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广义经典力学中 Hamilton-Tabarrok-Leech 正则方程的对称性理论*

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提出广义 Hamilton-Tabarrok-Leech 正则方程的对称性理论.列写系统的运动方程.研究系统的 Noether 对称性、形式不变性和 Lie 对称性,并求出相应的守恒量.举例说明结果的应用.

关键词:广义经典力学,H-T-L 正则方程,对称性,守恒量

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1. 引 言

在广义经典力学中 Lagrange 函数含有二阶导数的运动微分方程在物理学和力学中具有重要的实用意义.例如在物理学和场论中研究带二阶导数的电磁理论[1].在工程力学中,研究定梁轴弯曲形状时,问题化为求梁的总位能的极小.此时 Lagrange 函数中含有广义坐标的二阶导数[2].

2002 年,由 Tabarrok 和 Leech 研究了具有二阶导数的泛函的 Hamilton 力学[3].沿用普通分析力学中的研究思路,引入两个新广义动量和新 Hamilton 函数,将四阶 Euler-Lagrange 方程化为四个一阶的运动方程,称为 H-T-L 正则方程,用它来求梁的挠度.这种方法对求解高阶运动微分方程提供了一条新的途径.

力学系统的对称性是其本质特征的反映.用对称性来求守恒量是数学物理的一种近代方法,主要有 Noether 理论[4],Lie 对称性[5],形式不变性[6]和积分因子理论[7]等.近年来这些方法的研究已取得一系列重要成果[8—14].本文研究 H-T-L 正则方程的 Noether 对称性,形式不变性和 Lie 对称性.

2. 系统的运动方程

假设力学系统的位形由 n 个广义坐标 $q_1,q_2,\cdots,q_n$ 确定.

考虑下面的变分:

$$\delta\int_{x_0}^{x_1} L(x,q^j,q_x^j,q_{xx}^j)\mathrm{d}x = 0,\quad (j=1,2,\cdots,n),\tag{1}$$

其中

$$q_x^j = \dot{q}^j = \frac{\mathrm{d}q^j}{\mathrm{d}x},$$

$$q_{xx}^j = \frac{\mathrm{d}}{\mathrm{d}t}q_x^j = \frac{\mathrm{d}^2 q^j}{\mathrm{d}x^2}.\tag{2}$$

文中其余各变量的导数表示记号与(2)式相同.则系统的 Euler-Lagrange 方程为

$$\frac{\partial L}{\partial q^j} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial L}{\partial q_x^j}\right) + \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(\frac{\partial L}{\partial q_{xx}^j}\right) = 0\quad (j=1,2,\cdots,n).\tag{3}$$

定义广义动量为

$$r^j = \frac{\partial L(x,q^j,q_x^j,q_{xx}^j)}{\partial q_{xx}^j}\quad (j=1,2,\cdots,n),\tag{4}$$

于是方程(3)可写为

$$\frac{\partial L}{\partial q^j} = \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\partial L}{\partial q_x^j} - \frac{\mathrm{d}r^j}{\mathrm{d}x}\right].\tag{5}$$

定义另一广义动量为

$$p^j = \frac{\partial L}{\partial q_x^j} - \frac{\mathrm{d}r^j}{\mathrm{d}x},\tag{6}$$

则方程(5)成为

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$$\frac{\partial L}{\partial q^j}=\frac{\mathrm{d}p^j}{\mathrm{d}x}. \tag{7}$$

引入新 Hamilton 函数为

$$H(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r})=P^jq_x^j+r^jq_{xx}^j-L, \tag{8}$$

于是新正则方程表为[3]

$$\frac{\partial H}{\partial q^j}=-\frac{\mathrm{d}p^j}{\mathrm{d}x},\frac{\partial H}{\partial p^j}=\frac{\mathrm{d}q^j}{\mathrm{d}x},$$
$$\frac{\partial H}{\partial q_x^j}=-\frac{\mathrm{d}r^j}{\mathrm{d}x},\frac{\partial H}{\partial r^j}=\frac{\mathrm{d}q_x^j}{\mathrm{d}x}. \tag{9}$$

方程(9)称为 Hamilton-Tabarrok-Leech 正则方程.

方程(9)可展开为显式

$$p_x^j=\frac{\mathrm{d}p^j}{\mathrm{d}x}=h^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$q_x^j=\frac{\mathrm{d}q^j}{\mathrm{d}x}=g^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$r_x^j=\frac{\mathrm{d}r^j}{\mathrm{d}x}=k^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$\dot{q}_x^j=\frac{\mathrm{d}q_x^j}{\mathrm{d}x}=f^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$(j=1,2,\cdots,n). \tag{10}$$

3. H-T-L 系统的 Noether 对称性

3.1. Hamilton 作用量的变分

Hamilton 作用量表为

$$A=\int_{x_0}^{x_1}\left[p^jq_x^j+r^jq_{xx}^j-H\right]\mathrm{d}x. \tag{11}$$

引入无限小变换:

$$\tilde{x}=x+\varepsilon\tau(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$\tilde{q}^j=q^j+\varepsilon\xi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$\tilde{q}_x^j=q_x^j+\varepsilon\varphi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$\tilde{p}^j=p^j+\varepsilon\eta^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$\tilde{r}^j=r^j+\varepsilon\psi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),$$
$$(j=1,2,\cdots,n), \tag{12}$$

其中 ε 为无限小参数,$\tau,\xi^j,\varphi^j,\eta^j$ 和 ψ^j 称为无限小变换的生成元.Hamilton 作用量的变分为

$$\Delta A=\delta A+\dot{A}\Delta x=\int_{x_0}^{x_1}\Big\{p^j\delta q_x^j+q_x^j\delta p^j+r^j\delta q_{xx}^j+q_{xx}^j\delta r^j-\frac{\partial H}{\partial q^j}\delta q^j-\frac{\partial H}{\partial q_x^j}\delta q_x^j-\frac{\partial H}{\partial p^j}\delta p^j-\frac{\partial H}{\partial r^j}\delta r^j\Big\}\mathrm{d}x+(p^jq_x^j+r^jq_{xx}^j-H)\Delta x$$

$$=\int_{x_0}^{x_1}\Big\{\frac{\mathrm{d}}{\mathrm{d}x}\left[p^j\delta q^j+r^j\delta q_x^j+(p^jq_x^j+r^jq_{xx}^j-H)\Delta x\right]+\left(\frac{\mathrm{d}q_x^j}{\mathrm{d}x}-\frac{\partial H}{\partial r^j}\right)\delta r^j-\left(\frac{\partial H}{\partial q_x^j}+\frac{\mathrm{d}r^j}{\mathrm{d}x}\right)\delta q_x^j-\left(\frac{\partial H}{\partial q^j}+\frac{\mathrm{d}p^j}{\mathrm{d}x}\right)\delta q^j+\left(-\frac{\partial H}{\partial p^j}+\frac{\mathrm{d}q^j}{\mathrm{d}x}\right)\delta p^j\Big\}\mathrm{d}x. \tag{13}$$

3.2. 准对称变换

3.2.1. 定义

如果相空中 Hamilton 作用量是无限小变换的准不变量,即对每一个无限小变换(12),始终成立

$$\Delta A=-\int_{x_0}^{x_1}\frac{\mathrm{d}}{\mathrm{d}x}(\Delta G)\mathrm{d}x, \tag{14}$$

其中 $G=G(t,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r})$ $(\Delta G=\varepsilon G)$ 称为规范函数,则无限小变换(12)称为 Noether 意义下的准对称变换.

3.2.2. 判据

如果无限小变换(12)满足方程

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p^j\bar{\xi}^j+r^j\bar{\varphi}^j+(p^jq_x^j+r^jq_{xx}^j-H)\tau\right]+\left(\frac{\mathrm{d}q_x^j}{\mathrm{d}x}-\frac{\partial H}{\partial r^j}\right)\bar{\psi}^j-\left(\frac{\mathrm{d}r^j}{\mathrm{d}x}+\frac{\partial H}{\partial q_x^j}\right)\bar{\varphi}^j-\left(\frac{\mathrm{d}p^j}{\mathrm{d}x}+\frac{\partial H}{\partial q^j}\right)\bar{\xi}^j+\left(\frac{\mathrm{d}q^j}{\mathrm{d}x}-\frac{\partial H}{\partial p^j}\right)\bar{\eta}^j=-G_x, \tag{15}$$

其中

$$\bar{\xi}^j=\xi^j-q_x^j\tau,\bar{\varphi}^j=\varphi^j-q_{xx}^j\tau,$$
$$\bar{\psi}^j=\psi^j-r_x^j\tau,\bar{\eta}^j=\eta^j-p_x\tau. \tag{16}$$

则变换(12)为给定系统的准对称变换.

3.2.3. 判据

如果无限小变换(12)满足 Noether 等式

$$p^j\xi_x^j+r^j\varphi_x^j-\frac{\partial H}{\partial x}\tau-\frac{\partial H}{\partial q^j}\xi^j-\frac{\partial H}{\partial q_x^j}\varphi^j-H\tau_x=-G_x. \tag{17}$$

则变换(12)为给定系统的准对称变换.

3.3. H-T-L 系统的 Noether 守恒定理

3.3.1. 定理

假设给定的无限小变换(12)是 H-T-L 系统(9)的准对称变换,则系统存在如下的守恒量(初积分):

$$I=p^j\xi^j+r^j\varphi^j-H\tau+G=\mathrm{const}. \tag{18}$$

证明　对 H-T-L 系统有方程(9),将其代入(15)式,有积分

$$p^j\bar{\xi}^j+r^j\bar{\varphi}^j+(p^jq_x^j+r^jq_{xx}^j-H)\tau+G=\mathrm{const}. \tag{19}$$

将(16)式代入上式,便得守恒量(18).

3.4. H-T-L 系统的 Noether 逆定理

假设系统有初积分

$$I = I(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}) = \text{const}. \tag{20}$$

求与之相应的 Noether 对称性. 由(20)式得

$$\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\partial I}{\partial x} + \frac{\partial I}{\partial q^j}q^j_x + \frac{\partial I}{\partial q^j_x}q^j_{xx} + \frac{\partial I}{\partial p^j}p^j_x + \frac{\partial I}{\partial r^j}r^j_x = 0. \tag{21}$$

将方程(9)的第一式乘($\xi^j - q_x\tau$),第三式乘($\varphi^j - q^j_{xx}\tau$),分别对 j 求和后相加,有

$$\left(\frac{\partial H}{\partial q^j} + \frac{\mathrm{d}p^j}{\mathrm{d}x}\right)(\xi^j - q_x\tau) + \left(\frac{\partial H}{\partial q^j_x} + \frac{\mathrm{d}r^j}{\mathrm{d}x}\right)(\varphi^j - q^j_{xx}\tau) = 0. \tag{22}$$

比较(21)式与(22)式中含 p^j_x 和 r^j_x 的项,得到

$$\xi^j = \frac{\partial H}{\partial p^j}\tau + \frac{\partial I}{\partial p^j},\varphi^j = \frac{\partial H}{\partial r^j}\tau + \frac{\partial I}{\partial r^j}. \tag{23}$$

再令初积分(20)等于守恒量(18),即

$$p^j\xi^j + r^j\varphi^j - H\tau + G = I. \tag{24}$$

3.4.1. 定理

如果已知 H-T-L 系统(9)有一个初积分(20),则与积分(20)相应的无限小生成元由方程(23)和(24)确定.

4. H-T-L 系统的形式不变性

假设在经历无限小变换(12)后,Hamilton 函数 H 变为 H^*,有

$$H^* = H(x^*,\boldsymbol{q}^*,\boldsymbol{q}^*_x,\boldsymbol{p}^*,\boldsymbol{r}^*) = H(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}) + \varepsilon X^{(0)}(H) + O(\varepsilon^2), \tag{25}$$

其中

$$X^{(0)} = \tau\frac{\partial}{\partial x} + \xi^j\frac{\partial}{\partial q^j} + \varphi^j\frac{\partial}{\partial q^j_x} + \eta^j\frac{\partial}{\partial p^j} + \psi^j\frac{\partial}{\partial r^j}. \tag{26}$$

4.1. 定义

如果用无限小变换后的 H^* 代替变换前的 H 时,方程(9)的形式保持不变,即

$$\frac{\partial H^*}{\partial q^j} = -\frac{\mathrm{d}p^j}{\mathrm{d}x},\frac{\partial H^*}{\partial p^j} = \frac{\mathrm{d}q^j}{\mathrm{d}x},$$
$$\frac{\partial H^*}{\partial q^j_x} = -\frac{\mathrm{d}r^j}{\mathrm{d}x},\frac{\partial H^*}{\partial r^j} = \frac{\mathrm{d}q^j_x}{\mathrm{d}x} \tag{27}$$

成立,则称这种不变性为 H-T-L 系统正则方程(9)的形式不变性.

4.2. 判据

对于 H-T-L 系统(9),如果无限小生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$ 满足

$$\frac{\partial}{\partial q^j}[X^{(0)}(H)] = 0,\frac{\partial}{\partial q^j_x}[X^{(0)}(H)] = 0,$$
$$\frac{\partial}{\partial p^j}[X^{(0)}(H)] = 0,\frac{\partial}{\partial r^j}[X^{(0)}(H)] = 0. \tag{28}$$

则相应不变性为系统的形式不变性.

证明　将(25)式代入方程(27)的第一式,忽略 ε^2 及更高阶小项,并利用方程(9),有

$$\frac{\partial H^*}{\partial q^j} = \frac{\partial}{\partial q^j}[H(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}) + \varepsilon X^{(0)}(H)] = \frac{\partial H}{\partial q^j} + \varepsilon\frac{\partial}{\partial q^j}[X^{(0)}(H)] = \frac{\partial H}{\partial q^j} = -\frac{\mathrm{d}p^j}{\mathrm{d}x}. \tag{29}$$

同理可证方程(28)中的其余各式.

对 H-T-L 系统(9),形式不变性通过 Noether 对称性可间接导出 Noether 守恒量.

4.3. 定理

如果 H-T-L 系统(9)的形式不变性的生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$ 和规范函数 $G = G(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r})$ 满足如下结构方程:

$$p^j\xi^j_x + r^j\varphi^j_x - \frac{\partial H}{\partial x}\tau - \frac{\partial H}{\partial q^j}\xi^j - \frac{\partial H}{\partial q^j_x}\varphi^j - H\tau_x = -G_x, \tag{30}$$

则形式不变性导致 Noether 守恒量

$$I = p^j\xi^j + r^j\varphi^j - H\tau + G = \text{const}. \tag{31}$$

5. H-T-L 系统的 Lie 对称性

5.1. Lie 对称性的确定方程

根据微分方程在无限小变换下的不变性理论知,H-T-L 方程(9)在无限小变换(12)下的不变性归为

$$X^{(1)}\left(p^j_x + \frac{\partial H}{\partial q^j}\right) = 0,X^{(1)}\left(q^j_x - \frac{\partial H}{\partial p^j}\right) = 0,$$
$$X^{(1)}\left(r^j_x + \frac{\partial H}{\partial q^j_x}\right) = 0,X^{(1)}\left(q^j_{xx} - \frac{\partial H}{\partial r^j}\right) = 0, \tag{32}$$

其中

$$X^{(0)}=\tau\frac{\partial}{\partial x}+\xi^k\frac{\partial}{\partial q^k}+\varphi^k\frac{\partial}{\partial q_x^k}+\eta^k\frac{\partial}{\partial p^k}+\psi^k\frac{\partial}{\partial r^k},$$

$$X^{(1)}=X^{(0)}+\left(\xi_x^k-\frac{\partial H}{\partial p^k}\tau_x\right)\frac{\partial}{\partial\dot q^k}+\left(\varphi_x^k-\frac{\partial H}{\partial r^k}\tau_x\right)\frac{\partial}{\partial\dot q_k}+\left(\eta_x^k+\frac{\partial H}{\partial q^k}\tau_x\right)\frac{\partial}{\partial\dot p^k}+\left(\psi_x^k+\frac{\partial H}{\partial q_x^k}\tau_x\right)\frac{\partial}{\partial\dot r^k}.\tag{33}$$

将(33)式代入方程(32),得

$$\eta_x^j+\frac{\partial H}{\partial q^j}\tau_x=-\frac{\partial^2 H}{\partial q^j\partial x}\tau-\frac{\partial^2 H}{\partial q^j\partial q^k}\xi^k-\frac{\partial^2 H}{\partial q^j\partial q_x^k}\varphi^k-\frac{\partial^2 H}{\partial q^j\partial p^k}\eta^k-\frac{\partial^2 H}{\partial q^j\partial r^k}\psi^k,$$

$$\xi_x^j-\frac{\partial H}{\partial p^j}\tau_x=\frac{\partial^2 H}{\partial p^j\partial x}\tau+\frac{\partial^2 H}{\partial p^j\partial q^k}\xi^k+\frac{\partial^2 H}{\partial p^j\partial q_x^k}\varphi^k+\frac{\partial^2 H}{\partial p^j\partial p^k}\eta^k+\frac{\partial^2 H}{\partial p^j\partial r^k}\psi^k,$$

$$\psi_x^j+\frac{\partial H}{\partial q_x^j}\tau_x=-\frac{\partial^2 H}{\partial q_x^j\partial x}\tau-\frac{\partial^2 H}{\partial q_x^j\partial q^k}\xi^k+\frac{\partial^2 H}{\partial q_x^j\partial q_x^k}\varphi^k+\frac{\partial^2 H}{\partial q_x^j\partial p^k}\eta^k+\frac{\partial^2 H}{\partial q_x^j\partial r^k}\psi^k,$$

$$\varphi_x^j-\frac{\partial H}{\partial r^j}\tau_x=\frac{\partial^2 H}{\partial r^j\partial x}\tau+\frac{\partial^2 H}{\partial r^j\partial q^k}\xi^k+\frac{\partial^2 H}{\partial r^j\partial q_x^k}\varphi^k+\frac{\partial^2 H}{\partial r^j\partial p^k}\eta^k+\frac{\partial^2 H}{\partial r^j\partial r^k}\psi^k.\tag{34}$$

H-T-L 方程(10)在无限小变换(12)下的不变性归为

$$\eta_x^j-h^j\tau_x=X^{(0)}(h^j),\ \xi_x^j-g^j\tau_x=X^{(0)}(g^j),$$

$$\psi_x^j-k^j\tau_x=X^{(0)}(k^j),\ \varphi_x^j-f^j\tau_x=X^{(0)}(f^j).\tag{35}$$

方程(34)和(35)称为 Lie 对称性的确定方程.

5.2. Lie 对称变换

如果无限小变换的生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$ 满足确定方程(34)或(35),则称相应的变换是 H-T-L 系统的 Lie 对称变换.

5.3. 结构方程与守恒量

5.3.1. 定理

对于满足确定方程(35)的无限小生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$,如果存在规范函数 $G=G(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r})$ 满足如下结构方程:

$$p^j\xi_x^j+r^j\varphi_x^j+\frac{\partial H}{\partial p^j}\eta^j+\frac{\partial H}{\partial r^j}\psi^j-H\tau_x-X^{(0)}(H)+G_x=0,\tag{36}$$

则 H-T-L 系统存在如下形式的守恒量:

$$I=p^j\xi^j+r^j\varphi^j-H\tau+G=\text{const}.\tag{37}$$

证明

$$\frac{\mathrm{d}I}{\mathrm{d}t}=p_x^j\xi^j+p^j\xi_x^j+r_x^j\varphi^j+r^j\varphi_x^j-\dot H\tau-H\tau_x-p^j\xi_x^j-r^j\varphi_x^j-\frac{\partial H}{\partial p^j}\eta^j-\frac{\partial H}{\partial r^j}\psi^j+H\tau_x+X^{(0)}(H)=-\left(\frac{\partial H}{\partial q_x^j}\dot q_x^j+\frac{\partial H}{\partial r^j}r_x^j\right)\tau-\left(\frac{\partial H}{\partial q^j}q_x^j+\frac{\partial H}{\partial p^j}p_x^j\right)\tau=0.\tag{38}$$

5.4. Lie 对称性逆问题

H-T-L 系统 Lie 对称性逆问题的提法是由已知初积分(守恒量)来求与其相应的 Lie 对称性.

第一步,根据已知初积分求相应的 Noether 对称性,假设系统有初积分

$$I=I(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),\tag{39}$$

则由方程(23)和(24)可求得无限小生成元 τ,ξ^j 和 φ^j.

其次,求生成元 η^j 和 ψ^j.

由于

$$\Delta p^j=\varepsilon\eta^j.\tag{40}$$

根据方程(6),有 $p^j=p^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{r},\boldsymbol{r}_x)$,于是

$$\Delta p^j=\frac{\partial p^j}{\partial x}\Delta x+\frac{\partial p^j}{\partial q^k}\Delta q^k+\frac{\partial p^j}{\partial q_x^k}\Delta q_x^k+\frac{\partial p^j}{\partial r^k}\Delta r^k+\frac{\partial p^j}{\partial r_x^k}\Delta r_x^k=\varepsilon\left[\frac{\partial p^j}{\partial x}\tau+\frac{\partial p^j}{\partial q^k}\xi^k+\frac{\partial p^j}{\partial q_x^k}\varphi^k+\frac{\partial p^j}{\partial r^k}\psi^k+\frac{\partial p^j}{\partial r_x^k}(\psi_x^k-r_x^k\tau_x)\right],$$

故有

$$\eta^j=\frac{\partial p^j}{\partial x}\tau+\frac{\partial p^j}{\partial q^k}\xi^k+\frac{\partial p^j}{\partial q_x^k}\varphi^k+\frac{\partial p^j}{\partial r^k}\psi^k+\frac{\partial p^j}{\partial r_x^k}(\psi_x^k-r_x^k\tau_x).\tag{41}$$

又 $\Delta r^j=\varepsilon\psi^j$,由方程(4)知,$r^j=r^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{q}_{xx})$,于是,仿上运算,得

$$\psi^j=\frac{\partial r^j}{\partial x}\tau+\frac{\partial r^j}{\partial q^k}\xi^k+\frac{\partial r^j}{\partial q_x^k}\varphi^k+\frac{\partial r^j}{\partial q_{xx}^k}(\varphi_x^k-q_{xx}^k\tau_x).\tag{42}$$

这样,便找到了与积分(39)相应的 Noether 对称性.

第二步,由 Noether 对称性求 Lie 对称性.将(23),(24),(41)和(42)式求得的 Noether 对称性生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$ 代入 Lie 对称性的确定方程(35),如果它们满足,则所得对称性是 Lie 的,否则不是 Lie 的.

5.4.1 定理

如果由(23),(24),(41)和(42)式确定的无限小生成元 $\tau,\xi^j,\varphi^j,\eta^j,\psi^j$ 满足确定方程(35),则无限小变换是 Lie 对称的.

6. Lie 对称性与 Hojman 守恒量

H-T-L 系统(9)的 Lie 对称性可直接导出 Hojman 守恒量.取 x 不变的特殊无限小变换.

$$\begin{aligned}\tilde{x} &= x,\\ \tilde{q}^j &= q^j + \varepsilon\xi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),\\ \tilde{q}_x^j &= q_x^j + \varepsilon\varphi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),\\ \tilde{p}^j &= p^j + \varepsilon\eta^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}),\\ \tilde{r}^j &= r^j + \varepsilon\psi^j(x,\boldsymbol{q},\boldsymbol{q}_x,\boldsymbol{p},\boldsymbol{r}).\end{aligned} \tag{43}$$

此时,Lie 对称性的确定方程(34)给出

$$\begin{aligned}\frac{\bar{d}}{dx}\eta^j &= \eta_x^j = -\frac{\partial^2 H}{\partial q^j\partial q^k}\xi^k - \frac{\partial^2 H}{\partial q^j\partial q_x^k}\varphi^k - \frac{\partial^2 H}{\partial q^j\partial p^k}\eta^k - \frac{\partial^2 H}{\partial q^j\partial r^k}\psi^k,\\ \frac{\bar{d}}{dx}\xi^j &= \xi_x^j = \frac{\partial^2 H}{\partial p^j\partial q^k}\xi^k + \frac{\partial^2 H}{\partial p^j\partial q_x^k}\varphi^k + \frac{\partial^2 H}{\partial p^j\partial p^k}\eta^k + \frac{\partial^2 H}{\partial p^j\partial r^k}\psi^k,\\ \frac{\bar{d}}{dx}\psi^j &= \psi_x^j = -\frac{\partial^2 H}{\partial q_x^j\partial q^k}\xi^k - \frac{\partial^2 H}{\partial q_x^j\partial q_x^k}\varphi^k + \frac{\partial^2 H}{\partial q_x^j\partial p^k}\eta^k + \frac{\partial^2 H}{\partial q_x^j\partial r^k}\psi^k,\\ \frac{\bar{d}}{dx}\varphi^j &= \varphi_x^j = \frac{\partial^2 H}{\partial r^j\partial q^k}\xi^k + \frac{\partial^2 H}{\partial r^j\partial q_x^k}\varphi^k + \frac{\partial^2 H}{\partial r^j\partial p^k}\eta^k + \frac{\partial^2 H}{\partial r^j\partial r^k}\psi^k,\end{aligned} \tag{44}$$

其中

$$\frac{\bar{d}}{dx} = \frac{\partial}{\partial x} + \frac{\partial H}{\partial q^j}\frac{\partial}{\partial p^j} + \frac{\partial H}{\partial q_x^j}\frac{\partial}{\partial r^j} - \frac{\partial H}{\partial p^j}\frac{\partial}{\partial q^j} - \frac{\partial H}{\partial r^j}\frac{\partial}{\partial q_x^j}. \tag{45}$$

于是,有如下结果.

6.1. 定理

对 H-T-L 系统(9),在特殊无限小变换(43)下,如果生成元 ξ^j,φ^j,η^j 和 ψ^j 满足方程(44),则系统的 Lie 对称性导致 Hojman 守恒量

$$I_H = \frac{\partial\xi^j}{\partial q^j} + \frac{\partial\varphi^j}{\partial q_x^j} + \frac{\partial\eta^j}{\partial p^j} + \frac{\partial\psi^j}{\partial r^j} = \text{const}. \tag{46}$$

证明 将(46)式对 x 求导数,有

$$\frac{\bar{d}I}{dx} = \frac{\bar{d}}{dx}\left(\frac{\partial\xi^j}{\partial q^j}\right) + \frac{\bar{d}}{dx}\left(\frac{\partial\varphi^j}{\partial q_x^j}\right) + \frac{\bar{d}}{dx}\left(\frac{\partial\eta^j}{\partial p^j}\right) + \frac{\bar{d}}{dx}\left(\frac{\partial\psi^j}{\partial r^j}\right). \tag{47}$$

由于

$$\begin{aligned}\frac{\partial\eta_x^j}{\partial p^j} =& -\frac{\partial^3 H}{\partial q^j\partial q^k\partial p^j}\xi^k - \frac{\partial^3 H}{\partial q^j\partial q_x^k\partial p^j}\varphi^k - \frac{\partial^3 H}{\partial q^j\partial p^k\partial p^j}\eta^k - \frac{\partial^3 H}{\partial q^j\partial r^k\partial p^j}\psi^k\\ &- \frac{\partial^2 H}{\partial q^j\partial q^k}\frac{\partial\xi^k}{\partial p^j} - \frac{\partial^2 H}{\partial q^j\partial q_x^k}\frac{\partial\varphi^k}{\partial p^j} - \frac{\partial^2 H}{\partial q^j\partial p^k}\frac{\partial\eta^k}{\partial p^j} - \frac{\partial^2 H}{\partial q^j\partial r^k}\frac{\partial\varphi^k}{\partial p^j},\\ \frac{\partial\xi_x^j}{\partial q^j} =& \frac{\partial^3 H}{\partial p^j\partial q^k\partial q^j}\xi^k + \frac{\partial^3 H}{\partial p^j\partial q_x^k\partial q^j}\varphi^k + \frac{\partial^3 H}{\partial p^j\partial p^k\partial q^j}\eta^k + \frac{\partial^3 H}{\partial p^j\partial r^k\partial q^j}\psi^k\\ &+ \frac{\partial^2 H}{\partial p^j\partial q^k}\frac{\partial\xi^k}{\partial q^j} + \frac{\partial^2 H}{\partial p^j\partial q_x^k}\frac{\partial\varphi^k}{\partial q^j} + \frac{\partial^2 H}{\partial p^j\partial p^k}\frac{\partial\eta^k}{\partial q^j} + \frac{\partial^2 H}{\partial p^j\partial r^k}\frac{\partial\varphi^k}{\partial q^j},\\ \frac{\partial\psi_x^j}{\partial r^j} =& -\frac{\partial^3 H}{\partial q_x^j\partial q^k\partial r^j}\xi^k - \frac{\partial^3 H}{\partial q_x^j\partial q_x^k\partial r^j}\varphi^k - \frac{\partial^3 H}{\partial q_x^j\partial p^k\partial r^j}\eta^k - \frac{\partial^3 H}{\partial q_x^j\partial r^k\partial r^j}\psi^k\\ &- \frac{\partial^2 H}{\partial q_x^j\partial q^k}\frac{\partial\xi^k}{\partial r^j} - \frac{\partial^2 H}{\partial q_x^j\partial q_x^k}\frac{\partial\varphi^k}{\partial r^j} - \frac{\partial^2 H}{\partial q_x^j\partial p^k}\frac{\partial\eta^k}{\partial r^j} - \frac{\partial^2 H}{\partial q_x^j\partial r^k}\frac{\partial\varphi^k}{\partial r^j},\\ \frac{\partial\varphi_x^j}{\partial q_x^j} =& \frac{\partial^3 H}{\partial r^j\partial q^k\partial q_x^j}\xi^k + \frac{\partial^3 H}{\partial r^j\partial q_x^k\partial q_x^j}\varphi^k + \frac{\partial^3 H}{\partial r^j\partial p^k\partial q_x^j}\eta^k + \frac{\partial^3 H}{\partial r^j\partial r^k\partial q_x^j}\psi^k\\ &+ \frac{\partial^2 H}{\partial r^j\partial q^k}\frac{\partial\xi^k}{\partial q_x^j} + \frac{\partial^2 H}{\partial r^j\partial q_x^k}\frac{\partial\varphi^k}{\partial q_x^j} + \frac{\partial^2 H}{\partial r^j\partial p^k}\frac{\partial\eta^k}{\partial q_x^j} + \frac{\partial^2 H}{\partial r^j\partial r^k}\frac{\partial\varphi^k}{\partial q_x^j}.\end{aligned} \tag{48}$$

注意到

$$
\begin{aligned}
\frac{\partial \eta^j_x}{\partial p^j} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \eta^j}{\partial p^j}\right) + \frac{\partial \eta^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial p^j} + \frac{\partial \eta^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial p^j} \\
&\quad - \frac{\partial \eta^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial p^j} - \frac{\partial \eta^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial p^j}, \\
\frac{\partial \xi^j_x}{\partial q^j} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \xi^j}{\partial q^j}\right) + \frac{\partial \xi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial q^j} + \frac{\partial \xi^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial q^j} \\
&\quad - \frac{\partial \xi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial q^j} - \frac{\partial \xi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial q^j}, \\
\frac{\partial \psi^j_x}{\partial r^j} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\partial \psi^j}{\partial r^j}\right) + \frac{\partial \psi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial r^j} + \frac{\partial \psi^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial r^j} \\
&\quad - \frac{\partial \psi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial r^j} - \frac{\partial \psi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial r^k}, \\
\frac{\partial \varphi^j_x}{\partial q^j_x} &= \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\partial \varphi^j}{\partial q^j_x}\right) + \frac{\partial \varphi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial q^j_x} + \frac{\partial \varphi^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial q^j_x} \\
&\quad - \frac{\partial \varphi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial q^j_x} - \frac{\partial \varphi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^j_x \partial q^k_x}.
\end{aligned}
\tag{49}
$$

将方程(49)代入方程(48)，得

$$
\begin{aligned}
-\frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \eta^j}{\partial p^j}\right) &= \frac{\partial \eta^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial p^j} + \frac{\partial \eta^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial p^j} \\
&\quad - \frac{\partial \eta^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial p^j} - \frac{\partial \eta^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial p^j} \\
&\quad + \frac{\partial^3 H}{\partial q^j \partial q^k \partial p^j}\xi^k + \frac{\partial^3 H}{\partial q^j \partial q^k_x \partial p^j}\varphi^k \\
&\quad + \frac{\partial^3 H}{\partial q^j \partial p^k \partial p^j}\eta^k + \frac{\partial^3 H}{\partial q^j \partial r^k \partial p^j}\psi^k \\
&\quad + \frac{\partial^2 H}{\partial q^j \partial q^k}\frac{\partial \xi^k}{\partial p^j} + \frac{\partial^2 H}{\partial q^j \partial q^k_x}\frac{\partial \varphi^k}{\partial p^j} \\
&\quad + \frac{\partial^2 H}{\partial q^j \partial p^k}\frac{\partial \eta^k}{\partial p^j} + \frac{\partial^2 H}{\partial q^j \partial r^k}\frac{\partial \psi^k}{\partial p^j}, \\
-\frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \xi^j}{\partial q^j}\right) &= \frac{\partial \xi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial q^j} + \frac{\partial \xi^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial q^j} \\
&\quad - \frac{\partial \xi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial q^j} - \frac{\partial \xi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial q^j} \\
&\quad - \frac{\partial^3 H}{\partial p^j \partial q^k \partial q^j}\xi^k - \frac{\partial^3 H}{\partial p^j \partial q^k_x \partial q^j}\varphi^k \\
&\quad - \frac{\partial^3 H}{\partial p^j \partial p^k \partial q^j}\eta^k - \frac{\partial^3 H}{\partial p^j \partial r^k \partial q^j}\psi^k \\
&\quad - \frac{\partial^2 H}{\partial p^j \partial q^k}\frac{\partial \xi^k}{\partial q^j} - \frac{\partial^2 H}{\partial p^j \partial q^k_x}\frac{\partial \varphi^k}{\partial q^j} \\
&\quad - \frac{\partial^2 H}{\partial p^j \partial p^k}\frac{\partial \eta^k}{\partial q^j} - \frac{\partial^2 H}{\partial p^j \partial r^k}\frac{\partial \psi^k}{\partial q^j}, \\
-\frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \psi^j}{\partial r^j}\right) &= \frac{\partial \psi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial r^j} + \frac{\partial \psi^j}{\partial q^k_x}\frac{\partial^2 H}{\partial r^k \partial r^j} \\
&\quad - \frac{\partial \psi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial r^j} - \frac{\partial \psi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial r^j} \\
&\quad + \frac{\partial^3 H}{\partial q^j_x \partial q^k \partial r^j}\xi^k + \frac{\partial^3 H}{\partial q^j_x \partial q^k_x \partial r^j}\varphi^k \\
&\quad + \frac{\partial^3 H}{\partial q^j_x \partial p^k \partial r^j}\eta^k + \frac{\partial^3 H}{\partial q^j_x \partial r^k \partial r^j}\psi^k \\
&\quad + \frac{\partial^2 H}{\partial q^j_x \partial q^k}\frac{\partial \xi^k}{\partial r^j} + \frac{\partial^2 H}{\partial q^j_x \partial q^k_x}\frac{\partial \varphi^k}{\partial r^j} \\
&\quad + \frac{\partial^2 H}{\partial q^j_x \partial p^k}\frac{\partial \eta^k}{\partial r^j} + \frac{\partial^2 H}{\partial q^j_x \partial r^k}\frac{\partial \psi^k}{\partial r^j}, \\
-\frac{\bar{\mathrm{d}}}{\mathrm{d}x}\left(\frac{\partial \varphi^j}{\partial q^j_x}\right) &= \frac{\partial \varphi^j}{\partial q^k}\frac{\partial^2 H}{\partial p^k \partial q^j_x} + \frac{\partial \varphi^j}{\partial q^j_x}\frac{\partial^2 H}{\partial r^k \partial q^j_x} \\
&\quad - \frac{\partial \varphi^j}{\partial p^k}\frac{\partial^2 H}{\partial q^k \partial r^j_x} - \frac{\partial \varphi^j}{\partial r^k}\frac{\partial^2 H}{\partial q^k_x \partial q^j_x} \\
&\quad - \frac{\partial^3 H}{\partial r^j \partial q^k \partial q^j_x}\xi^k - \frac{\partial^3 H}{\partial r^j \partial q^k_x \partial q^j_x}\varphi^k \\
&\quad - \frac{\partial^3 H}{\partial r^j \partial p^k \partial q^j_x}\eta^k - \frac{\partial^3 H}{\partial r^j \partial r^k \partial q^j_x}\psi^k \\
&\quad - \frac{\partial^2 H}{\partial r^j \partial q^k}\frac{\partial \xi^k}{\partial q^j_x} - \frac{\partial^2 H}{\partial r^j \partial q^k_x}\frac{\partial \varphi^k}{\partial q^j_x} \\
&\quad - \frac{\partial^2 H}{\partial r^j \partial p^k}\frac{\partial \eta^k}{\partial q^j_x} - \frac{\partial^2 H}{\partial r^j \partial r^k}\frac{\partial \psi^k}{\partial q^j_x}.
\end{aligned}
\tag{50}
$$

将方程(50)代入(47)式，有

$$
\frac{\mathrm{d}I_{\mathrm{H}}}{\mathrm{d}x} = 0,\text{即 } I_{\mathrm{H}} = \mathrm{const}. \tag{51}
$$

7. 举　　例

一棱柱形梁受轴向压力 F 和横向均匀分布载荷 ω，对于小挠度理论，系统的 Lagrange 函数为

$$
L = \frac{EI}{2}q^2_{xx} - \omega q - \frac{F}{2}q^2_x. \tag{52}
$$

其中 EI 是抗弯刚度，$EI=\mathrm{const}$. 试求系统的守恒量.

解 1　先研究正问题. 应用 3.3.1 定理来求守恒量. 由于

$$
r = \frac{\partial L}{\partial q_{xx}} = EIq_{xx},
$$

$$
p = \frac{\partial L}{\partial q_x} - \frac{\mathrm{d}r}{\mathrm{d}x} = -Fq_x - EI\dot{q}_{xx}, \tag{53}
$$

$$
H = pq_x + rq_{xx} - L = pq_x + \omega q + \frac{F}{2}q^2_x + \frac{r^2}{2EI}. \tag{54}
$$

Noether 对称性的判据(16)式给出

$$
\begin{aligned}
&(-Fq_x - EI\dot{q}_{xx})\xi_x + EIq_{xx}\varphi \\
&- \omega\xi - (p + Fq_x)\varphi - H\tau_x = -G_x.
\end{aligned}
\tag{55}
$$

方程(55)有解

$$\xi = 1, \tau = \varphi = 0, G = \omega x. \tag{56}$$

守恒量(18)给出

$$I = P + \omega x = \text{const}.$$

或

$$I = (-Fq_x - EI\dot{q}_{xx}) + \omega x = \text{const}. \tag{57}$$

其次,研究逆问题.假设系统有初积分(57),来求与之相应的 Noether 准对称变换.由(23),(24)式得

$$\xi = q_x\tau + 1, \varphi = \frac{r}{EI}\tau,$$

$$p\xi + r\varphi - H\tau + G = P + \omega x. \tag{58}$$

若取 $G = \omega x$,由此解得

$$\xi = 1, \tau = \varphi = 0. \tag{59}$$

解 2 应用形式不变性 4.3 定理来求守恒量.

做计算

$$X^{(0)}(H) = \xi\omega + \varphi(p + Fq_x) + \eta q_x + \psi\frac{r}{EI}. \tag{60}$$

取生成元

$$\tau = 1, \xi = \varphi = \psi = \eta = 0. \tag{61}$$

它们都使 $X^{(0)}(H) = 0$,并满足 4.2 判据式(28),因此变换(61)是系统的形式不变性变换.将方程(61)代入结构方程(30),求得 $G = 0$.于是守恒量(31)给出

$$I = -H = \text{const}. \tag{62}$$

解 3 应用 Lie 对称性 5.3.1 定理来求守恒量.

确定方程(34)给出

$$\eta_x + \omega\tau_x = 0, \xi_x - q_x\tau_x = \varphi,$$

$$\psi_x + (P + Fq_x)\tau_x = -F\varphi + \eta,$$

$$\varphi_x - \frac{r}{EI}\tau_x = \frac{1}{EI}\psi. \tag{63}$$

方程(63)有解

$$\tau = 1, \varphi = \xi = \eta = \psi = 0. \tag{64}$$

将方程(64)代入结构方程(36),求得 $G = 0$.于是守恒量(37)给出

$$I = -H = \text{const}, \tag{65}$$

其次,研究 Lie 对称性逆问题.假设系统(52)有初积分(65),求与之相应的 Lie 对称性,(23),(24),(41)和(42)式给出

$$\xi = q_x\tau - q_x, \varphi = \frac{r}{EI}\tau - \frac{r}{EI},$$

$$p\xi + r\varphi - H\tau + G = -H,$$

$$\eta = -p\varphi, \psi = EI(\varphi_x - q_{xx}\tau_x). \tag{66}$$

若取 $G = 0$,则由方程(66)求得

$$\tau = 1, \varphi = \xi = \eta = \psi = 0. \tag{67}$$

将(67)式代入确定方程(34),显然成立.因此,由 5.3.2 定理知,生成元(67)对应的变换是 Lie 对称的.

解 4 应用 6.1 定理求 Hojman 守恒量.

Lie 对称性确定方程(44)给出

$$\eta_x = 0, \xi_x = \varphi, \psi_x = -F\varphi + \eta,$$

$$\varphi_x = \frac{1}{EI}\psi. \tag{68}$$

方程(68)有解

$$\xi = \left(pq_x + \omega q + \frac{F}{2}q_x^2 + \frac{r^2}{2EI}\right)^2,$$

$$\eta = \varphi = \psi = 0. \tag{69}$$

于是守恒量(46)给出

$$I_{\mathrm{H}} = 2\omega\left(pq_x + \omega q + \frac{F}{2}q_x^2 + \frac{r^2}{2EI}\right) = \text{const}. \tag{70}$$

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Symmetry theory of the Hamilton-Tabarrok-Leech's canonical equations in generalized classical mechanics*

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Abstract

Symmetry theory of the generalized Hamilton-Tabarrok-Leech's canonical equations is presented. The differential equations of motion of systems are written. The Noether symmetry, the form invariance and the Lie symmetry of systems are studied, and the corresponding conserved quantities are found. Finally, an example is given to illustrate the application of the results.

Keywords: generalized classical mechanics, H-T-L canonical equation, symmetry, conserved quantity

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