视界面上的入射波与出射波分别为

$$\Phi_\omega^{\mathrm{in}} = \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}\mathrm{e}^{\mathrm{i}k_\theta\theta_* + \mathrm{i}k_\varphi\varphi_*},$$
$$\Phi_\omega^{\mathrm{out}} = \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}\mathrm{e}^{-Dr_* + \mathrm{i}[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]r_*}\mathrm{e}^{\mathrm{i}k_\theta\theta_* + \mathrm{i}k_\varphi\varphi_*}$$
$$= \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}(r - r_h)^{-D/2\kappa}$$
$$\times (r - r_h)^{\mathrm{i}[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]/2\kappa}\mathrm{e}^{\mathrm{i}k_\theta\theta_* + \mathrm{i}k_\varphi\varphi_*}. \qquad (28)$$

$\Phi_\omega^{\mathrm{out}}$ 在 $r = r_h$ 处奇异,只能描述视界外的出射粒子,不能描述视界内的出射粒子,为此我们通过复平面绕过视界解析延拓到视界内部,此时变量为 $r - r_h \to |r - r_h|\mathrm{e}^{-\mathrm{i}\pi} = (r_h - r)\mathrm{e}^{-\mathrm{i}\pi}$,于是,在视界内的出射波为

$$R_\omega^{\mathrm{out}}(r < r_h) = \mathrm{e}^{-\mathrm{i}\omega\upsilon_* + \mathrm{i}k_\theta\theta_* + \mathrm{i}k_\varphi\varphi_*}[(r_h - r)\mathrm{e}^{-\mathrm{i}\pi}]^{-D/2\kappa}$$
$$\times [(r_h - r)\mathrm{e}^{-\mathrm{i}\pi}]^{\mathrm{i}[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]/2\kappa}$$
$$= \mathrm{e}^{-\mathrm{i}\omega\upsilon_*}\mathrm{e}^{-Dr_* + i[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]r_*}$$
$$\times \mathrm{e}^{\mathrm{i}k_\theta\theta_* + \mathrm{i}k_\varphi\varphi_* + \mathrm{i}D\pi/2\kappa}$$
$$\times \mathrm{e}^{\pi[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]/2\kappa}, \qquad (29)$$

所以出射波在视界处的散射概率为

$$\left|\frac{\Phi_\omega^{\mathrm{out}}(r > r_h)}{\Phi_\omega^{\mathrm{out}}(r < r_h)}\right|^2 = \mathrm{e}^{-\pi[2(\omega-\omega_0) - Bk_\theta - Ck_\varphi]/\kappa}. \qquad (30)$$

根据 Damour-Ruffini[12] 和 Sannan[13] 的方法,可以得到出射波的黑体谱为

$$N_\omega = \frac{1}{\mathrm{e}^{[(\omega-\omega_0) - Bk_\theta/2 - Ck_\varphi/2]/k_B T} - 1}, \qquad (31)$$

其中 k_B 为玻尔兹曼常数,$T = \frac{\kappa}{2\pi k_B}$是辐射温度.

4. 拖曳角速度

对度规(1)做 $R = r - r_h$, $\mathrm{d}R = \mathrm{d}r - r_{h\upsilon}\mathrm{d}\upsilon - r_{h\theta}\mathrm{d}\theta - r_{h\varphi}\mathrm{d}\varphi$ 的坐标变换[14],可得

$$\mathrm{d}s^2 = \left(\hat{g}_{00} - \frac{\hat{g}_{02}^2}{\hat{g}_{22}} - \frac{\hat{g}_{03}^2}{\hat{g}_{33}}\right)\mathrm{d}\upsilon^2 + 2\mathrm{d}\upsilon\mathrm{d}R$$

$$+\hat{g}_{22}\left(\mathrm{d}\theta+\frac{\hat{g}_{02}}{\hat{g}_{22}}\mathrm{d}\upsilon\right)^2+\hat{g}_{33}\left(\mathrm{d}\varphi+\frac{\hat{g}_{03}}{\hat{g}_{33}}\mathrm{d}\upsilon\right)^2,\tag{32}$$

其中

$$\begin{aligned}\hat{g}_{00}&=g_{00}+2r_{h\upsilon},\\ \hat{g}_{02}&=g_{02}+r_{h\theta},\\ \hat{g}_{03}&=g_{03}+r_{h\varphi},\\ \hat{g}_{22}&=g_{22},\\ \hat{g}_{33}&=g_{33}.\end{aligned}\tag{33}$$

显见,拖曳速度为

$$\Omega_\theta=\frac{\mathrm{d}\theta}{\mathrm{d}\upsilon}=-\frac{\hat{g}_{02}}{\hat{g}_{22}},\Omega_\varphi=\frac{\mathrm{d}\varphi}{\mathrm{d}\upsilon}=-\frac{\hat{g}_{03}}{\hat{g}_{33}}.\tag{34}$$

比较(21),(22)和(34)式,可知

$$\frac{B}{2}=(\Omega_\theta)_{r=r_h},\frac{C}{2}=(\Omega_\varphi)_{r=r_h},\tag{35}$$

可见,(21),(22)式中的参数 B,C 与黑洞的拖曳角速度有关.

5. 黑洞的熵

根据文献[15]提供的方法,我们用薄膜模型计算黑洞的熵.在此模型中黑洞的熵来源于视界附近 $r_h+\varepsilon\to r_h+\varepsilon+\delta$ 区域场的贡献,薄膜中的温度是随位置和角度发生变化的,因此我们把薄膜分成许多小的部分,$r_h(\upsilon,\theta_i,\varphi_i)+\varepsilon\to r_h(\upsilon,\theta_i,\varphi_i)+\varepsilon+\delta$,$\theta_i\to\theta_i+\Delta\theta_i$,$\varphi_i\to\varphi_i+\Delta\varphi_i$,$i=1,2,3,\cdots,n$,这样在每个部分里场是准平衡的,且统计规律是有效的.

度规(32)的不为零的逆变度规分量为

$$\hat{g}^{01}=1,\hat{g}^{22}=\frac{1}{r^2},\hat{g}^{33}=\frac{1}{r^2\sin^2\theta},$$

$$\hat{g}^{12}=-\frac{\hat{g}_{02}}{\hat{g}_{22}}=\Omega_\theta,\hat{g}^{13}=-\frac{\hat{g}_{03}}{\hat{g}_{33}}=\Omega_\varphi,$$

$$\hat{g}^{11}=1-\frac{2m}{r}-2ar_h\cos\theta+\frac{Q^2}{r^2}-4a\frac{Q^2}{r}\cos\theta-2r_{h\upsilon}+2fr_{h\theta}+2hr_{h\varphi}+\frac{r_{h\theta}^2}{r^2}+\frac{r_{h\varphi}^2}{r^2\sin^2\theta},\tag{36}$$

可见,这里的 $\hat{g}^{11}$ 就是(16)式中的 $\hat{g}^{11}$,因此同样的 $\hat{g}^{11}(r_h)=0$,$\left.\frac{\partial\hat{g}^{11}}{\partial r}\right|_{r=r_h}=2\kappa$.

把(36)式代入 Klein-Gordon 方程

$$\frac{1}{\sqrt{-\hat{g}}}\left(\frac{\partial}{\partial x^\mu}-\mathrm{i}e\hat{A}_\mu\right)\left[\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\left(\frac{\partial}{\partial x^\nu}-\mathrm{i}e\hat{A}_\nu\right)\right]\Phi-\mu^2\Phi=0,$$

其中 $\hat{A}_\mu$ 为原度规下电磁四矢 A_μ 的变换后的形式,令 $\Phi=\mathrm{e}^{-\mathrm{i}E\upsilon+\mathrm{i}G(R,\theta,\varphi)}$ 进行分离变量,并利用 Lorentz 条件和 WKB 近似得

$$\begin{aligned}&\hat{g}^{11}k_R^2-2[E-\Omega_\theta k_\theta-\Omega_\varphi k_\varphi+e(\hat{A}_0+\hat{A}_1\hat{g}^{11}\\&+\hat{A}_2\Omega_\theta+\hat{A}_3\Omega_\varphi)]k_R+\hat{g}^{22}k_\theta^2+\hat{g}^{33}k_\varphi^2\\&-2e(\hat{A}_1\Omega_\theta+\hat{A}_2\hat{g}^{22})k_\theta-2e(\hat{A}_1\Omega_\varphi+\hat{A}_3\hat{g}^{33})k_\varphi\\&+2e\hat{A}_1E+e^2(2\hat{A}_0\hat{A}_1+2\Omega_\theta\hat{A}_1\hat{A}_2+2\Omega_\varphi\hat{A}_1\hat{A}_3\\&+\hat{g}^{11}\hat{A}_1^2+\hat{g}^{22}\hat{A}_2^2+\hat{g}^{33}\hat{A}_3^2)+\mu^2=0,\end{aligned}\tag{37}$$

其中 $k_R=\frac{\partial G}{\partial R}$,$k_\theta=\frac{\partial G}{\partial\theta}$,$k_\varphi=\frac{\partial G}{\partial\varphi}$.

解方程(37)可得

$$k_R^\pm=\frac{E'+e\hat{A}_1\hat{g}^{11}}{\hat{g}^{11}}\pm\frac{1}{\hat{g}^{11}}\times\sqrt{E'^2-\hat{g}^{11}[\hat{g}^{22}(k_\theta-e\hat{A}_2)^2+\hat{g}^{33}(k_\varphi-e\hat{A}_3)^2+\mu^2]},\tag{38}$$

其中 $E'=E-\Omega_\theta k_\theta-\Omega_\varphi k_\varphi+e(\hat{A}_0+\hat{A}_2\Omega_\theta+\hat{A}_3\Omega_\varphi)$.

根据量子统计理论,把薄膜分成许多小的子系统,则第 i 个子系统的自由能为

$$\Delta F_i=-\int_0^\infty\mathrm{d}E'\frac{\Gamma(E')}{e^{\beta E'}-1},\tag{39}$$

其中 $\Gamma(E')$ 是能量小于 E' 的状态总数.根据半经典量子化条件和薄膜 brick-wall 模型,有

$$\Gamma(E')=\frac{1}{4\pi^3}\int\mathrm{d}k_\theta\int\mathrm{d}k_\varphi\int_{\theta_i}^{\theta_i+\Delta\theta_i}\mathrm{d}\theta\int_{\varphi_i}^{\varphi_i+\Delta\varphi_i}\mathrm{d}\theta\left(\int_\varepsilon^{\varepsilon+\delta}k_R^+\mathrm{d}R+\int_{\varepsilon+\delta}^{\varepsilon}k_R^-\mathrm{d}R\right),\tag{40}$$

把(38)式代入(40)式,考虑到 $E'^2-\hat{g}^{11}[\hat{g}^{22}(k_\theta-e\hat{A}_2)^2+\hat{g}^{33}(k_\varphi-e\hat{A}_3)^2+\mu^2]\geqslant0$ 限制了 k_θ,k_φ 的积分上下限,因此

$$\begin{aligned}\Gamma(E')&=\frac{E'^3}{6\pi^2}\int\mathrm{d}\theta\int\mathrm{d}\varphi\int_\varepsilon^{\varepsilon+\delta}(\hat{g}^{11})^{-2}(\hat{g}^{22}\hat{g}^{33})^{-1/2}\mathrm{d}R\\&=\frac{E'^3}{6\pi^2}\int\mathrm{d}A_i\int_\varepsilon^{\varepsilon+\delta}(\hat{g}^{11})^{-2}\mathrm{d}R,\end{aligned}\tag{41}$$

其中 $\int\mathrm{d}A_i=\int_{\varphi_i}^{\varphi_i+\Delta\varphi_i}\int_{\theta_i}^{\theta_i+\Delta\theta_i}(\hat{g}^{22}\hat{g}^{33})^{-1/2}\mathrm{d}\theta\mathrm{d}\varphi$ 是第 i 个子系统在视界上的小面积,记为 ΔA_i.

把(41)式代入(39)式,得

$$\Delta F_i=-\frac{\Delta A_i}{6\pi^2}\int_\varepsilon^{\varepsilon+\delta}(\hat{g}^{11})^{-2}\mathrm{d}R\int_0^\infty\frac{E'^3}{\mathrm{e}^{\beta E'}-1}\mathrm{d}E'.\tag{42}$$

因为 $\hat{g}^{11}(r_h)=0$,$\hat{g}^{11}$ 可展开为 $\hat{g}^{11}=p(\upsilon,r,\theta,\varphi)(r-r_h)$,于是

$$\Delta F_i = \frac{-\Delta A_i}{6\pi^2}\int_0^\infty \frac{E'^3 \mathrm{d}E'}{\mathrm{e}^{\beta E'}-1}\int_\varepsilon^{\varepsilon+\delta}\frac{1}{p^2(\upsilon,r,\theta,\varphi)(r-r_h)^2}\mathrm{d}R$$

$$\approx -\Delta A_i \frac{\pi^2}{90\beta^4 p^2(r_h)}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{43}$$

由熵和自由能的关系,可得子系统的熵为

$$\Delta S_i = \beta^2 \frac{\partial \Delta F_i}{\partial \beta}\bigg|_{\beta=\beta_h}$$

$$= \Delta A_i \frac{4\pi^2}{90\beta_h^3 p^2(r_h)}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{44}$$

考虑到(19)式,有 $p(r_h) = \frac{\partial \hat{g}^{11}}{\partial r}\Big|_{r=r_h} = 2\kappa$,而 $\beta_h = \frac{2\pi}{\kappa}$,因此

$$\Delta S_i = \frac{1}{90\beta_h}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\frac{1}{4}\Delta A_i. \tag{45}$$

选取适当截断因子和薄层厚度,使得

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_h, \tag{46}$$

则

$$\Delta S_i = \frac{1}{4}\Delta A_i, \tag{47}$$

进而,黑洞的总熵为

$$S = \sum_i \Delta S_i = \frac{1}{4}\sum_i \Delta A_i = \frac{1}{4}A_h. \tag{48}$$

可见,在新坐标变换下,通过约化视界附近的 K-G 方程,得到了黑洞的温度参数 κ.并且在用这种新坐标变换求得与面积成正比的 Bekenstein-Hawking 熵的过程中,不但得到了和静态、稳态一样简单的截断因子,而且此截断因子不再依赖于时空度规的种类.

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A new Tortoise coordinate transformation and entropy of arbitrarily accelerating charged black hole*

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Abstract

Considering the Klein-Gordon equation near the event horizon, Hawking temperature is calculated under a new Tortoise coordinate transformation. In the meantime, the cut-off parameter in thin membrane model is also simplified to the same as that in the static and stationary cases.

Keywords: Tortoise coordinate transformation, black hole, Hawking temperature, entropy

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