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热带海-气耦合振子的摄动解*

莫嘉琪[1)]† 林一骅[2)] 林万涛[2)]

[1)](安徽师范大学数学系,芜湖 241000)

[2)](中国科学院大气物理研究所,北京 100029)

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研究了一类热带海-气耦合振子的模型.利用摄动方法求出了相应模式的渐近解.

关键词:非线性,摄动,海气耦合模式,厄尔尼诺-南方涛动

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1.引　　言

海气相互作用,是大气和海洋这两种介质同时相互对来自大气对海洋强迫的响应和海洋对大气强迫的响应.这是两个同时发生且相互影响的过程.例如,厄尔尼诺/拉尼娜和南方涛动(ENSO)是分别发生在热带太平洋大气和海洋中的异常现象.它的发生,严重地影响全球各地区气候和生态等方面的变化.它的气候异常,带来了许多灾害,对全球的经济发展和人类生活都有严重的影响.因此对它的规律和预防的研究,为当前国际学术界所关注的对象.

赤道地区大气和海洋中的波动是大气和海洋中的一类独特的运动形式.热带海洋的表层运动,基本上可以看成是表层海水对风应力的响应,而这种响应正是热带海气耦合过程的一个重要方面.这种运动形式是由于在赤道附近的运动速度,再加上地球自转的影响而出现的.这就引导人们更有兴趣去研究热带海气耦合系统.

许多学者已经用了不同的方法对大气、海洋的耦合系统的局部性和整体性的性态作了多方位的研究[1—7].在大气物理、海洋气候、动力系统等方面,作者也研究了某些相关的海气振子问题[8—20].

摄动问题在国际学术界的研究中是一个十分关注的对象[21].近十年来许多近似方法被发展,包括平均法,边界层法,渐近展开法和多重尺度法等[22,23].本文是讨论一类海-气耦合振子模型.在一定的情况下,从数学物理理论的角度,利用摄动方法较简捷地得到了相应问题的渐近解,从而可以较直接地讨论问题某些相关物理量的定性、定量方面的特征.

2.耦合振子的基本模型

考虑如下海洋对大气的加热及风应力的作用,描述赤道地区大气和海洋中的耦合运动的方程[8]

$$\frac{\partial u^{\mathrm{a}}}{\partial t}+\frac{\partial \phi}{\partial x}+Du^{\mathrm{a}}=0,$$

$$\frac{\partial \phi}{\partial t}+C_{\mathrm{a}}^{2}\frac{\partial u^{\mathrm{a}}}{\partial x}+D\phi=-Q,$$

$$\beta yu^{\mathrm{a}}+\frac{\partial \phi}{\partial y}=0,$$

$$\frac{\partial u^{s}}{\partial t}+\frac{\partial \eta}{\partial x}+du^{\mathrm{a}}=\tau^{x},$$

$$\frac{\partial \eta}{\partial t}+C_{\mathrm{s}}^{2}\frac{\partial u^{s}}{\partial x}+d\eta=0,$$

$$\beta yu^{s}+\frac{\partial \eta}{\partial y}=0,$$

其中 u^{a} 和 u^{s} 分别为大气和海洋的温度距平,ϕ 为海洋的重力位势,η 为海洋的混合层厚度,C_{a} 和 C_{s} 分别为大气和海洋重力波速,β 为 Rossby 参数,Q 为海洋对大气的加热率,τ^{x} 为风应力,D 和 d 分别为大气和海洋的 Rayleigh 和 Newton 冷却系数.

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†E-mail:mojiaqi@mail.ahnu.edu.cn

海洋对大气的加热率 Q 和风应力 τ^x 一般的耦合关系为 $Q = Q(u^s, T(\eta), \varepsilon)$，$\tau^x = \tau^x(u^a, \phi, \varepsilon)$ 其中 T 为海温距平 SST. 我们假设 Q，τ^x 为充分光滑的函数，且 $Q = O(\varepsilon)$，$\tau^x = O(\varepsilon)$，其中 ε 为正的小参数.

引入无量纲特征量

$$x = L_x x',\ t = (L_x / C_s) t',$$
$$y_a = (C_a/2\beta)^{1/2} y',\ y_s = (C_s/2\beta)^{1/2} y',$$
$$u^a = C_a u'^a,\ u^s = C_s u'^s,\ \phi = C_a^2 \phi',\ \eta = C_s^2 \eta',$$

其中 $L_x = C_s/2\beta$. 因此原系统可化为无量纲形式（符号′已省略）

$$\frac{\partial u^a}{\partial t} + \alpha^{-1}\frac{\partial \phi}{\partial x} + \tilde{D} u^a = 0,$$
$$\frac{\partial \phi}{\partial t} + \alpha^{-1}\frac{\partial u^a}{\partial x} + \tilde{D}\phi = -\varepsilon\tilde{Q}, \tag{1}$$
$$\frac{\partial u^s}{\partial t} + \frac{\partial \eta}{\partial x} + \tilde{d} u^a = \varepsilon\tilde{\tau}^x,$$
$$\frac{\partial \eta}{\partial t} + \frac{\partial u^s}{\partial x} + \tilde{d}\phi = 0, \tag{2}$$
$$\frac{1}{2} y u^a + \frac{\partial \phi}{\partial y} = 0,$$
$$\frac{1}{2} y u^s + \frac{\partial \eta}{\partial y} = 0, \tag{3}$$

其中

$$\alpha = C_s / C_a,\ \tilde{D} = D/(2\beta C_a),\ \tilde{d} = C_s^2 d/(2\beta),$$
$$\varepsilon\tilde{Q} = Q/(2\beta C_a),\ \varepsilon\tilde{\tau}^x = \tau^x/(2\beta).$$

3. 系统的摄动解

下面用摄动理论和方法来求得系统(1)—(3)的渐近解. 设

$$u^a = \sum_{i=0}^{\infty} u_i^a \varepsilon^i,\ u^s = \sum_{i=0}^{\infty} u_i^s \varepsilon^i,$$
$$\phi = \sum_{i=0}^{\infty} \phi_i \varepsilon^i,\ \eta = \sum_{i=0}^{\infty} \eta_i \varepsilon^i. \tag{4}$$

将(4)式代入(1)—(3)式，按 ε 展开非线性项 Q，τ^x，合并 ε 的同次幂项，分别令 ε^i，$i = 0,1,2,\cdots$，的系数为零. 对应于 $i = 0$，有

$$L_1(u_0^a, \phi_0) = 0,\ L_2(u_0^a, \phi_0) = 0, \tag{5}$$
$$L_3(u_0^s, \eta_0) = 0,\ L_4(u_0^s, \eta_0) = 0, \tag{6}$$
$$L_5(u_0^a, \phi_0) = 0,\ L_6(u_0^s, \eta_0) = 0. \tag{7}$$

对应于 $i = 1,2,\cdots$，有

$$L_1(u_i^a, \phi_i) = 0,\ L_2(u_i^a, \phi_i) = -Q_{i-1}, \tag{8}$$
$$L_3(u_i^s, \eta_i) = \tau_{i-1}^x,\ L_4(u_i^s, \eta_i) = 0, \tag{9}$$
$$L_5(u_i^a, \eta_i) = 0,\ L_6(u_i^s, \eta_i) = 0, \tag{10}$$

其中算子 L_i 为

$$L_1(u^a, \phi) = \frac{\partial u^a}{\partial t} + \alpha^{-1}\frac{\partial \phi}{\partial x} + \tilde{D} u^a,$$
$$L_2(u^a, \phi) = \frac{\partial \phi}{\partial t} + \alpha^{-1}\frac{\partial u^a}{\partial x} + \tilde{D}\phi,$$
$$L_3(u^s, \eta) = \frac{\partial u^s}{\partial t} + \frac{\partial \eta}{\partial x} + \tilde{d} u^s,$$
$$L_4(u^s, \eta) = \frac{\partial \eta}{\partial t} + \frac{\partial u^s}{\partial x} + \tilde{d}\eta,$$
$$L_5(u^a, \phi) = \frac{1}{2} y u^a + \frac{\partial \phi}{\partial y},$$
$$L_6(u^s, \eta) = \frac{1}{2} y u^s + \frac{\partial \eta}{\partial y},$$

而 Q_i，τ_i^x，$i = 1,2,\cdots$，为

$$Q_i = \frac{1}{i!}\left[\frac{\partial^i}{\partial \varepsilon^i}\left(\tilde{Q}\left(\sum_{j=0}^{\infty} u_j^a \varepsilon^j, \sum_{j=0}^{\infty} \eta_j \varepsilon^j, \varepsilon\right)\right]_{\varepsilon=0},$$
$$\tau_i^x = \frac{1}{i!}\left[\frac{\partial^i}{\partial \varepsilon^i}\left(\tilde{\tau}^x\left(\sum_{j=0}^{\infty} u_j^a \varepsilon^j, \sum_{j=0}^{\infty} \eta_j \varepsilon^j, \varepsilon\right)\right]_{\varepsilon=0}.$$

由两组一阶偏微分系统(5)和(6)，并考虑到方程(7)，不难得到对应的解(u_0^a, ϕ_0)，(u_0^a, ϕ_0)分别为

$$u_0^a = \frac{1}{2}\left[\exp\left(\tilde{D}t + \Psi_{01}(x - \alpha^{-1}t) - \frac{y^2}{4}\right)\right.$$
$$\left. + \exp\left((\tilde{D}t + \Psi_{02}(x + \alpha^{-1}t) + \frac{y^2}{4}\right)\right], \tag{11}$$
$$u_0^s = \frac{1}{2}\left[\exp\left(\tilde{d}t + \Psi_{03}(x - t) - \frac{y^2}{4}\right)\right.$$
$$\left. + \exp\left(\tilde{d}t + \Psi_{03}(x + t) + \frac{y^2}{4}\right)\right], \tag{12}$$
$$\phi_0 = \frac{1}{2}\left[\exp\left(\tilde{D}t + \Psi_{01}(x - \alpha^{-1}t) - \frac{y^2}{4}\right)\right.$$
$$\left. - \exp\left(\tilde{D}t + \Psi_{02}(x + \alpha^{-1}t) + \frac{y^2}{4}\right)\right], \tag{13}$$
$$\eta_0 = \frac{1}{2}\left[\exp\left(\tilde{d}t + \Psi_{03}(x - t) - \frac{y^2}{4}\right)\right.$$
$$\left. - \exp\left(\tilde{d}t + \Psi_{04}(x + t) + \frac{y^2}{4}\right)\right], \tag{14}$$

其中 Ψ_{0i}，$i = 1,2,3,4$ 为任意函数.

为了求得系统(1)—(3)的一阶渐近解，在(8)—(10)式中取 $i = 1$，这时有

$$L_1(u_1^a, \phi_1) = 0,\ L_2(u_1^a, \phi_1) = -Q_0$$
$$= -\tilde{Q}(u_0^a, \phi_0, 0), \tag{15}$$
$$L_3(u_1^s, \eta_1) = \tau_0^x = \tilde{\tau}^x(u_0^s, \tau_0, 0),$$
$$L_4(u_1^s, \eta_1) = 0, \tag{16}$$
$$L_5(u_1^a, \phi_1) = 0,\ L_6(u_1^s, \eta_1) = 0. \tag{17}$$

由(15),(16)式,可得到

$$\frac{\partial(u_1^{a}+\phi_1)}{\partial t}+\alpha^{-1}\frac{\partial(u_1^{a}+\phi_1)}{\partial x}+\tilde{D}(u_1^{a}+\phi_1)+\tilde{Q}(u_0^{s},\eta_0,0)=0, \tag{18}$$

$$\frac{\partial(u_1^{a}-\phi_1)}{\partial t}-\alpha^{-1}\frac{\partial(u_1^{a}-\phi_1)}{\partial x}+\tilde{D}(u_1^{a}-\phi_1)-\tilde{Q}(u_0^{s},\eta_0,0)=0, \tag{19}$$

$$\frac{\partial(u_1^{s}+\eta_1)}{\partial t}+\frac{\partial(u_1^{s}+\eta_1)}{\partial x}+\tilde{d}(u_1^{s}+\eta_1)-\tilde{\tau}^{x}(u_0^{a},\phi_0,0)=0, \tag{20}$$

$$\frac{\partial(u_1^{s}-\eta_1)}{\partial t}-\frac{\partial(u_1^{s}-\eta_1)}{\partial x}+\tilde{d}(u_1^{s}-\eta_1)-\tilde{\tau}^{x}(u_0^{a},\phi_0,0)=0. \tag{21}$$

(18)式所对应的特征方程为

$$\frac{\mathrm{d}t}{1}=\frac{\mathrm{d}x}{\alpha^{-1}}=\frac{\mathrm{d}(u_1^{a}+\phi_1)}{\tilde{D}(u_1^{a}+\phi_1)+\tilde{Q}(u_0^{s},\eta_0,0)}. \tag{22}$$

显然,上述常微分系统(22)具有首次积分:$x-\alpha^{-1}t=c_1$.设系统的另一个独立的首次积分为$\Phi_{11}(x,y,t)=c_2$其中c_1,c_2为任意函数.由此可知,偏微分系统(18)的通解为

$$u_1^{a}+\phi_1=\Psi_{11}(x-\alpha^{-1}t,\Phi_{11}(x,t)),$$

同样,可得到偏微分系统(19)—(21)的通解

$$u_1^{a}-\phi_1=\Psi_{12}(x+\alpha^{-1}t,\Phi_{12}(x,t)),$$
$$u_1^{s}+\eta_1=\Psi_{13}(x-t,\Phi_{13}(x,t)),$$
$$u_1^{s}-\eta_1=\Psi_{14}(x+t,\Phi_{14}(x,t)).$$

于是有

$$u_1^{a}=\frac{1}{2}[\Psi_{11}(x-\alpha^{-1}t,\Phi_{11}(x,y,t))+\Psi_{12}(x+\alpha^{-1}t,\Phi_{12}(x,y,t))],$$

$$u_1^{a}=\frac{1}{2}[\Psi_{13}(x-t,\Phi_{13}(x,y,t))+\Psi_{14}(x+t,\Phi_{14}(x,y,t))],$$

$$\phi_1=\frac{1}{2}[\Psi_{11}(x-\alpha^{-1}t,\Phi_{11}(x,y,t))-\Psi_{12}(x+\alpha^{-1}t,\Phi_{12}(x,y,t))],$$

$$\eta_1=\frac{1}{2}[\Psi_{13}(x-t,\Phi_{13}(x,y,t))-\Psi_{14}(x+t,\Phi_{13}(x,y,t))],$$

其中$\Phi_{1i}=c$为对应的特征方程的首次积分,Ψ_{1i}为任意函数.但上述的Φ_{1i}和Ψ_{1i}尚需满足关系式(17).关于这方面的讨论,在此从略.

用类似的方法,可以依次地求出($u_i^{a},u_i^{s},\phi_i,\eta_i$),$i=2,3,\cdots$.于是,我们便得到了系统(1)—(3)如下直到$m$阶的渐近式:

$$\begin{aligned}u^{a}={}&\frac{1}{2}\Big[\exp\Big(\tilde{D}t+\Psi_{01}(x-\alpha^{-1}t)-\frac{y^2}{4}\Big)+\exp\Big(\tilde{D}t+\Psi_{02}(x+\alpha^{-1}t)+\frac{y^2}{4}\Big)\Big]\\&+\frac{1}{2}[\Psi_{11}(x-\alpha^{-1}t,\Phi_{11}(x,y,t))+\Psi_{12}(x+\alpha^{-1}t,\Phi_{12}(x,y,t))]\\&+\sum_{i=2}^{m}u_i^{a}\varepsilon^{i}+O(\varepsilon^{m+1}),\quad 0<\varepsilon\ll1,\end{aligned}$$

$$\begin{aligned}u^{s}={}&\frac{1}{2}\Big[\exp\Big(\tilde{d}t+\Psi_{03}(x-t)-\frac{y^2}{4}\Big)+\exp\Big(\tilde{d}t+\Psi_{04}(x+t)+\frac{y^2}{4}\Big)\Big]\\&+\frac{1}{2}[\Psi_{13}(x-t,\Phi_{13}(x,y,t))+\Psi_{14}(x+t,\Phi_{13}(x,y,t))]\\&+\sum_{i=1}^{m}u_i^{s}\varepsilon^{i}+O(\varepsilon^{m+1}),\quad 0<\varepsilon\ll1,\end{aligned}$$

$$\begin{aligned}\phi={}&\frac{1}{2}\Big[\exp\Big(\tilde{D}t+\Psi_{01}(x-\alpha^{-1}t)-\frac{y^2}{4}\Big)-\exp\Big(\tilde{D}t+\Psi_{02}(x+\alpha^{-1}t)+\frac{y^2}{4}\Big)\Big]\\&+\frac{1}{2}[\Psi_{11}(x-\alpha^{-1}t,\Phi_{11}(x,y,t))-\Psi_{12}(x+\alpha^{-1}t,\Phi_{12}(x,y,t))]\\&+\sum_{i=1}^{m}\phi_i\varepsilon^{i}+O(\varepsilon^{m+1}),\quad 0<\varepsilon\ll1,\end{aligned}$$

$$\begin{aligned}\eta={}&\frac{1}{2}\Big[\exp\Big(\tilde{d}t+\Psi_{03}(x-t)-\frac{y^2}{4}\Big)-\exp\Big(\tilde{d}t+\Psi_{04}(x+t)+\frac{y^2}{4}\Big)\Big]\\&+\frac{1}{2}[\Psi_{13}(x-t,\Phi_{13}(x,y,t))-\Psi_{14}(x+t,\Phi_{13}(x,y,t))]\\&+\sum_{i=1}^{m}\mu_i\varepsilon^{i}+O(\varepsilon^{m+1}),\quad 0<\varepsilon\ll1.\end{aligned}$$

4.结　　论

1.大气物理是一个复杂的自然现象.因此研究海气耦合振子只能通过简化成较简单的系统模型来研究它的主要规律.其中一个重要的研究方法,是通过问题的动力学系统,用数学的解析理论来得到相应的渐近解.本文就是利用摄动理论和方法,来求得热带海气耦合振子的一个模型的渐近解.

2. 本文提供的解法有别于单纯的数值解法. 它的一个优点是保留了得到的解的某些解析性态和渐近性态. 从而可以通过所求得物理量的渐近解,进一步去研究其自身和其他相关物理量的变化状态.

3. 摄动方法的优点还在于它的简捷有效. 可以用迭代的方法得到任意次精度的渐近解.

4. 从本文提出的海气振子耦合模型系统的结果可以看出,本耦合振子模型的解具有行波性态.

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The perturbative solution for the tropic sea-air coupled oscillators*

Mo Jia-Qi[1)] Lin Yi-Hua[2)] Lin Wan-Tao[2)]

[1)] (*Department of Mathematics*, *Anhui Normal University*, *Wuhu* 241000, *China*)

[2)] (*Institute of Atmospheric Physics*, *Chinese Academy of Sciences*, *Beijing* 100029, *China*)

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Abstract

A class of models of equator sea-air coupled oscillators is studied. Using the perturbative method the asymptotic solution of the corresponding model is obtained.

Keywords: nonlinear, perturbation, sea-air coupled model, El Niño-Southern Oscillator

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