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耦合 Klein-Gordon-Schrödinger 方程新的精确解

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用函数变换法将耦合 Klein-Gordon-Schrödinger 方程化成可求解的不定积分形式,进而求出其精确解,包括有理函数型解、三角函数型解、双曲函数型解,以及椭圆函数型解.所得结果包含了文献中用齐次平衡法所得的解为特例.

关键词:耦合 Klein-Gordon-Schrödinger 方程,精确解,椭圆函数,双曲函数
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1.引 言

耦合 Klein-Gordon-Schrödinger 方程(以下简称 cKGS 方程)

$$\mathrm{i}\frac{\partial}{\partial t}\psi+\frac{1}{2}(\psi_{xx}+\psi_{yy}+\psi_{zz})+\phi\psi=0, \quad (1)$$

$$\frac{\partial^2}{\partial t^2}\phi-(\phi_{xx}+\phi_{yy}+\phi_{zz})+m^2\phi-|\psi|^2=0, \quad (2)$$

描述了一个保守的复原子核场 ψ 与一个实中性介子场 ϕ 的相互作用的古典 Yukawa 型.ψ 和 ϕ 均为标量场,m 为介子的质量[1,2].Zhang[3] 用齐次平衡法得到了方程(1)和(2)的显示解

$$\phi(x,y,z,t)=\frac{3}{4}k^2\mathrm{sech}^2\left(\frac{kx+ly+pz\pm mt}{2}\right), \quad (3)$$

$$\varphi(x,y,z,t)=\frac{3}{4}k^2\exp\left[\mathrm{i}\left(\alpha_1x+\beta_1y+\gamma_1z+\frac{1}{2}\delta t\right)\right]\times\mathrm{sech}^2\left(\frac{kx+ly+pz\pm mt}{2}\right), \quad (4)$$

其中

$$k^2=\frac{6}{a}m^2,\quad \alpha_1k+\beta_1l+\gamma_1p=\pm m,$$
$$\alpha_1^2+\beta_1^2+\gamma_1^2+\delta=2m^2,\quad k^2+l^2+p^2=2m^2, \quad (5)$$

$a=3$(文献[3]中 a 是任一大于零的常数,这有误.将(3)和(4)式代入(1)和(2)式,利用(5)式即得 $a=3$)

本文利用函数变换法化方程(1)和(2)为可解的不定积分形式,求出其精确解,包括有理函数型解、三角函数型解、双曲函数型解和椭圆函数型解,并以(3)和(4)式作为特例.

2.cKGS 方程(1)和(2)的精确解

为求方程(1)和(2)的精确解,如文献[3],假设

$$\psi(x,y,z,t)=\phi(x,y,z,t)\exp[\mathrm{i}g(x,y,z,t)], \quad (6)$$

$$g(x,y,z,t)=\alpha_1x+\beta_1y+\gamma_1z+\frac{1}{2}\delta t, \quad (7)$$

则方程(1)和(2)成为

$$\phi_{tt}-(\phi_{xx}+\phi_{yy}+\phi_{zz})+m^2\phi-\phi^2=0, \quad (8)$$

$$\frac{1}{2}(\phi_{xx}+\phi_{yy}+\phi_{zz})-\frac{1}{2}\left(\alpha_1^2+\beta_1^2+\gamma_1^2+\frac{1}{2}\delta\right)\phi+\phi^2=0, \quad (9)$$

$$\phi_t+\alpha_1\phi_x+\beta_1\phi_y+\gamma_1\phi_z=0. \quad (10)$$

设(8)—(10)式的行波解为

$$\phi(x,y,z,t)=\phi(\xi), \quad (11)$$

其中

$$\xi=kx+ly+pz+\omega t, \quad (12)$$

下面求方程(1)和(2)的形如(6),(7),(11)和(12)式的精确解.将(11)式代入(8)—(10)式,有

$$(\omega^2-(k^2+l^2+p^2))\phi_{\xi\xi}+m^2\phi-\phi^2=0, \quad (13)$$

$$\frac{1}{2}(k^2+l^2+p^2)\phi_{\xi\xi}-\frac{1}{2}(\alpha_1^2+\beta_1^2+\gamma_1^2+\delta)\phi+\phi^2=0, \quad (14)$$

$$(\omega+\alpha_1k+\beta_1l+\gamma_1p)\phi_\xi=0. \quad (15)$$

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由(13)和(14)式相对照,再令(15)式中 $\phi_\xi \neq 0$ 知,若

$$k^2 + l^2 + p^2 = 2\omega^2, \tag{16}$$

$$\frac{1}{2}(\alpha_1^2 + \beta_1^2 + \gamma_1^2 + \delta) = m^2, \tag{17}$$

$$\omega + \alpha_1 k + \beta_1 l + \gamma_1 p = 0 \tag{18}$$

成立,则只需考虑:

$$\phi_{\xi\xi} = -\frac{1}{\omega^2}\phi^2 + \frac{m^2}{\omega^2}\phi. \tag{19}$$

对(19)式积分一次,有

$$\phi_\xi^2 = -\frac{2}{3\omega^2}\phi^3 + \frac{m^2}{\omega^2}\phi^2 + c, \tag{20}$$

c 为积分常数. 令

$$W = \left(-\frac{2}{3\omega^2}\right)^{1/3}\phi, \quad \xi_1 = \left(-\frac{2}{3\omega^2}\right)^{1/3}\xi, \tag{21}$$

则(20)式写成

$$\left(\frac{\mathrm{d}W}{\mathrm{d}\xi_1}\right)^2 = F(W) = W^3 + aW^2 + c, \tag{22}$$

其中

$$a = \frac{m^2}{\omega^2}\left(-\frac{2}{3\omega^2}\right)^{-2/3}. \tag{23}$$

下面再分两种情形讨论:

情形 1 $\Delta = 0$ 这里 $\Delta = -27\left(\frac{2}{27}a^3 + c\right)^2 - 4\times\left(-\frac{a^2}{3}\right)^3$ 为 $F(W)$ 的判别式. 此时 $F(W) = 0$ 有重根,设为 $F(W) = (W-\alpha)^2(W-\beta)$,由(22)式知,当 $W > \beta$ 时,有以下三种情形:

$$\pm(\xi_1 - \xi_0) = \int\frac{\mathrm{d}W}{(W-\alpha)\sqrt{W-\beta}} = \frac{1}{\sqrt{\alpha-\beta}}\ln\left|\frac{\sqrt{W-\beta}-\sqrt{\alpha-\beta}}{\sqrt{W-\beta}+\sqrt{\alpha-\beta}}\right|, \quad \alpha > \beta, \tag{24}$$

$$\pm(\xi_1 - \xi_0) = \frac{2}{\sqrt{\beta-\alpha}}\arctan\sqrt{\frac{W-\beta}{\beta-\alpha}}, \quad \alpha < \beta, \tag{25}$$

$$\pm(\xi_1 - \xi_0) = -\frac{2}{\sqrt{W-\beta}}, \quad \alpha = \beta, \tag{26}$$

其中 ξ_0 为积分常数. 相应地,(19)式的解为

$$\phi_1 = \left(-\frac{2}{3\omega^2}\right)^{-1/3} \times\left[\left(\frac{2\sqrt{\alpha-\beta}}{1+\exp\left[\pm\sqrt{\alpha-\beta}\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0)\right]} - \sqrt{\alpha-\beta}\right)^2 + \beta\right] = \left(-\frac{2}{3\omega^2}\right)^{-1/3} \times\left[(\alpha-\beta)\mathrm{th}^2\left(\frac{1}{2}\sqrt{\alpha-\beta}\left(-\frac{2}{3\omega^2}\right)^{1/3} \times(\xi-\xi_0)\right)+\beta\right], \quad \alpha > \beta, \tag{27}$$

$$\phi_2 = \left(-\frac{2}{3\omega^2}\right)^{-1/3} \times\left[\left(\frac{2\sqrt{\alpha-\beta}}{1-\exp\left[\pm\sqrt{\alpha-\beta}\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0)\right]} - \sqrt{\alpha-\beta}\right)^2 + \beta\right] = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[(\alpha-\beta)\mathrm{cth}^2\left(\frac{1}{2}\sqrt{\alpha-\beta} \times\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0)\right)+\beta\right], \quad \alpha > \beta, \tag{28}$$

$$\phi_3 = \left(-\frac{2}{3\omega^2}\right)^{-1/3}(\beta-\alpha) \times\sec^2\left(\frac{\sqrt{\beta-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0)\right) + \left(-\frac{2}{3\omega^2}\right)^{1/3}\alpha, \quad \alpha < \beta, \tag{29}$$

$$\phi_4 = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[\beta + \frac{4}{\left(-\frac{2}{3\omega^2}\right)^{2/3}(\xi-\xi_0)^2}\right], \quad \alpha = \beta. \tag{30}$$

(27)—(30)式中积分常数 ξ_0 已经重新标度,仍用 ξ_0 表示,下同. 这里(27)和(28)式是(19)式的孤波解,(29)式是三角函数型周期解,(30)式是其有理函数解,相应地可以写出方程(1)和(2)的显式精确解

$$\phi_1(x,y,z,t) = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[(\alpha-\beta)\mathrm{th}^2\left(\frac{1}{2}\sqrt{\alpha-\beta}\left(-\frac{2}{3\omega^2}\right)^{1/3} \times(\xi-\xi_0)\right)+\beta\right], \quad \alpha > \beta, \tag{31}$$

$$\psi_1(x,y,z,t) = \phi_1(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1 x + \beta_1 y + \gamma_1 z + \frac{1}{2}\delta t\right)\right], \tag{32}$$

$\phi_2(x,y,z,t)$

$$= \left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[(\alpha-\beta)\mathrm{cth}^2\left(\frac{1}{2}\sqrt{\alpha-\beta}\left(-\frac{2}{3\omega^2}\right)^{1/3}\times(\xi-\xi_0)\right)+\beta\right],\quad \alpha>\beta, \tag{33}$$

$$\psi_2(x,y,z,t) = \phi_2(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1 x+\beta_1 y+\gamma_1 z+\frac{1}{2}\delta t\right)\right], \tag{34}$$

$$\phi_3(x,y,z,t) = \left(-\frac{2}{3\omega^2}\right)^{-1/3}(\beta-\alpha)\sec^2\left[\frac{\sqrt{\beta-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{1/3}\times(\xi-\xi_0)\right]+\left(-\frac{2}{3\omega^2}\right)^{1/3}\alpha,\quad \alpha<\beta, \tag{35}$$

$$\psi_3(x,y,z,t) = \phi_3(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1 x+\beta_1 y+\gamma_1 z+\frac{1}{2}\delta t\right)\right], \tag{36}$$

$$\phi_4 = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\times\left(\beta+\frac{4}{\left(-\frac{2}{3\omega^2}\right)^{-2/3}(\xi-\xi_0)^2}\right),\quad \alpha=\beta, \tag{37}$$

$$\psi_4(x,y,z,t) = \phi_4(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1 x+\beta_1 y+\gamma_1 z+\frac{1}{2}\delta t\right)\right]. \tag{38}$$

注 1 事实上,(24)和(25)式对应于 $a\neq0$;(26)式对应于 $a=0$,此时 $F(W)=W^3$,即 $\alpha=\beta=0$.

注 2 当 $c=0$ 时,有 $\Delta=0$. 此时有 $\alpha=0,\beta=-\frac{m^2}{\omega^2}\left(-\frac{2}{3\omega^2}\right)^{-2/3}$. 若实数 $m\neq0$,则 $\alpha\neq\beta$. 由(27)—(29)式,有

$$\phi(\xi) = \frac{3}{2}m^2\left[1-\mathrm{th}^2\left(\frac{m}{\omega}(\xi-\xi_0)\right)\right],\quad \omega^2>0, \tag{39}$$

$$\phi(\xi) = \frac{3}{2}m^2\left[1-\mathrm{cth}^2\left(\frac{m}{\omega}(\xi-\xi_0)\right)\right],\quad \omega^2>0, \tag{40}$$

$$\phi(\xi) = \frac{3}{2}m^2\sec^2\left(\frac{1}{2}\sqrt{-\frac{m^2}{\omega^2}}(\xi-\xi_0)\right),\quad \omega^2<0. \tag{41}$$

在(39)式中,令 $\omega=m,k^2=2m^2,\xi_0=0$,有

$$\phi(\xi) = \frac{3}{4}k^2\mathrm{sech}^2\left(\frac{1}{2}(kx+ly+pz\pm\omega t)\right). \tag{42}$$

此式即为(3)式. 因此文献[3]中的结果只是(31)和(32)式中的参数取特殊值时的特例.

情形 2 $\Delta\neq0$,再分两种情形讨论.

1) $\Delta>0,a\neq0$. 此时 $F(W)=0$ 有三个不同的实根 $\alpha<\beta<\gamma$,当 $\alpha<W<\beta$ 时,作变量替换

$$W = \alpha+(\beta-\alpha)\sin^2\varphi. \tag{43}$$

由(22)式,有

$$\pm(\xi_1-\xi_0) = \int\frac{\mathrm{d}W}{\sqrt{F(W)}} = \int\frac{2(\beta-\alpha)\sin\varphi\cos\varphi\,\mathrm{d}\varphi}{\sqrt{\gamma-\alpha}(\beta-\alpha)\sin\varphi\cos\varphi\sqrt{1-k^2\sin^2\varphi}} = \frac{2}{\sqrt{\gamma-\alpha}}\int\frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}}, \tag{44}$$

其中 $k^2=\frac{\beta-\alpha}{\gamma-\alpha}$. 由(44)式,有

$$\pm\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0) = \int\frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}}. \tag{45}$$

由(43)和(45)式及 Jacobi 椭圆正弦函数的定义[5]知,

$$\mathrm{sn}^2\left(\pm\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0),k\right) = \sin^2\varphi = \frac{W-\alpha}{\beta-\alpha}, \tag{46}$$

即

$$W = \alpha+(\beta-\alpha)\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}(\xi_1-\xi_0),k\right), \tag{47}$$

相应地方程(19)的椭圆周期解为

$$\phi_5(\xi) = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\alpha+\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\beta-\alpha)\times\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right). \tag{48}$$

由此得 cKGS 方程(1)和(2)的含椭圆函数的精确解为

$$\phi_5(\xi) = \left(-\frac{2}{3\omega^2}\right)^{-1/3}\alpha+\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\beta-\alpha)\times\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right), \tag{49}$$

$$\psi_5(x,y,z,t) = \phi_5(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1 x+\beta_1 y+\gamma_1 z+\frac{1}{2}\delta t\right)\right]. \tag{50}$$

若 $W>\gamma$,则作变量代换

$$W = \frac{-\beta\sin^2\phi+\gamma}{\cos^2\phi}, \tag{51}$$

相似地，方程(19)的解为

$$\phi_6(\xi)=\left(-\frac{2}{3\omega^2}\right)^{-1/3}\times\frac{-\beta\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right)+\gamma}{\mathrm{cn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right)},\tag{52}$$

其中 $k^2=\frac{\beta-\alpha}{\gamma-\alpha}$，由此得 cKGS 方程(1)和(2)的含椭圆函数的精确解为

$$\phi_6(\xi)=\left(-\frac{2}{3\omega^2}\right)^{-1/3}\times\frac{-\beta\mathrm{sn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right)+\gamma}{\mathrm{cn}^2\left(\frac{\sqrt{\gamma-\alpha}}{2}\left(-\frac{2}{3\omega^2}\right)^{-1/3}(\xi-\xi_0),k\right)},\tag{53}$$

$$\psi_6(x,y,z,t)=\phi_6(x,y,z,t)\times\exp\left[\mathrm{i}\left(\alpha_1x+\beta_1y+\gamma_1z+\frac{1}{2}\delta t\right)\right].\tag{54}$$

2) $\Delta<0$，此时 $F(W)=0$ 只有一个实根，设为 $F(W)=(W-\alpha)(W^2+p_0W+q_0)$，且 $p_0^2-4q_0<0$，当 $W>\alpha$ 时，作变量替换

$$W=\alpha+\sqrt{\alpha^2+p_0\alpha+q_0}\tan^2\frac{\varphi}{2}.\tag{55}$$

由(22)式，有

$$\pm(\xi_1-\xi_0)=\int\frac{\mathrm{d}W}{\sqrt{(W-\alpha)(W^2+p_0W+q_0)}}=\int\frac{\sqrt{\alpha^2+p_0\alpha+q_0}\dfrac{\tan\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2}}\mathrm{d}\varphi}{(\alpha^2+p_0\alpha+q_0)^{3/4}\dfrac{\tan\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2}}\sqrt{1-k^2\sin^2\varphi}}=\frac{1}{(\alpha^2+p_0\alpha+q_0)^{1/4}}\int\frac{\mathrm{d}\varphi}{\sqrt{1-k^2\sin^2\varphi}},\tag{56}$$

其中 $k^2=\frac{1}{2}\left(1-\frac{\alpha+\frac{p_0}{2}}{\sqrt{\alpha^2+p_0\alpha+q_0}}\right)$. 由(56)式及 Jacobi 椭圆余弦函数的定义[5]知，

$$\mathrm{cn}((\alpha^2+p_0\alpha+q_0)^{1/4}(\xi_1-\xi_0),k)=\cos\varphi.\tag{57}$$

又由变量代换(55)式，有

$$\cos\varphi=\frac{2\sqrt{\alpha^2+p_0\alpha+q_0}}{W-\alpha+\sqrt{\alpha^2+p_0\alpha+q_0}}-1.\tag{58}$$

由(57)和(58)式知，当 $W>\alpha$ 时，有

$$W=\alpha+\frac{2\sqrt{\alpha^2+p^0\alpha+q_0}}{1+\mathrm{cn}((\alpha^2+p_0\alpha+q_0)^{1/4}(\xi_1-\xi_0),k)}-\sqrt{\alpha^2+p_0\alpha+q_0}.\tag{59}$$

相应地，方程(19)的椭圆周期解为

$$\phi_7(\xi)=\left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[\alpha+\frac{2\sqrt{\alpha^2+p_0\alpha+q_0}}{1+\mathrm{cn}\left((\alpha^2+p_0\alpha+q_0)^{1/4}\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0),k\right)}-\sqrt{\alpha^2+p_0\alpha+q_0}\right].\tag{60}$$

由(60)式可写出 cKGS 方程(1)和(2)的另一含椭圆函数的精确解

$$\phi_7(\xi)=\left(-\frac{2}{3\omega^2}\right)^{-1/3}\left[\alpha+\frac{2\sqrt{\alpha^2+p_0\alpha+q_0}}{1+\mathrm{cn}\left((\alpha^2+p_0\alpha+q_0)^{1/4}\left(-\frac{2}{3\omega^2}\right)^{1/3}(\xi-\xi_0),k\right)}-\sqrt{\alpha^2+p_0\alpha+q_0}\right],\tag{61}$$

$$\psi_7(x,y,z,t)=\phi_7(x,y,z,t)\exp\left[\mathrm{i}\left(\alpha_1x+\beta_1y+\gamma_1z+\frac{1}{2}\delta t\right)\right].\tag{62}$$

3. 结 束 语

尽管已经建立了许多求解非线性方程的方法，如齐次平衡法[4,6,7]、试探函数法[8,9]、双曲函数展开法[10—12]、椭圆函数展开法[13—15]，函数变换法[16]、sine-cosine 法[17—19]等，这一领域依然充满挑战. 本文的方法得到的结果是系统的，得到了在条件(16)—

(18)式下,形如(6),(7),(11)和(12)式的全部精确解 ϕ_i,ψ_i($i=1,\cdots,7$).

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New exact solutions of coupled Klein-Gordon-Schrödinger equations

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Abstract

By using a function-transformation method, coupled Klein-Gordorn-Schrödinger equations are reduced to the integrals that can be solved with elementary functions and elliptic functions. Its exact solutions that include discontinuous solutions, hyperbolic function type solutions and elliptic function type solutions, and a special solutions obtained in a reference, are given.

Keywords: coupled Klein-Gordon-Schrödinger equations, exact solutions, elliptic function, hyperbolic function

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