第 54 卷 第 2 期 2005 年 2 月
1000-3290/2005/54(02)/0522-08

物 理 学 报
ACTA PHYSICA SINICA

Vol.54,No.2,February,2005


# 含时耦合谐振子系统的时间演化与双模压缩态

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（2004 年 4 月 7 日收到;2004 年 5 月 21 日收到修改稿）

运用 Lewis-Riesenfeld 不变量理论,通过适当地选取厄米不变量,得到了含驱动项和双模耦合项的含时耦合谐振子系统薛定谔方程的封闭解,给出了系统的演化算符及其产生双模光场的压缩态的条件,并得出系统压缩态的量子涨落与驱动项无关但与系统所处的初态有关的结论.

**关键词**:含时耦合谐振子系统,Lewis-Riesenfeld 不变量理论,双模压缩态

**PACC**:0365,4250

## 1. 引　　言

含时哈密顿量 $\hat{H}$ 的动力学系统的时间演化,因其广泛的应用价值,成为近年来人们十分感兴趣的研究课题之一.由于对于 $\hat{H}$ 含时的系统,能量既不是守恒量,也非物理观测量,以它的瞬时本征态为基矢往往得不到含时薛定谔方程封闭形式的解[1],使得精确求解此类问题显得困难重重.为此,人们寻求各种不同的方法[2—5],对含时系统进行研究.其中,一种行之有效的方法是运用 Lewis 和 Riesenfeld 提出的量子不变量理论[6,7],人们已对此理论及其各种应用进行了广泛的探讨[1,8—21].文献[8]用其研究了一个具有较普遍意义(有压缩和驱动项)的含时单模谐振子系统的时间演化和压缩态.但对于耦合谐振子模型,人们仅就一些特殊的情况进行了求解.如文献[4,5]运用李代数方法对如下简单模型进行了研究:

$$\hat{H}(t) = \omega_1\hat{a}_1^+\hat{a}_1 + \omega_2\hat{a}_2^+\hat{a}_2 + \mathrm{i}g\{\hat{a}_1\hat{a}_2\exp[\mathrm{i}(\omega t+\phi)] - \hat{a}_1^+\hat{a}_2^+\exp[-\mathrm{i}(\omega t+\phi)]\} + G. \tag{1}$$

这一模型(文献[4]中的 $\omega_1=\Omega_1^0$,$\omega_2=\Omega_2^0$,$g=-k\varepsilon/2$,$\omega=2\Omega$,$G=0$,文献[5]中的 $\phi=0$,$G=(\omega_1+\omega_2)/2$)能描述非线性量子光学中经典抽运两模非简并参量下转换过程,其中 $\omega_1$,$\omega_2$ 和 $\omega$ 分别为信号光、闲置光和抽运光的频率,$\hat{a}_1$,$\hat{a}_2$ 分别为信号光和闲置光的湮没算符,$g$ 为耦合常数,其值正比于介质的二阶非线性极化率和抽运光的振幅[22].文献[4,5]研究了此动力学系统的时间演化以及在失谐($\Omega=\omega-\omega_1-\omega_2\neq0$)情况下双模量子系统的一些非经典性质.最近,文献[23]借助于量子力学纠缠态表象,求出了一有驱动作用下的双模耦合系统量子态的时间演化.但也仅仅在对某些参数进行了较苛刻的限制的特殊情形下进行了求解.本文将运用 Lewis-Riesenfeld 不变量理论,通过适当选取厄米不变量,对有驱动项和双模耦合项的一般含时耦合谐振子系统进行讨论,得到了系统精确的量子态时间演化,给出系统双模光场的压缩态产生的条件,并分析了影响压缩态的有关因素.

## 2. 量子态时间演化和 Lewis-Riesenfeld 相位

本文讨论的含时系统哈密顿算符的形式为(取 $\hbar=1$)

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1 + \hat{H}_2, \tag{2a}$$

$$\hat{H}_0 = \sum_{j=1}^{2}\omega_j(t)\hat{a}_j^+\hat{a}_j + G(t), \tag{2b}$$

$$\hat{H}_1 = \sum_{j=1}^{2}G_j(t)[\hat{a}_j^+\exp(\mathrm{i}\varphi_j(t)) + \hat{a}_j\exp(-\mathrm{i}\varphi_j(t))], \tag{2c}$$

$$\hat{H}_2 = G_{12}(t)[\hat{a}_1^+\hat{a}_2^+\exp(\mathrm{i}\phi(t)) + \hat{a}_1\hat{a}_2\exp(-\mathrm{i}\phi(t))]. \tag{2d}$$

而 $\hat{a}_j$ 和 $\hat{a}_j^+$ 满足如下对易关系

$$[\hat{a}_j,\hat{a}_k^+] = \delta_{jk},[\hat{a}_j,\hat{a}_k] = [\hat{a}_j^+,\hat{a}_k^+] = 0 \quad (j,k=1,2), \tag{3}$$

这里 $\omega_j(t)$,$G_{12}(t)$,$\phi(t)$,$G(t)$,$G_j(t)$和 $\varphi_j(t)$($j=1,2$,下同)为任意的实时间函数,$\hat{a}_j$ 和 $\hat{a}_j^+$ 为不显含时间的玻色子湮没算符和产生算符;$\hat{H}_0$ 表示两模光场的自由哈密顿量,$\hat{H}_1$ 为驱动项,$\hat{H}_2$ 为双模耦合项.当(2)式中的 $\hat{H}_2=0$,哈密顿算符 $\hat{H}(t)=\hat{H}_0+\hat{H}_1$ 可以描述经典广义力作用下两受迫量子振子的行为,这一过程将导致相干态的产生[24,25].当(2)式中的 $\hat{H}_1=0$,并使哈密顿算符 $\hat{H}(t)=\hat{H}_0+\hat{H}_2$ 取(1)式的特殊形式时,则可以描述非简并参量下转换过程,并将导致双模压缩态的产生[4,5].当(2)式中的 $\omega_1(t)=\omega_2(t)=G(t)=\omega'$,$G_1(t)=\lambda\cos\omega_1 t$,$G_2(t)=\sigma\cos\omega_2 t$,$\varphi_1(t)=\varphi_2(t)=0$,$\phi(t)=-2\omega_0 t$,$G_{12}(t)=g=\omega'-\omega_0$(这里 $\omega'$,$\lambda$,$\sigma$,$\omega_1$,$\omega_2$,$\omega_0$ 均为常量)时,本文的模型可退化为文献[23]的情形,这时属于薛定谔方程的一个最简单的解是压缩相干态.本文讨论(2)式中所有的参数均为任意的含时函数的情形,这时,$G_j(t)\exp(\mathrm{i}\varphi_j(t))$表示作用在第 $j$ 个模上的瞬变经典广义力[25],$G_{12}(t)\exp(\mathrm{i}\phi(t))$是与介质、光场的性质及其耦合强度有关的任意的含时函数,例如,若令 $\phi(t)=\pi/2-2\Omega t-\phi$,$G_{12}(t)$(正比于介质的二阶非线性极化率和经典抽运光的振幅)与 $t$ 有关,则表示此物理过程中经典抽运光的位相与两模压缩光场的总位相之差 $\phi$[4] 以及抽运光的频率稳定,但其振幅随时间变化;当然,若 $G_{12}(t)$为常量,则表示抽运光的振幅不变.因此,本文讨论的模型能普遍地描述非线性量子光学中含有参量下转换相互作用的两模受迫量子振子的行为[23],对其进行深入的研究具有重要的理论和实际意义.

系统量子态的时间演化遵从 Schrödinger 方程

$$\mathrm{i}\frac{\partial}{\partial t}|\psi(t)\rangle=\hat{H}(t)|\psi(t)\rangle, \tag{4}$$

假定存在一含时厄米不变量 $\hat{I}(t)$,满足条件

$$\mathrm{i}\frac{\partial}{\partial t}\hat{I}(t)+[\hat{I}(t),\hat{H}(t)]=0. \tag{5}$$

本文利用厄米算符 $\hat{K}_0=C_1\hat{a}_1^+\hat{a}_1+C_2\hat{a}_2^+\hat{a}_2$ 的幺正变换来构造这个厄米不变量,这里 $C_1$,$C_2$ 为任意实常数.引入

$$\hat{I}(t)=\hat{D}_1(z_1(t))\hat{D}_2(z_2(t))\hat{S}(\xi(t))\hat{K}_0\times\hat{S}^+(\xi(t))\hat{D}_2^+(z_2(t))\hat{D}_1^+(z_1(t)), \tag{6}$$

其中 $\hat{D}_j(z_j(t))$是平移算符,其定义为

$$\hat{D}_j(z_j(t))=\exp[z_j(t)\hat{a}_j^+-z_j^*(t)\hat{a}_j]\quad(j=1,2), \tag{7a}$$

$\hat{S}(\xi(t))$是双模压缩算符,其定义为

$$\hat{S}(\xi(t))=\exp[\xi^*(t)\hat{a}_1\hat{a}_2-\xi(t)\hat{a}_1^+\hat{a}_2^+]. \tag{7b}$$

此外(6)式中的

$$z_j(t)=r_j(t)\exp(\mathrm{i}\delta_j(t))\qquad(j=1,2), \tag{8a}$$

$$\xi(t)=s(t)\exp(\mathrm{i}\theta(t)), \tag{8b}$$

并且设 $r_j(t)$,$\delta_j(t)$,$s(t)$,$\theta(t)$均是实时间函数,为简单起见,将其简记为 $r_j$,$\delta_j$,$s$,$\theta$(将 $t=0$ 时刻的量简记为 $r_{j0}$,$\delta_{j0}$,$S_0$,$\theta_0$).这六个含时参量,可由(2)和(5)式来确定.利用下面关系式(略去其厄米共轭形式)

$$\hat{D}_j(z_j)\hat{a}_j\hat{D}_j^+(z_j)=\hat{a}_j-z_j\qquad(j=1,2), \tag{9a}$$

$$\hat{S}(\xi)\hat{a}_1\hat{S}^+(\xi)=\hat{a}_1\cosh s+\hat{a}_2^+\exp(\mathrm{i}\theta)\sinh s, \tag{9b}$$

$$\hat{S}(\xi)\hat{a}_2\hat{S}^+(\xi)=\hat{a}_2\cosh s+\hat{a}_1^+\exp(\mathrm{i}\theta)\sinh s. \tag{9c}$$

由(6)和(9)式得

$$\begin{aligned}\hat{I}(t)&=(C_1\cosh^2 s+C_2\sinh^2 s)\hat{a}_1^+\hat{a}_1\\&+(C_2\cosh^2 s+C_1\sinh^2 s)\hat{a}_2^+\hat{a}_2\\&+\frac{1}{2}(C_1+C_2)[\hat{a}_1^+\hat{a}_2^+\exp(\mathrm{i}\theta)\\&+\hat{a}_1\hat{a}_2\exp(-\mathrm{i}\theta)]\sinh 2s-\hat{a}_1^+f_1\\&-\hat{a}_1f_1^*-\hat{a}_2^+f_2-\hat{a}_2f_2^*+f_0,\end{aligned} \tag{10}$$

其中

$$f_1=z_1(C_1\cosh^2 s+C_2\sinh^2 s)+\frac{1}{2}(C_1+C_2)z_1^*\exp(\mathrm{i}\theta)\sinh 2s, \tag{11a}$$

$$f_2=z_2(C_2\cosh^2 s+C_1\sinh^2 s)+\frac{1}{2}(C_1+C_2)z_1^*\exp(\mathrm{i}\theta)\sinh 2s, \tag{11b}$$

$$\begin{aligned}f_0&=(C_1r_1^2+C_2r_2^2)\cosh^2 s+(C_2r_1^2+C_1r_2^2)\sinh^2 s\\&+(C_1+C_2)[r_1r_2\sinh 2s\cos(\delta_1+\delta_2-\theta)\\&+\sinh^2 s].\end{aligned} \tag{11c}$$

由(2)和(10)式得 $\hat{I}(t)$和 $\hat{H}(t)$的对易关系为

$$\begin{aligned}&[\hat{I}(t),\hat{H}(t)]\\&=\Big\{(C_1+C_2)\Big[G_{12}(t)\cosh 2s\exp(\mathrm{i}\phi)\\&-\frac{\omega_1(t)+\omega_2(t)}{2}\sinh 2s\exp(\mathrm{i}\theta)\Big]\hat{a}_1^+\hat{a}_2^+\\&+[\omega_1(t)f_1-G_{12}(t)f_2^*\exp(\mathrm{i}\phi)+G_1(t)\\&\times(C_1\cosh^2 s+C_2\sinh^2 s)\exp(\mathrm{i}\varphi_1)\\&-\frac{1}{2}(C_1+C_2)G_2(t)\sinh 2s\exp(i(\theta-\varphi_2))]\hat{a}_1^+\\&+[\omega_2(t)f_2-G_{12}(t)f_1^*\exp(\mathrm{i}\phi)+G_2(t)\end{aligned}$$

$$\times (C_2\cosh^2 s + C_1\sinh^2 s)\exp(\mathrm{i}\varphi_2) - \frac{1}{2}(C_1 + C_2)G_1(t)\sinh 2s \times \exp(\mathrm{i}(\theta - \varphi_1))]\hat{a}_2^+\Big\} - \mathrm{H.c.} - \mathrm{i}(C_1 + C_2)G_{12}(t)\sinh 2s \times \sin(\theta - \phi)(\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1) + G_1(t)[f_1\exp(-\mathrm{i}\varphi_1) - f_1^*\exp(\mathrm{i}\varphi_1)] + G_2(t)[f_2\exp(-\mathrm{i}\varphi_2) - f_2^*\exp(\mathrm{i}\varphi_2)], \tag{12}$$

H.c.表前一项的厄米共轭.将(10)和(12)代入(5)式得

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 2G_{12}(t)\coth 2s\cos(\theta - \phi) - \omega_1(t) - \omega_2(t), \tag{13a}$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = G_{12}(t)\sin(\theta - \phi), \tag{13b}$$

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \mathrm{i}[-\omega_1(t)f_1 + G_{12}(t)f_2^*\exp(\mathrm{i}\phi) - G_1(t)(C_1\cosh^2 s + C_2\sinh^2 s)\exp(\mathrm{i}\varphi_1) + \frac{1}{2}(C_1 + C_2)G_2(t)\sinh 2s\exp(\mathrm{i}(\theta - \varphi_2))], \tag{14a}$$

$$\frac{\mathrm{d}f_2}{\mathrm{d}t} = \mathrm{i}[-\omega_2(t)f_2 + G_{12}(t)f_1^*\exp(\mathrm{i}\phi) - G_2(t)(C_2\cosh^2 s + C_1\sinh^2 s)\exp(\mathrm{i}\varphi_2) + \frac{1}{2}(C_1 + C_2)G_1(t)\sinh 2s\exp(\mathrm{i}(\theta - \varphi_1))], \tag{14b}$$

$$\frac{\mathrm{d}f_0}{\mathrm{d}t} = (C_1 + C_2)G_{12}(t)\sinh 2s\sin(\theta - \phi) + \mathrm{i}G_1(t)[f_1\exp(-\mathrm{i}\varphi_1) - f_1^*\exp(\mathrm{i}\varphi_1)] + \mathrm{i}G_2(t)[f_2\exp(-\mathrm{i}\varphi_2) - f_2^*\exp(\mathrm{i}\varphi_2)]. \tag{14c}$$

由(13)和(14)式可求出 $s,\theta$ 和 $r_1,r_2,\delta_1,\delta_2$(或 $z_1$,$z_2$),将所求出的结果代入(10)式,即可求出含时不变量 $\hat{I}(t)$.

设 $|n_1,n_2\rangle$ 为算符 $\hat{K}_0$ 的本征态,即

$$\hat{K}_0|n_1,n_2\rangle = (C_1n_1 + C_2n_2)|n_1,n_2\rangle. \tag{15}$$

显然 $\hat{I}(t)$ 的本征态为 $\hat{D}_1(z_1)\hat{D}_2(z_2)S(\xi)|n_1,n_2\rangle$,将其简记为 $|z_1,z_2,\xi,n_1,n_2\rangle$,即

$$\hat{D}_1(z_1)\hat{D}_2(z_2)S(\xi)|n_1,n_2\rangle = |z_1,z_2,\xi,n_1,n_2\rangle, \tag{16}$$

这是因为

$$\hat{I}(t)|z_1,z_2,\xi,n_1,n_2\rangle = \hat{D}_1(z_1)\hat{D}_2(z_2)\hat{S}(\xi)\hat{K}_0\hat{S}^+(\xi)\hat{D}_2^+(z_2) \times \hat{D}_1^+(z_1)\hat{D}_1(z_1)\hat{D}_2(z_2)S(\xi)|n_1,n_2\rangle = (C_1n_1 + C_2n_2)|z_1,z_2,\xi,n_1,n_2\rangle. \tag{17}$$

不难看出,$\hat{I}(t)$ 的本征态 $|z_1,z_2,\xi,n_1,n_2\rangle$ 是完备的,这是因为

$$\sum_{n_1,n_2}|z_1,z_2,\xi,n_1,n_2\rangle\langle z_1,z_2,\xi,n_1,n_2| = \hat{D}_1(z_1)\hat{D}_2(z_2)\hat{S}(\xi)\sum_{n_1,n_2}|n_1,n_2\rangle \times \langle n_1,n_2|\hat{S}^+(\xi)\hat{D}_2^+(z_2)\hat{D}_1^+(z_1) = \hat{D}_1(z_1)\hat{D}_2(z_2)\hat{S}(\xi)\cdot\hat{S}^+(\xi) \times \hat{D}_2^+(z_2)\hat{D}_1^+(z_1) = 1. \tag{18}$$

按照 Lewis-Riesenfeld 不变量理论[6],含时 Schrödinger 方程(4)的通解可用 $\hat{I}(t)$ 的本征态表示为

$$|\psi(t)\rangle = \sum_{n_1,n_2}C_{n_1n_2}\exp(\mathrm{i}\alpha_{n_1n_2})|z_1,z_2,\xi,n_1,n_2\rangle = \sum_{n_1,n_2}C_{n_1n_2}\exp(\mathrm{i}\alpha_{n_1n_2})\hat{D}_1(z_1) \times \hat{D}_2(z_2)\hat{S}(\xi)|n_1,n_2\rangle, \tag{19}$$

式中 $\alpha_{n_1n_2}$ 称为 Lewis-Riesenfeld 相位,它可分解为几何部分 $\gamma_{n_1n_2}$ 和动力学部分 $\beta_{n_1n_2}$,即 $\alpha_{n_1n_2} = \gamma_{n_1n_2} + \beta_{n_1n_2}$,而

$$\gamma_{n_1n_2} = \int_0^t\langle z_1,z_2,\xi,n_1,n_2|\mathrm{i}\frac{\partial}{\partial t} \times |z_1,z_2,\xi,n_1,n_2\rangle\mathrm{d}t, \tag{20a}$$

$$\beta_{n_1n_2} = -\int_0^t\langle z_1,z_2,\xi,n_1,n_2|\hat{H}(t) \times |z_1,z_2,\xi,n_1,n_2\rangle\mathrm{d}t, \tag{20b}$$

利用下面的关系(略去其厄米共轭形式)

$$\hat{S}^+(\xi)\hat{a}_1\hat{S}(\xi) = \hat{a}_1\cosh s - \hat{a}_2^+\exp(\mathrm{i}\theta)\sinh s, \tag{21a}$$

$$\hat{S}^+(\xi)\hat{a}_2\hat{S}(\xi) = \hat{a}_2\cosh s - \hat{a}_1^+\exp(\mathrm{i}\theta)\sinh s, \tag{21b}$$

$$\hat{D}_j^+(z_j)\hat{a}_j\hat{D}_j(z_j) = \hat{a}_j + z_j \qquad (j = 1,2) \tag{21c}$$

及[26]

$$\hat{D}_j(z_j) = \exp(z_j\hat{a}_j^+)\exp(-z_j^*\hat{a}_j) \times \exp\left(-\frac{1}{2}z_j^*z_j\right) \qquad (j = 1,2), \tag{22a}$$

$$\hat{S}(\xi) = \exp(-\hat{a}_1^+\hat{a}_2^+\mathrm{e}^{\mathrm{i}\theta}\tanh s) \times \exp[-(\hat{a}_1^+\hat{a}_1 + \hat{a}_2^+\hat{a}_2 + 1)\ln(\cosh s)] \times \exp(\hat{a}_1\hat{a}_2\mathrm{e}^{-\mathrm{i}\theta}\tanh s). \tag{22b}$$

通过繁杂的运算后得

$$\gamma_{n_1 n_2} = \int_0^t \Big[ \frac{\mathrm{i}}{2}(\dot{z}_1 z_1^* + \dot{z}_2 z_2^* - z_1 \dot{z}_1^* - z_2 \dot{z}_2^*) - (n_1 + n_2 + 1)\dot{\theta}\sinh^2 s \Big]\mathrm{d}t, \tag{23a}$$

$$\begin{aligned}\beta_{n_1 n_2} = &-\int_0^t [(\omega_1 n_1 + \omega_2 n_2)\cosh^2 s + (\omega_1 n_2 + \omega_2 n_1 + \omega_1 + \omega_2)\sinh^2 s + \omega_1 z_1 z_1^* + \omega_2 z_2 z_2^*]\mathrm{d}t \\ &-\int_0^t \{G_{12}[z_1^* z_2^* \exp(\mathrm{i}\phi) + z_1 z_2 \exp(-\mathrm{i}\phi) - (n_1 + n_2 + 1)\cosh(\theta - \phi)\sinh 2s] \\ &+ G_1[z_1^* \exp(\mathrm{i}\varphi_1) + z_1 \exp(-\mathrm{i}\varphi_1)] \\ &+ G_2[z_2^* \exp(\mathrm{i}\varphi_2) + z_2 \exp(-\mathrm{i}\varphi_2)] + G(t)\}\mathrm{d}t,\end{aligned} \tag{23b}$$

其中 $\dot{z}_1 = \frac{\mathrm{d}z_1}{\mathrm{d}t}$, $\dot{z}_2 = \frac{\mathrm{d}z_2}{\mathrm{d}t}$, $\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$. 也可将 Lewis-Riesenfeld 相位写成

$$\alpha_{n_1 n_2} = \gamma_{n_1 n_2} + \beta_{n_1 n_2} = -(\varepsilon_1 n_1 + \varepsilon_2 n_2) + \sigma, \tag{24}$$

其中

$$\varepsilon_1 = \int_0^t [\dot{\theta}\sinh^2 s + \omega_1 \cosh^2 s + \omega_2 \sinh^2 s - G_{12}\cos(\theta - \phi)\sinh 2s]\mathrm{d}t, \tag{25a}$$

$$\varepsilon_2 = \int_0^t [\dot{\theta}\sinh^2 s + \omega_2 \cosh^2 s + \omega_1 \sinh^2 s - G_{12}\cos(\theta - \phi)\sinh 2s]\mathrm{d}t, \tag{25b}$$

$$\begin{aligned}\sigma = \int_0^t \{ &\frac{\mathrm{i}}{2}(\dot{z}_1 z_1^* + \dot{z}_2 z_2^* - z_1 \dot{z}_1^* - z_2 \dot{z}_2^*) - \omega_1 z_1 z_1^* - \omega_2 z_2 z_2^* - (\dot{\theta} + \omega_1 + \omega_2)\sinh^2 s \\ &+ G_{12}[\cos(\theta - \phi)\sinh 2s - z_1^* z_2^* \exp(\mathrm{i}\phi) - z_1 z_2 \exp(-\mathrm{i}\phi)] - G_1[z_1^* \exp(\mathrm{i}\varphi_1) \\ &+ z_1 \exp(-\mathrm{i}\varphi_1)] - G_2[z_2^* \exp(\mathrm{i}\varphi_2) + z_2 \exp(-\mathrm{i}\varphi_2)] - G(t)\}\mathrm{d}t.\end{aligned} \tag{26}$$

## 3. 时间演化算符

当 $t = 0$ 时,系统的量子态为

$$|\psi(0)\rangle = \hat{D}_1(z_{10})\hat{D}_2(z_{20})\hat{S}(\xi_0) \times \sum_{n_1, n_2} C_{n_1 n_2} |n_1, n_2\rangle, \tag{27}$$

所以

$$\sum_{n_1 n_2} C_{n_1 n_2} |n_1, n_2\rangle = \hat{S}^+(\xi_0)\hat{D}_2^+(z_{20})\hat{D}_1^+(z_{10})|\psi(0)\rangle. \tag{28}$$

将(24)式和(28)式代入(19)式得时间演化量子态为

$$\begin{aligned}|\psi(t)\rangle = &\exp(\mathrm{i}\sigma)\hat{D}_1(z_1)\hat{D}_2(Z_2)\hat{S}(\xi) \times \exp[-\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)] \\ &\times \hat{S}^+(\xi_0)\hat{D}_2^+(z_{20})\hat{D}_1^+(z_{10})|\psi(0)\rangle.\end{aligned} \tag{29}$$

这时,系统的时间演变算符为

$$\begin{aligned}\hat{U}(t,0) = &\exp(\mathrm{i}\sigma)\hat{D}_1(z_1)\hat{D}_2(z_2)\hat{S}(\xi) \times \exp[-\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)] \\ &\times \hat{S}^+(\xi_0)\hat{D}_2^+(z_{20})\hat{D}_1^+(z_{10}),\end{aligned} \tag{30a}$$

$$\begin{aligned}\hat{U}^+(t,0) = &\exp(-\mathrm{i}\sigma)\hat{D}_1(z_{10})\hat{D}_2(z_{20})\hat{S}(\xi_0) \times \exp[\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)] \\ &\times \hat{S}^+(\xi)\hat{D}_2^+(z_2)\hat{D}_1^+(z_1).\end{aligned} \tag{30b}$$

在 Heisenberg 表象中,任意算符的时间演变由下式给出:

$$\hat{A}(t) = \hat{U}^+(t,0)\hat{A}(0)\hat{U}(t,0). \tag{31}$$

## 4. 系统的压缩态

对于由(2)式给出的含时耦合谐振子系统,使用(30)式给出的时间演变算符及(21)式, 并注意到

$$\begin{aligned}&\exp[\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)]\hat{a}_j \times \exp[-\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)] = \exp(-\mathrm{i}\varepsilon_j)\hat{a}_j, \\ &\exp[\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)]\hat{a}_j^+ \times \exp[-\mathrm{i}(\varepsilon_1 \hat{a}_1^+ \hat{a}_1 + \varepsilon_2 \hat{a}_2^+ \hat{a}_2)] = \exp(\mathrm{i}\varepsilon_j)\hat{a}_j^+.\end{aligned} \tag{32}$$

经运算不难得出光子的产生和湮没算符的时间演变为

$$\hat{a}_1(t) = \hat{U}^+(t,0)\hat{a}_1\hat{U}(t,0) = h_1(t)\hat{a}_1 + h'_1(t)\hat{a}_2^+ + z_1 - z_{10}h_1(t) + z_{20}^* h'_1(t), \tag{33a}$$

$$\hat{a}_1^+(t) = \hat{U}^+(t,0)\hat{a}_1^+\hat{U}(t,0) = h_1^*(t)\hat{a}_1^+ + h_1'^*(t)\hat{a}_2 + z_1^* - z_{10}^* h_1^*(t) + z_{20}h_1'^*(t), \tag{33b}$$

$$\hat{a}_2(t) = \hat{U}^+(t,0)\hat{a}_2\hat{U}(t,0) = h_2(t)\hat{a}_2 + h'_2(t)\hat{a}_1^+ + z_2 - z_{20}h_2(t) + z_{10}^* h'_2(t), \tag{34a}$$

$$\hat{a}_2^+(t) = \hat{U}^+(t,0)\hat{a}_2^+\hat{U}(t,0) = h_2^*(t)\hat{a}_2^+ + h_2'^*(t)\hat{a}_1 + z_2^* - z_{20}^* h_2^*(t) + z_{10}h_2'^*(t), \tag{34b}$$

其中

$$h_1(t) = \cosh s_0 \cosh s \exp(-\mathrm{i}\varepsilon_1) - \sinh s_0 \sinh s \exp[\mathrm{i}(\varepsilon_2 + \theta - \theta_0)], \tag{35a}$$

$$h'_1(t) = \sinh s_0 \cosh s \exp(-\mathrm{i}(\varepsilon_1 - \theta_0)] - \cosh s_0 \sinh s \exp[\mathrm{i}(\varepsilon_2 + \theta)], \tag{35b}$$

$$h_2(t) = \cosh s_0 \cosh s \exp(-\mathrm{i}\varepsilon_2) - \sinh s_0 \sinh s \exp[\mathrm{i}(\varepsilon_1 + \theta - \theta_0)], \tag{35c}$$

$$h'_2(t) = \sinh s_0 \cosh s \exp(-\mathrm{i}(\varepsilon_2 - \theta_0)] - \cosh s_0 \sinh s \exp[\mathrm{i}(\varepsilon_1 + \theta)]. \tag{35d}$$

对于双模光场而言,通常所感兴趣的光场两正交分量算符可定义为[26,27]

$$\hat{X} = (\hat{a}_1 + \hat{a}_1^+ + \hat{a}_2 + \hat{a}_2^+)/2^{3/2},$$
$$\hat{Y} = (\hat{a}_1 - \hat{a}_1^+ + \hat{a}_2 - \hat{a}_2^+)/(2^{3/2}\mathrm{i}). \tag{36}$$

不难证明,它们满足对易关系

$$[\hat{X}, \hat{Y}] = \mathrm{i}/2. \tag{37}$$

因而 $\hat{X}$,$\hat{Y}$ 的量子涨落所满足的不确定关系式为

$$(\Delta\hat{X})^2(\Delta\hat{Y})^2 \geqslant 1/16. \tag{38}$$

如果光场两正交分量的某一分量的量子涨落值小于1/4(如$\Delta\hat{X})^2 < 1/4$),就说双模光场在该正交分量(如$\hat{X}$)上的量子噪声被压缩.从(29)式可以看出,含驱动项的双模耦合系统的量子态,一般可以处于一种所谓的广义双模压缩态[28],并随初态$|\psi(0)\rangle$选取的不同而不同.下面讨论初态处于不同情形下双模光场的压缩情况.

### 4.1. 初态处于相干态或真空态情形

设系统初始时刻处于相干态$|\psi(0)\rangle = |\alpha_1, \alpha_2\rangle$,则 $t>0$ 后系统将处于时间演化量子态

$$|\psi(t)\rangle = \hat{U}(t,0)|\alpha_1, \alpha_2\rangle. \tag{39}$$

利用(33),(34)和(36)式及 $\hat{a}_1|\alpha_1, \alpha_2\rangle = \alpha_1|\alpha_1, \alpha_2\rangle$,$\hat{a}_2|\alpha_1, \alpha_2\rangle = \alpha_2|\alpha_1, \alpha_2\rangle$,可算出光场 $\hat{X}$ 和 $\hat{Y}$ 分量的量子涨落值为

$$\begin{aligned}(\Delta\hat{X})^2 &= \langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2 \\ &= \frac{1}{8}(h_1 h_1^* + h'_1 h_1'^* + h_2 h_2^* + h'_2 h_2'^* \\ &\quad + h_1 h'_2 + h_1^* h_2'^* + h_2 h'_1 + h_2^* h_1'^*),\end{aligned} \tag{40a}$$

$$\begin{aligned}(\Delta\hat{Y})^2 &= \langle\hat{Y}^2\rangle - \langle\hat{Y}\rangle^2 \\ &= \frac{1}{8}(h_1 h_1^* + h'_1 h_1'^* + h_2 h_2^* + h'_2 h_2'^* \\ &\quad - h_1 h'_2 - h_1^* h_2'^* - h_2 h'_1 - h_2^* h_1'^*).\end{aligned} \tag{40b}$$

因设初始时刻系统处于相干态,故初始压缩因子 $s(t)|_{t=0} = s_0 = 0$,这时由(35)式得

$$\begin{aligned} h_1(t) &= \cosh s \exp(-\mathrm{i}\varepsilon_1), \\ h'_1(t) &= -\sinh s \exp[\mathrm{i}(\varepsilon_2 + \theta)], \\ h_2(t) &= \cosh s \exp(-\mathrm{i}\varepsilon_2), \\ h'_2(t) &= -\sinh s \exp[\mathrm{i}(\varepsilon_1 + \theta)]. \end{aligned} \tag{41}$$

将(41)代入(40)式得

$$\begin{aligned}(\Delta\hat{X})^2 &= \frac{1}{4}(\cosh 2s - \sinh 2s\cos\theta) \\ &= \frac{1}{4}\left[\exp(-2s)\cos^2\frac{\theta}{2} + \exp(2s)\sin^2\frac{\theta}{2}\right],\end{aligned} \tag{42a}$$

$$\begin{aligned}(\Delta\hat{Y})^2 &= \frac{1}{4}(\cosh 2s + \sinh 2s\cos\theta) \\ &= \frac{1}{4}\left[\exp(-2s)\sin^2\frac{\theta}{2} + \exp(2s)\cos^2\frac{\theta}{2}\right].\end{aligned} \tag{42b}$$

显然,当系统初态处于真空态$|\psi(0)\rangle = |0,0\rangle$时,也可得到与(42)式相同的结果.

由(42)式可知,$\hat{X}$ 分量被压缩的条件为 $\cos\theta > \tanh s$,$\hat{Y}$ 分量被压缩的条件为 $\cos\theta < -\tanh s$.当 $\theta = 0$ 或 $\pi$ 时,$(\Delta\hat{X})^2(\Delta\hat{Y})^2 = 1/16$,系统处于最小不确定态.其中,当 $\theta = 0$ 时,$(\Delta\hat{X})^2 = \frac{1}{4}\exp(-2s)$,$(\Delta\hat{Y})^2 = \frac{1}{4}\exp(2s)$,显然,这时 $\hat{X}$ 分量的量子涨落被压缩,$\hat{Y}$ 分量的量子涨落被放大;而当 $\theta = \pi$ 时,情况刚好相反.对于其他角度不确定量较大,极大条件是 $\theta = \pi/2$ 或 $3\pi/2$,极大值为$(\Delta\hat{X})^2(\Delta\hat{Y})^2 = \frac{1}{16}\cosh^2 2s$.

### 4.2. 初态处于 Fock 态情形

设系统初始时刻处于 Fock 态,并有$|\psi(0)\rangle = |n_1, n_2\rangle$,则 $t>0$ 以后系统将处于时间演化量子态

$$|\psi(t)\rangle = \hat{U}(t,0)|n_1, n_2\rangle. \tag{43}$$

利用(33),(34)和(36)式,可算出光场 $\hat{X}$ 和 $\hat{Y}$ 分量的量子涨落值为

$$\begin{aligned}(\Delta\hat{X})^2 &= \langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2 \\ &= \frac{1}{8}(h_1^* + h'_2)(h_1 + h_2'^*)(1 + 2n_1) \\ &\quad + \frac{1}{8}(h_1'^* + h_2)(h'_1 + h_2^*)(1 + 2n_2),\end{aligned} \tag{44a}$$

$$\begin{aligned}(\Delta\hat{Y})^2 &= \langle\hat{Y}^2\rangle - \langle\hat{Y}\rangle^2 \\ &= \frac{1}{8}(h_1^* - h'_2)(h_1 - h_2'^*)(1 + 2n_1)\end{aligned}$$

$$+\frac{1}{8}(h_1'^{*}-h_2)(h_1'-h_2^{*})(1+2n_2). \tag{44b}$$

设初始时刻系统无压缩,即压缩因子 $s(t)|_{t=0}=s_0=0$,这时(44)式简化为

$$\begin{aligned}(\Delta\hat{X})^2&=\frac{1}{4}(n_1+n_2+1)(\cosh 2s-\sinh 2s\cos\theta)\\&=\frac{1}{4}(n_1+n_2+1)\left[\exp(-2s)\cos^2\frac{\theta}{2}\right.\\&\quad\left.+\exp(2s)\sin^2\frac{\theta}{2}\right],\end{aligned}\tag{45a}$$

$$\begin{aligned}(\Delta\hat{Y})^2&=\frac{1}{4}(n_1+n_2+1)(\cosh 2s+\sinh 2s\cos\theta)\\&=\frac{1}{4}(n_1+n_2+1)\left[\exp(-2s)\sin^2\frac{\theta}{2}\right.\\&\quad\left.+\exp(2s)\cos^2\frac{\theta}{2}\right],\end{aligned}\tag{45b}$$

$$\begin{aligned}(\Delta\hat{X})^2(\Delta\hat{Y})^2&=\frac{1}{16}(n_1+n_2+1)^2[(\cosh^2 2s\\&\quad-\sinh^2 2s\cos^2\theta)].\end{aligned}\tag{46}$$

由(45)式可知,$\hat{X}$ 分量被压缩的条件为 $\cos\theta>\tanh s+\frac{n_1+n_2}{(n_1+n_2+1)\sinh 2s}$,$\hat{Y}$ 分量被压缩的条件为 $\cos\theta<-\left[\tanh s+\frac{n_1+n_2}{(n_1+n_2+1)\sinh 2s}\right]$.当 $\theta=0$ 或 $\pi$ 时,$(\Delta\hat{X})^2(\Delta\hat{Y})^2=\frac{1}{16}(n_1+n_2+1)^2$,取最小值,但系统不处于最小不确定态.其中,当 $\theta=0$,$s>\frac{1}{2}\ln(n_1+n_2+1)$时,$(\Delta\hat{X})^2=\frac{1}{4}(n_1+n_2+1)\exp(-2s)<\frac{1}{4}$,即 $\hat{X}$ 分量的量子涨落被压缩,而$(\Delta\hat{Y})^2=\frac{1}{4}(n_1+n_2+1)\exp(2s)$,显然,这时 $\hat{Y}$ 分量的量子涨落被放大;而当 $\theta=\pi$ 时,情况刚好相反.对于其他角度不确定量较大,极大条件是 $\theta=\pi/2$ 或 $3\pi/2$,极大值为 $(\Delta\hat{X})^2(\Delta\hat{Y})^2=\frac{1}{16}(n_1+n_2+1)^2\cosh^2 2s$.

## 5. 结论与讨论

从以上推导不难看出,无论系统初始时刻处于真空态、相干态还是 Fock 态,在由(13),(25),(35),(40)和(44)式计算$(\Delta\hat{X})^2$ 和$(\Delta\hat{Y})^2$ 过程中,均没有涉及到驱动项参数 $G_j(t)$,$\varphi_j(t)$,因此不难得出光场压缩态两正交分量 $\hat{X}$ 和 $\hat{Y}$ 的量子涨落值与驱动项无关的结论.另外,由(29),(42)和(45)式可以看出,系统随时间演化的量子态$|\psi(t)\rangle$,一般处于广义双模压缩态,其量子涨落与系统所处的初态$|\psi(0)\rangle$有关,对于初态处于真空态和相干态的耦合系统,其量子涨落性质相同,并可以处于最小不确定态,但初态处于 Fock 态的耦合系统,其压缩态两正交分量的量子涨落总是大于最小不确定态的量子涨落.不过,不管系统初始时刻处于哪一个状态,适当调节哈密顿量 $\hat{H}$ 中的非驱动项参数,可达到调整双模压缩态压缩深度以及压缩方位角的目的.如调节参数使得 $\theta=0$ 或 $\pi$ 时,总可以使其中的一个正交分量的量子涨落得到最有效的压缩.

需要指出的是,本文所给出的时间演化量子态(29)式和时间演化算符(30)式,还可以计算由(2)式所描述的系统的其他物理效应,如玻色子分布、聚束和反聚束效应、信噪比以及光场不同模式之间非经典性质的转移[16]等.此外,由于(2)式假设的各参量可以是任意的含时函数,因此该模型具有较普遍的意义,在特殊情况下,本文的解可退化为前人已有的结果.如当(2)式中取 $\omega_1=\Omega_1^0$,$\omega_2=\Omega_2^0$,$G_{12}=k\varepsilon/2$,$\phi(t)=\pi/2-2\Omega t-\phi$,$G_1=G_2=G=0$,$2\Omega=\omega_1+\omega_2$ 可简化为文献[4]在($\Delta\Omega_i^0=0$)的模型(其中 $\phi$ 为常数),这时方程(13),(14)的一个特解为 $s(t)=k\varepsilon t/2$,$\theta(t)=\pi-2\Omega t-\phi$,$z_1=z_2=z_{10}=z_{20}=0$,将其代入(25)和(26)式得(设 $s_0=0$),$\varepsilon_1=\omega_1 t$,$\varepsilon_2=\omega_2 t$,$\sigma=0$,最后由(30a)得时间演化算符为

$$\begin{aligned}\hat{U}(t,0)&=\hat{S}(\xi)\exp[-\mathrm{i}(\omega_1\hat{a}_1^+\hat{a}_1+\omega_2\hat{a}_2^+\hat{a}_2)t]\\&=\exp\{(k\varepsilon t/2)[-\hat{a}_1\hat{a}_2\mathrm{e}^{\mathrm{i}(2\Omega t+\phi)}+\hat{a}_1^+\hat{a}_2^+\mathrm{e}^{-\mathrm{i}(2\Omega t+\phi)}]\}\\&\quad\times\exp[-\mathrm{i}(\omega_1\hat{a}_1^+\hat{a}_1+\omega_2\hat{a}_2^+\hat{a}_2)t]\\&=\exp[-\mathrm{i}(\omega_1\hat{a}_1^+\hat{a}_1+\omega_2\hat{a}_2^+\hat{a}_2)t]\\&\quad\times\exp[-(k\varepsilon t/2)(\hat{a}_1\hat{a}_2\mathrm{e}^{\mathrm{i}\phi}-\hat{a}_1^+\hat{a}_2^+\mathrm{e}^{-\mathrm{i}\phi})].\end{aligned}\tag{47}$$

由(35),(33)和(34)式得

$$\begin{aligned}\hat{a}_1(t)&=[\hat{a}_1\cosh(k\varepsilon t/2)+\hat{a}_2^+\sinh(k\varepsilon t/2)\\&\quad\times\exp(-\mathrm{i}\phi)]\exp(-\mathrm{i}\omega_1 t),\end{aligned}\tag{48a}$$

$$\begin{aligned}\hat{a}_2(t)&=[\hat{a}_2\cosh(k\varepsilon t/2)+\hat{a}_1^+\sinh(k\varepsilon t/2)\\&\quad\times\exp(-\mathrm{i}\phi)]\exp(-\mathrm{i}\omega_2 t).\end{aligned}\tag{48b}$$

显然(47)式与文献[4]的(1.6),(1.11)式相同.当令(48)式中 $k\varepsilon/2=\chi$,$\phi=0$,则可得到与文献[22]的(5.30)和(5.31)式一致的结果.当在(2)式中取 $\phi(t)=\pi/2-\omega t$,$G_{12}(t)=-g$,$G_1=G_2=0$,$G=(\omega_1+\omega_2)/2$ 可简化为文献[5]的模型.在 $\Omega=\omega-(\omega_1+\omega_2)\neq0$ 的失谐情况下,可求出方程(13)的解析显式

解，代入(30a)可得到与文献[5]的(10)式相同的演化算符. 对于如何得出方程(13)在失谐条件下的解析解，以及在有驱动项存在的情形下，一些其他非经典性质的讨论，将在另文中涉及.


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# The evolution and two-mode squeezed states of the time-dependent two coupled harmonic oscillators

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(Received 7 April 2004; revised manuscript received 21 May 2004)

Abstract

A closed solution of the Schrödinger equation for a time-dependent two coupled harmonic oscillators including a driving part and a two-mode coupled part is derived by adopting the Lewis-Riesenfeld invariant theory and properly chosen Hermitian invariant operators. The evolution operator of the system and the condition for generating two-mode squeezed states are obtained. It is shown that the quantum fluctuation of squeezed states of the system is independent of the driving part, but depends on the initial states of the system.

**Keywords**: time-dependent coupled harmonic oscillators, Lewis-Riesenfeld invariant theory, two-mode squeezed states

**PACC**: 0365, 4250