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含外规范场和动力学费米子自能的狄拉克算符及费米子凝聚*

杨 华[1)2)] 刘增平[2)] 王学雷[2)] 张胜海[1)]

1)(信息工程大学理学院数理系,郑州 450001)

2)(河南师范大学物理系,新乡 453002)

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将非阿贝尔规范理论中狄拉克算符行列式的计算从传统的只能含有硬费米子质量项的情况推广到可以含有动量相关的费米子自能的情况,并且行列式与费米子凝聚的计算都被推广到使之能够含有任意的外规范场.

关键词:费米子自能,外规范场,狄拉克算符的行列式,费米子凝聚

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1. 引 言

4 维欧式空间可重整的量子场理论中,含有外规范场的费米子系统内部的定域相互作用是通过狄拉克算符 D 来实现的双线性场作用,如下所示:

$$D \equiv \nabla\!\!\!/ - s + \mathrm{i}p\gamma_5,$$
$$\nabla_\mu \equiv \partial_\mu - \mathrm{i}v_\mu - \mathrm{i}a_\mu\gamma_5 = -\nabla_\mu^+, \tag{1}$$

其中 s, p, v_μ, a_μ 为厄米的标量、赝标量、矢量和轴矢量外规范场. 由此算符构成的可重整化的作用量为

$$\int \mathrm{d}^4x \bar{\psi} D\psi, \tag{2}$$

ψ 和 $\bar{\psi}$ 为狄拉克旋量. 显然,此作用量在下面的定域手征变换下是不变的:

$$\psi(x) \to \psi'(x) = [V_\mathrm{L}(x)P_\mathrm{L} + V_\mathrm{R}(x)P_\mathrm{R}]\psi(x),$$
$$J(x) \to J'(x) = [V_\mathrm{L}(x)P_\mathrm{R} + V_\mathrm{R}(x)P_\mathrm{L}] \times [J(x) + \mathrm{i}\partial\!\!\!/_x] \times [V_\mathrm{L}^+(x)P_\mathrm{L} + V_\mathrm{R}^+(x)P_\mathrm{R}]; \tag{3}$$

$V_\mathrm{L}(x)$ 和 $V_\mathrm{R}(x)$ 为左、右手旋转矩阵,$P_{\mathrm{R,L}} \equiv \frac{1}{2}(1 \pm \gamma_5)$ 为投影算符. 外场 $J(x)$ 的定义如下:

$$J(x) = -\mathrm{i}\,v\!\!\!/(x) - \mathrm{i}\,a\!\!\!/(x)\gamma_5 - s(x) + \mathrm{i}p(x)\gamma_5. \tag{4}$$

如果费米子系统中有 N_f 个狄拉克旋量,则理论遵从 $U_\mathrm{L}(N_\mathrm{f}) \otimes U_\mathrm{R}(N_\mathrm{f})$ 定域手征对称性.

在 4 维时空的量子场理论中,作用量(2)在路径积分中的贡献为

$$\int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, \mathrm{e}^{\int \mathrm{d}^4x[\bar{\psi}D\psi + \bar{I}\psi + \bar{\psi}I]} = \mathrm{e}^{\mathrm{trln}D - \int \mathrm{d}^4x\int \mathrm{d}^4y \bar{I}(x)D^{-1}(x,y)I(y)}, \tag{5}$$

$\bar{I}$ 和 I 为费米子系统的外源. 由于费米子系统的双线性特征,上式中的费米子场可以被积掉. 结果只依赖于含外规范场的费米子行列式 $\mathrm{trln}D$ 和传播子 $D^{-1}(x,y) = \langle x | D^{-1} | y\rangle$,一旦能够计算它们,费米子系统在路径积分中的贡献就变成是完全知道的了.

实际计算中,考虑到物理上的需要和红外发散的正规化,裸费米子质量项 m 被引进理论中,这是一个常量,等价于从标量场 s 中抽取出一凝聚项 $s \to s - m$. 引入这一费米子质量项后,相互作用量(2)变为

$$\int \mathrm{d}^4x \bar{\psi}(D + m)\psi. \tag{6}$$

这样再在路径积分中积掉费米子场后,结果变成是依赖于含一个硬质量项和外规范场的费米子行列式 $\mathrm{trln}(D + m)$ 和费米子传播子 $(D + m)^{-1}(x, y)$.

费米子质量项的引入明显地破坏了体系原始地手征对称性 $U_\mathrm{L}(N_\mathrm{f}) \otimes U_\mathrm{R}(N_\mathrm{F})$,而只遵从 $U_\mathrm{V}(N_\mathrm{F})$ 对称性,后面的推广中始终要求理论遵从 $U_\mathrm{V}(N_\mathrm{f})$

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对称性,本文最后将讨论如何通过非线性实现使推广后的理论仍然遵从 $U_L(N_f)\otimes U_R(N_f)$ 对称性.

除常数质量外,有文献将常数的硬质量项推广为一个矩阵[1]来进一步讨论对称性破缺.在这样的讨论中,由于硬质量通常是手征对称性放进理论中的,因此无法涉及所谓的动力学对称性破缺的情况,而只能讨论明显的对称性破坏的情况,而且,量子场理论中,手征对称性只是在领头阶计算中有可能被硬质量项破缺,如果包括进高阶量子修正,即使只引入硬质量,理论中也会自然地出现动量依赖的费米子自能项,因此有必要将前面提到的加入硬费米子质量的计算进一步推广到更一般的含费米子自能的情形.相比较而言,硬质量只是费米子自能的一种特殊情况,而且单纯引入硬质量项通常可能会造成不必要的紫外发散,而由动力学决定的依赖动量的费米子自能在很多情形下在紫外区域会有压低,因而有可能改善体系的紫外行为.含有硬质量项及外规范场的费米子行列式的计算方法最早是由 Schwinger 提出的[2,3],本文将讨论如何根据对称性将它和费米子传播子推广为含有费米子自能项 $\Sigma(k^2)$ 的情况.为简单起见,对费米子行列式,我们只研究其实部.对费米子传播子,我们只考虑两个时空点收缩到同一点,即费米子凝聚[4](传播子取同一时空点的极限)的情形.与反常相关的行列式的虚部和费米子传播子非定域部分在本文中将暂不讨论.费米子的凝聚通常作为理论中对称性序参量是不考虑外场的,但在有些情况下外场本身也可能会对对称性的实现方式造成决定性的影响,例如外磁场可能会引发手征对称性的自发破缺[5—8].建立含外规范场的费米子的凝聚的计算方法可以使我们有能力去探索外规范场所引发的各种物理问题.

2. 含有硬质量的费米子行列式与费米子凝聚

计算费米子行列式与费米子凝聚需要用到 Schwinger proper-time 公式[2,9],这是一种可以保持计算过程协变性的计算方法,在求解手征规范理论时,由于 γ_5 的存在而使传统的维数正规化方法不可用,这种方法就显得十分重要.作为回顾,我们先用这种方法直接计算含有硬质量项的费米子行列式的实部和费米子凝聚.

根据传统的 Schwinger proper-time 公式,费米子行列式的实部可分解为

$$\begin{aligned}
&\mathrm{Re}\ln\det(D+m)\\
&=\frac{1}{2}\mathrm{tr}\ln[(D^{+}+m)(D+m)]\\
&=\frac{1}{2}\lim_{\Lambda\to\infty}\Big[-\gamma+\ln\Lambda^2-\int\mathrm{d}^4x\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{\mathrm{d}\tau}{\tau}\\
&\quad\times\mathrm{tr}\,\mathrm{e}^{-m^2\tau}\langle x|\,\mathrm{e}^{-\tau(E-\nabla^2)}\,|x\rangle\Big],
\end{aligned}\tag{7}$$

tr 是对内部对称性指标求迹,如旋量、味道等.

$$E-\nabla^2=D^{+}D+D^{+}m+mD,\tag{8}$$

其中 E 定义为

$$\begin{aligned}
E&=-2ms-2\mathrm{i}m\not{a}\gamma_5+\frac{\mathrm{i}}{4}[\gamma^{\mu},\gamma^{\nu}]R_{\mu\nu}\\
&\quad+\gamma_{\mu}d^{\mu}(s-\mathrm{i}p\gamma_5)+\mathrm{i}\gamma^{\mu}[a_{\mu}\gamma_5(s-\mathrm{i}p\gamma_5)\\
&\quad+(s-\mathrm{i}p\gamma_5)a_{\mu}\gamma_5]+s^2+p^2-[s,p]\mathrm{i}\gamma_5 .
\end{aligned}\tag{9}$$

$$\begin{aligned}
R_{\mu\nu}&\equiv\mathrm{i}[\nabla_{\mu},\nabla_{\nu}]\\
&=[d_{\mu}a_{\nu}-d_{\nu}a_{\mu}]\gamma_5+V_{\mu\nu}-\mathrm{i}[a_{\mu},a_{\nu}],\\
d_{\mu}f&=\partial_{\mu}f-\mathrm{i}[\nu_{\mu},f],\\
V_{\mu\nu}&=\partial_{\mu}\nu_{\nu}-\partial_{\nu}\nu_{\mu}-\mathrm{i}[\nu_{\mu},\nu_{\nu}].
\end{aligned}$$

方程(7)中用到了下述关系:

$$\begin{aligned}
\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{\mathrm{e}^{-A\tau}}{\tau}\mathrm{d}\tau&=-E\mathrm{i}\Big(-\frac{A}{\Lambda^2}\Big)\\
&=-\gamma-\ln\Big(\frac{A}{\Lambda^2}\Big)-\sum_{n=1}^{\infty}\frac{(-1)^n}{n\,!n}\Big(\frac{A}{\Lambda^2}\Big)^n .
\end{aligned}\tag{10}$$

此外,矩阵 $A\equiv(D^{+}+m)(D+m)=E-\nabla^2+m^2$.由于 τ 是一个与对称性无关的参数(prorer time),这种计算方法就能保持系统原有的对称性.

根据标准的 Seely-DeWitt 展开[9],

$$\begin{aligned}
&\langle x|\,\mathrm{e}^{-\tau(E-\nabla^2)}\,|x\rangle\\
&=\frac{1}{16\pi^2}\Big[\frac{1}{\tau^2}-\frac{E}{\tau}+\frac{1}{2}E^2-\frac{1}{6}[\nabla_{\mu},[\nabla^{\mu},E]]\\
&\quad-\frac{1}{12}R_{\mu\nu}R^{\mu\nu}-\frac{\tau}{6}E^3+\frac{\tau}{12}\{E[\nabla^{\mu},[\nabla_{\mu},E]]\\
&\quad+[\nabla^{\mu},[\nabla_{\mu},E]]E+[\nabla^{\mu},E][\nabla_{\mu},E]\}\\
&\quad+\frac{\tau^2}{24}E^4+\cdots\Big].
\end{aligned}\tag{11}$$

(7)式中完成对 τ 的积分,Relndet$(D+m)$ 的展开式为

$$\begin{aligned}
&\mathrm{Re}\ln\det(D+m)\\
&=\mathrm{Re}\ln\det(\not{\partial}+m)-\frac{1}{32\pi^2}\lim_{\Lambda\to\infty}\int\mathrm{d}^4x\\
&\quad\times\mathrm{tr_f}\Big[8m\Big[\Lambda^2+m^2\Big(\ln\frac{m^2}{\Lambda^2}+\gamma-1\Big)\Big]s
\end{aligned}$$

$$
\begin{aligned}
&- 8m^2\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)a^2-\frac{4}{3}[d_\mu a^\mu]^2\\
&-\frac{2}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+1\right)(d_\mu a_\nu-d_\nu a_\mu)(d^\mu a^\nu-d^\nu a^\mu)\\
&-\left[\frac{4}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)+\frac{16}{3}\right]a^4\\
&+\left[\frac{4}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)+\frac{3}{3}\right]a_\mu a_\nu a^\mu a^\nu\\
&-4\left[\Lambda^2+m^2\left(3\ln\frac{m^2}{\Lambda^2}+3\gamma-1\right)\right]s^2\\
&-4\left[\Lambda^2+m^2\left(\ln\frac{m^2}{\Lambda^2}+\gamma-1\right)\right]p^2\\
&+\left(16\ln\frac{m^2}{\Lambda^2}+16\gamma+16\right)msa^2\\
&-\frac{2}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)V_{\mu\nu}V^{\mu\nu}\\
&+\mathrm{i}\left[\frac{8}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)+\frac{16}{3}\right]a^\mu a^\nu V_{\mu\nu}\\
&+8m\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)pd^\mu a_\mu\\
&+\varepsilon^{\sigma\rho\mu\nu}\left[-2\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)V_{\sigma\rho}d_\mu a_\nu\right.\\
&\left.+4\mathrm{i}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+2\right)a_\sigma a_\rho d_\mu a_\nu\right]+O(p^6)\Big],
\end{aligned}
\tag{12}
$$

式中 $\mathrm{tr_f}$ 是对味道指标求迹，$\mathrm{Relndet}(\not\partial+m)$是与外场无关的项：

$$
\begin{aligned}
&\mathrm{Relndet}(\not\partial+m)\\
&=-\frac{1}{2}\lim_{\Lambda\to\infty}\int \mathrm{d}^4x\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{\mathrm{d}\tau}{\tau}\mathrm{tr}\mathrm{e}^{-m^2\tau}\langle x\mid e^{\tau\partial^2}\mid x\rangle\\
&=-\frac{1}{32\pi^2}\lim_{\Lambda\to\infty}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{\mathrm{d}\tau}{\tau^3}\mathrm{e}^{-m^2\tau}\int \mathrm{d}^4x\,\mathrm{tr}l\\
&=-\frac{1}{32\pi^2}\lim_{\Lambda\to\infty}\Big[2\Lambda^4-4\Lambda^2m^2\\
&\quad-m^4\left(2\ln\frac{m^2}{\Lambda^2}+2\gamma-1\right)\Big]\int \mathrm{d}^4x\,\mathrm{tr}l.
\end{aligned}
\tag{13}
$$

(12)式是按照动量幂次展开的，其中 ∂_μ，ν_μ，a_μ 被认为 p 量级，s，p 为 p^2[10,11] 量级. 同理，利用上述方法可将费米子凝聚（传播子取同一时空点的极限）分解为

$$
\begin{aligned}
&(D+m)^{-1}(x,x)\\
&=\left[[(D^++m)(D+m)]^{-1}(D^++m)\right](x,x)\\
&=\lim_{\Lambda\to\infty}\int_{\frac{1}{\Lambda^2}}^{1}\mathrm{d}\tau\,\mathrm{e}^{-m^2\tau}\langle x|\mathrm{e}^{-\tau(E-\nabla^2)}(D^++m)|x\rangle\\
&=\lim_{\Lambda\to\infty}\int_{\frac{1}{\Lambda^2}}^{\infty}\mathrm{d}\tau\,\mathrm{e}^{-m^2\tau}\Big[-\langle x|\mathrm{e}^{-\tau(E-\nabla^2)}\not\nabla\mid x\rangle\\
&\quad+\langle x|\mathrm{e}^{-\tau(E-\nabla^2)}|x\rangle[m-s(x)-\mathrm{i}p(x)\gamma_5\\
&\quad-2i\not a(x)\gamma_5]\Big].
\end{aligned}
\tag{14}
$$

与(11)式相似，上式矩阵元

$$
\begin{aligned}
&\langle x|\mathrm{e}^{-\tau(E-\nabla^2)}\nabla_\mu|x\rangle\\
&=\frac{1}{16\pi^2}\Big[\frac{\mathrm{i}}{6\tau}[\nabla^\nu,R_{\nu\mu}]+\frac{1}{2\tau}[\nabla_\mu,E]\\
&\quad-\frac{1}{3}E[\nabla_\mu,E]-\frac{1}{6}[\nabla_\mu,E]E\Big]+\cdots,
\end{aligned}
\tag{15}
$$

完成对 τ 的积分，

$$
\begin{aligned}
&(D+m)^{-1}(x,x)\\
&=\frac{1}{16\pi^2}\lim_{\Lambda\to\infty}\Big\{\left[\Lambda^2+m^2\left(\ln\frac{m^2}{\Lambda^2}+\gamma-1\right)\right](m-\mathrm{i}p\gamma_5)\\
&-2\mathrm{i}m\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)\not a\gamma_5\\
&-\left[\Lambda^2+m^2\left(3\ln\frac{m^2}{\Lambda^2}+3\gamma-1\right)\right]s\\
&+\mathrm{i}m\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)d^\mu a_\mu\gamma_5+2m\left(\ln\frac{m^2}{\Lambda^2}+\gamma+1\right)a^2\\
&+\Big[\frac{\mathrm{i}}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)d^\nu V_{\mu\nu}\\
&+m\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)(pa^\mu-a^\mu p)\\
&-\frac{1}{3}[(d_\nu a^\nu)a^\mu-a^\mu(d_\nu a^\nu)]\\
&+\frac{1}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+1\right)[a_\nu(d^\mu a^\nu-d^\nu a^\mu)\\
&-(d^\mu a^\nu-d^\nu a^\mu)a_\nu]\\
&+\frac{1}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+2\right)d^\nu(a_\mu a_\nu-a_\nu a_\mu)\\
&+\varepsilon_\mu^{\sigma\rho\nu}\Big[\frac{\mathrm{i}}{2}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)d^\nu d_\sigma a_\rho\\
&+\frac{1}{4}\left(\ln\frac{m^2}{\Lambda^2}+\gamma-2\right)a^\nu V_{\sigma\rho}\\
&-\frac{1}{4}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+2\right)V_{\sigma\rho}a^\nu+\frac{2\mathrm{i}}{3}a_\sigma a_\rho a^\nu\Big]\Big]\gamma^\mu\\
&+\Big[\mathrm{i}m\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)d^\mu p\\
&-2\mathrm{i}m\left(\ln\frac{m^2}{\Lambda^2}+\gamma+1\right)(a^\mu s+sa^\mu)\\
&-\frac{\mathrm{i}}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+1\right)d^\nu(d_\nu a^\mu-d_\mu a^\nu)\\
&+\frac{1}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+2\right)(a^\nu V_{\mu\nu}-V_{\mu\nu}a^\nu)\\
&-\frac{\mathrm{i}}{3}d^\mu d^\nu a_\nu+\frac{\mathrm{i}}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+4\right)(a^2a^\mu+a^\mu a^2)
\end{aligned}
$$

$$-\frac{2\mathrm{i}}{3}\left(\ln\frac{m^2}{\Lambda^2}+\gamma+2\right)a_\nu a^\mu a^\nu$$
$$-\varepsilon_\mu^{\sigma\rho\nu}\left[-\frac{\mathrm{i}}{4}\left(\ln\frac{m^2}{\Lambda^2}+\gamma\right)d^\nu V_{\sigma\rho}-\frac{1}{3}a_\sigma d^\nu a_\rho\right.$$
$$\left.+\frac{1}{3}(d_\sigma a_\rho)a^\nu\right]\Big]\gamma_5\gamma_5^\mu\} + \text{tensor terms} + O(p^4). \tag{16}$$

上述结果是按照标量、赝标量、矢量和轴矢量给出的,理论中没有引进张量场,这里不能给出张量部分.

3. 从手征对称性明显破缺到动力学破缺的推广

非零的动量相关的费米子自能项 $\Sigma(k^2)$ 的存在,导致系统序参数$\langle 0|\bar{\psi}\psi|0\rangle\neq 0$,标志着对称性的动力学破缺. 由于 $\Sigma(k^2)$ 在坐标表象中的形式为$\Sigma(-\partial_x^2)\delta(x-y)$,将手征对称性明显破缺的拉氏量 $\bar{\psi}(D+m)\psi$ 简单推广至动力学手征对称性破缺的情况,拉氏量的形式应为 $\bar{\psi}[D+\Sigma(-\partial^2)]\psi$,然而,这样的简单推广是有问题的,其原因在于原始的$\bar{\psi}(D+m)\psi$ 在 $U_\mathrm{V}(N_\mathrm{f})$ 定域对称变换下是不变量.

$$\begin{aligned}\psi(x)&\to\psi'(x)=h^+(x)\psi(x),\\ s(x)&\to s'(x)=h^+(x)s(x)h(x),\\ p(x)&\to p'(x)=h^+(x)p(x)h(x),\\ \nu_\mu(x)&\to\nu'_\mu(x)=h^+(x)\nu_\mu(x)h(x)\\ &\qquad+h^+(x)[\mathrm{i}\partial_\mu h(x)],\\ a_\mu(x)&\to a'_\mu(x)=h^+(x)a_\mu(x)h(x);\end{aligned} \tag{17}$$

由此可知

$$\begin{aligned}&D_x+m\to(D_x+m)'\\ &\equiv\partial\!\!\!/_x-\mathrm{i}\,\nu\!\!\!/'(x)-\mathrm{i}\,a\!\!\!/(x)\gamma_5+m\\ &\quad-s'(x)+\mathrm{i}p'(x)\gamma_5\\ &=h^+(x)[\partial\!\!\!/_x-\mathrm{i}\,\nu\!\!\!/(x)-\mathrm{i}\,a\!\!\!/(x)\gamma_5\\ &\quad-s(x)+\mathrm{i}p(x)\gamma_5+m]h(x)\\ &=h^+(x)(D_x+m)h(x).\end{aligned} \tag{18}$$

上式的协变性是由于质量 m 是一个常数:

$$h^+(x)mh(x)=m, \tag{19}$$

所以,$\bar{\psi}(D+m)\psi$ 在 $U_\mathrm{V}(N_\mathrm{f})$变换下是不变量:

$$\begin{aligned}&\bar{\psi}(D+m)\psi\to\bar{\psi}'(D+m)'\psi'\\ &\equiv\bar{\psi}hh^+(D+m)hh^+\psi=\bar{\psi}(D+m)\psi.\end{aligned} \tag{20}$$

如果用 $\Sigma(-\partial^2)$ 来代替 m,由于依赖于 Σ 具体形式的微分算符的存在,(18) 式的相应变换式就不再是协变的了:

$$\begin{aligned}&h^+(x)\Sigma(-\partial_x^2)h(x)\\ &=\Sigma[-h^+(x)\partial_x^2h(x)]\\ &=\Sigma[-(\partial_\mu+h^+(x)[\partial_\mu h(x)])^2]\\ &\neq\Sigma(-\partial_x^2).\end{aligned} \tag{21}$$

所以,简单推广的表征动力学手征对称性破缺的拉氏量 $\bar{\psi}(D+\Sigma(-\partial^2))\psi$ 在$U_\mathrm{V}(N_\mathrm{f})$ 变换下不是不变的. 为了保持这种对称性,考虑$\Sigma(-\bar{\nabla}^2)$项

$$\bar{\nabla}^\mu\equiv\partial^\mu-\mathrm{i}\nu^\mu, \tag{22}$$

∇_μ上面有一短线以区别于我们在(1)式中引进的微商. 利用(17)式,$\bar{\nabla}_\mu$在 $U_\mathrm{V}(N_\mathrm{f})$变换下,

$$\bar{\nabla}_x^\mu\to\bar{\nabla}_x^{\mu'}\equiv\partial_x^\mu-\mathrm{i}\nu^{\mu'}(x)=h^+(x)\bar{\nabla}_x^\mu h(x), \tag{23}$$

即

$$\begin{aligned}\Sigma(-\bar{\nabla}_x^2)\to\Sigma(-\bar{\nabla}_x^{2'})&=\Sigma[-h^+(x)\bar{\nabla}_x^2h(x)]\\ &=h^+(x)\Sigma(-\bar{\nabla}_x^2)h(x),\end{aligned} \tag{24}$$

所以

$$\begin{aligned}&[D_x+\Sigma(-\bar{\nabla}_x^2)]\to[D'_x+\Sigma(-\bar{\nabla}_x^{2'})]\\ &=h^+(x)[D_x+\Sigma(-\bar{\nabla}_x^2)]h(x).\end{aligned} \tag{25}$$

这样,在(17)式变换下,$\bar{\psi}[D+\Sigma(-\bar{\nabla}^2)]\psi$ 就是不变的,是 $\bar{\psi}(D+m)\psi$ 的正确推广. 另外,原则上,我们也可以在$\bar{\nabla}^\mu$中增加一些(17)式变换下的协变量,如 $a_\mu\gamma_5$ 与一个常量的乘积项等,这些项不能由对称性本身完全确定,因此这些项可以认为是除了$\bar{\nabla}^\mu$和 $\Sigma(-\bar{\nabla}^2)$ 之外的相互作用. 这里给出的$\Sigma(-\bar{\nabla}^2)$是符合对称性要求的最小推广.

推广之后的费米子行列式的实部为

$$\begin{aligned}&\mathrm{Re}\ln\det[D+\Sigma(-\bar{\nabla}^2)]\\ &=\frac{1}{2}\mathrm{tr}\ln[[D^++\Sigma(-\bar{\nabla}^2)][D+\Sigma(-\bar{\nabla}^2)]]\\ &=-\frac{1}{2}\lim_{\Lambda\to\infty}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{\mathrm{d}\tau}{\tau}\\ &\quad\times\mathrm{tr}\,\mathrm{e}^{-\tau[\bar{E}-\bar{\nabla}^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-d\!\!\!/\Sigma(-\bar{\nabla}^2)]},\end{aligned} \tag{26}$$

式中

$$\begin{aligned}&\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)\\ &+\tilde{g}(\bar{\nabla}^2)K-d\!\!\!/\Sigma(-\bar{\nabla}^2)\\ &=[D^++\Sigma(-\bar{\nabla}^2)][D+\Sigma(-\bar{\nabla}^2)],\\ &[d\!\!\!/\Sigma(-\bar{\nabla}^2)]\\ &\equiv\gamma^\mu[d_\mu\Sigma(-\bar{\nabla}^2)]\\ &=\gamma^\mu(\partial_\mu\Sigma(-\bar{\nabla}^2)-\mathrm{i}[\nu_\mu,\Sigma(-\bar{\nabla}^2)]),\end{aligned}$$

$$
\begin{aligned}
\bar{E} \equiv & \frac{\mathrm{i}}{4}[\gamma^{\mu},\gamma^{\nu}]R_{\mu\nu}+\gamma_{\mu}d^{\mu}(s-\mathrm{i}p\gamma_5)\\
&+\mathrm{i}\gamma^{\mu}[a_{\mu}\gamma_5(s-\mathrm{i}p\gamma_5)+(s-\mathrm{i}p\gamma_5)a_{\mu}\gamma_5]\\
&+s^2+p^2-[s,p]\mathrm{i}\gamma_5,\\
g(x) = & \tilde{g}(x)\equiv\Sigma(-x),\\
J = & -\mathrm{i}\not{a}\gamma_5-s-\mathrm{i}p\gamma_5,\\
K = & -\mathrm{i}\not{a}\gamma_5-s+\mathrm{i}p\gamma_5.
\end{aligned}
\tag{27}
$$

推广后的费米子凝聚为

$$
\begin{aligned}
&[D+\Sigma(-\bar{\nabla}^2)]^{-1}(x,x)\\
&=[([D^{+}+\Sigma(-\bar{\nabla}^2)][D+\Sigma(-\bar{\nabla}^2)])^{-1}\\
&\quad\times[D^{+}+\Sigma(-\bar{\nabla}^2)]](x,x)\\
&=\lim_{\Lambda\to\infty}\int_{\frac{1}{\Lambda^2}}^{\infty}\mathrm{d}\tau\langle x|\mathrm{e}^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}\\
&\quad\times[-\not{\nabla}-s(x)-\mathrm{i}p(x)\gamma_5-2\mathrm{i}\not{a}(x)\gamma_5\\
&\quad+\Sigma(-\bar{\nabla}^2)]|x\rangle.
\end{aligned}
\tag{28}
$$

在以后的计算中，$\Sigma(k^2)$ 的紫外行为有以下限制：

$$
\frac{\Sigma^2(k^2)}{k^2}\xrightarrow{k^2\to\infty}0.
\tag{29}
$$

推广后的进一步计算中，由(26)式和(28)式可知关键是计算矩阵元

$$
\begin{aligned}
&\langle x|\mathrm{e}^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}|x\rangle,\\
&\langle x|\mathrm{e}^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}\\
&\times[-\not{\nabla}-s(x)-\mathrm{i}p(x)\gamma_5-2\mathrm{i}\not{a}(x)\gamma_5\\
&+\Sigma(-\bar{\nabla}^2)]|x\rangle.
\end{aligned}
\tag{30}
$$

这些矩阵元要比(11)式和(15)式复杂得多．原因在于现在指数上的算符不仅仅含有 2 阶椭圆微分算子，原则上，Σ 除受到(29)式限制外可以包括任意高阶的微分算子，这里不去关心 Σ 的具体形式．

下面将传统的 Schwinger proper-time 公式进行推广来计算上述矩阵元．

$$
\begin{aligned}
&\langle x|\mathrm{e}^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}|x\rangle,\\
&=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\exp\Big\{-\tau[\bar{E}(x)-\nabla_x^2-2\mathrm{i}k\cdot\nabla_x\\
&\quad+k^2+\Sigma^2(-\bar{\nabla}^2-2\mathrm{i}k\cdot\bar{\nabla}_x+k^2)\\
&\quad+Jg(\bar{\nabla}_x^2+2\mathrm{i}k\cdot\bar{\nabla}_x-k^2)\\
&\quad+\tilde{g}(\bar{\nabla}_x^2+2\mathrm{i}k\cdot\bar{\nabla}_x-k^2)K\\
&\quad-\not{d}\Sigma(-\bar{\nabla}_x^2-2\mathrm{i}k\cdot\bar{\nabla}_x+k^2)]\Big\},
\end{aligned}
\tag{31}
$$

其中 $\bar{E}$，J，K 为 p 量级．将(31)式进行低能展开，详细结果我们写在附录 B 中．同样，

$$
\begin{aligned}
&\langle x|\mathrm{e}^{-\tau[\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)]}\\
&\times[-\not{\nabla}-s(x)-\mathrm{i}p(x)\gamma_5-2\mathrm{i}\not{a}(x)\gamma_5\\
&+\Sigma(-\bar{\nabla}^2)]|x\rangle\\
&=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\exp\Big\{-\tau[\bar{E}(x)-\nabla_x^2-2\mathrm{i}k\cdot\nabla_x\\
&\quad+k^2+\Sigma^2(-\bar{\nabla}^2-2\mathrm{i}k\cdot\bar{\nabla}_x+k^2)\\
&\quad+Jg(\bar{\nabla}_x^2+2\mathrm{i}k\cdot\bar{\nabla}_x-k^2)\\
&\quad+\tilde{g}(\bar{\nabla}_x^2+2\mathrm{i}k\cdot\bar{\nabla}_x-k^2)K\\
&\quad-\not{d}\Sigma(-\bar{\nabla}_x^2-2\mathrm{i}k\cdot\bar{\nabla}_x+k^2)]\Big\}\\
&\quad\times[-\not{\nabla}_x-\mathrm{i}\not{k}-s(x)-\mathrm{i}p(x)\gamma_5-2\mathrm{i}\not{a}(x)\gamma_5\\
&\quad+\Sigma(-\bar{\nabla}_x^2-2\mathrm{i}k\cdot\bar{\nabla}_x+k^2)]|x\rangle.
\end{aligned}
\tag{32}
$$

本文第五节根据此矩阵元给出依赖于外场 s，p，ν_{μ}，a_{μ} 的费米子凝聚并对它进行讨论．

需要强调的是在传统的 Schwinger proper-time 公式中，结果的手征协变性是依赖于动量积分来完成的，而这里由于 $\Sigma(k^2)$ 的具体形式尚不清楚，动量积分不能直接完成，我们所采用的方法就必须是在不能首先完成积分的前提下仍能够明显保持手征协变性的处理方法．幸运的是我们发现可能的非协变项在动量空间都是全微商项(附录 B)，在(29)式限制下都不对最终结果给出贡献，使得我们在解析的完成动量积分前就已经知道能够像传统的 Schwinger proper-time 方法一样获得手征协变性结果．

4．含自能和外规范场的费米子行列式的实部

参数化费米子行列式的结果

$$
\begin{aligned}
&\mathrm{Relndet}[D+\Sigma(-\bar{\nabla}^2)]\\
&=\int\mathrm{d}^4x\,\mathrm{tr}_{\mathrm{f}}[C_0s+C_1a^2+C_2[d_{\mu}a^{\mu}]^2\\
&\quad+C_3(d^{\mu}a^{\nu}-d^{\nu}a^{\mu})(d_{\mu}a_{\nu}-d_{\nu}a_{\mu})+C_4a^4\\
&\quad+C_5a^{\mu}a^{\nu}a_{\mu}a_{\nu}+C_6s^2+C_7p^2+C_8sa^2\\
&\quad+C_9V^{\mu\nu}V_{\mu\nu}+C_{10}V^{\mu\nu}a_{\mu}a_{\nu}+C_{11}pd_{\mu}a^{\mu}]\\
&\quad+O(p^6),
\end{aligned}
\tag{33}
$$

将附录 B 中(B1)式的 $\bar{E}$，J，K，g 和 $\tilde{g}$ 的定义式代入，可求出上式中系数 C_i 对自能 $\Sigma(k^2)$ 的依赖关系：

$$
\begin{aligned}
C_0 &= -4\int\mathrm{d}\tilde{k}\;\Sigma_kX_k,\\
C_1 &= 2\int\mathrm{d}\tilde{k}\;\Big[(-2\Sigma_k^2+k^2\Sigma_k\Sigma_k')X_k^2\\
&\qquad+(-2\Sigma_k^2+k^2\Sigma_k\Sigma_k')\frac{X_k}{\Lambda^2}\Big],\\
C_2 &= \int\mathrm{d}\tilde{k}\;\Big[2A_KX_K^3+2A_k\frac{X_k^2}{\Lambda^2}+A_k\frac{X_k}{\Lambda^4}
\end{aligned}
$$

$$+\frac{k^2}{2}\Sigma_k'^2\frac{X_k}{\Lambda^2}+\frac{k^2}{2}\Sigma_k'^2X_k^2\Big],$$

$$C_3=\int\mathrm{d}\tilde{k}\Big[2B_kX_k^3+2B_k\frac{X_k^2}{\Lambda^2}+B_k\frac{X_k}{\Lambda^4}$$

$$+\frac{k^2}{2}\Sigma_k'^2\frac{X_k}{\Lambda^2}+\frac{k^2}{2}\Sigma_k'^2X_k^2\Big],$$

$$C_4=-2\int\mathrm{d}\tilde{k}\Big[\Big(\frac{4\Sigma_k^3}{3}+\frac{2k^2\Sigma_k^2}{3}+\frac{k^4}{18}\Big)$$

$$\times\Big(6X_k^4+\frac{6X_k^3}{\Lambda^2}+\frac{3X_k^2}{\Lambda^4}+\frac{X_k}{\Lambda^6}\Big)$$

$$-\Big(4\Sigma_k^2+\frac{k^2}{2}\Big)\Big(2X_k^3+\frac{2X_k^2}{\Lambda^2}+\frac{X_k}{\Lambda^4}\Big)+\frac{X_k}{\Lambda^2}+X_k^2\Big],$$

$$C_5=-\int\mathrm{d}\tilde{k}\Big[\Big(\frac{-4\Sigma_k^4}{3}-\frac{2k^2\Sigma_k^2}{3}+\frac{k^4}{18}\Big)$$

$$\times\Big(6X_k^4+\frac{6X_k^3}{\Lambda^2}+\frac{3X_k^2}{\Lambda^4}+\frac{X_k}{\Lambda^6}\Big)$$

$$+4\Sigma_k^2\Big(2X_k^3+\frac{2X_k^2}{\Lambda^2}+\frac{X_k}{\Lambda^4}\Big)-\frac{X_k}{\Lambda^2}-X_k^2\Big],$$

$$C_6=-2\int\mathrm{d}\tilde{k}\Big[(3\Sigma_k^2-2k^2\Sigma_k\Sigma_k')X_k^2$$

$$+(2\Sigma_k^2-k^2(1+2\Sigma_k\Sigma_k'))\frac{X_k}{\Lambda^2}\Big],$$

$$C_7=-2\int\mathrm{d}\tilde{k}\Big[(\Sigma_k^2-2k^2\Sigma_k\Sigma_k')X_k^2$$

$$-K^2(1+2\Sigma_k\Sigma_k')\frac{X_k}{\Lambda^2}\Big],$$

$$C_8=-4\int\mathrm{d}\tilde{k}\Big[(4\Sigma_k^3+k^2\Sigma_k)X_k^3+(4\Sigma_k^3+k^2\Sigma_k)\frac{X_k^2}{\Lambda^2}$$

$$+\Big(2\Sigma_k^3+\frac{1}{2}k^2\Sigma_k\Big)\frac{X_k}{\Lambda^4}-3\Sigma_k\frac{X_k}{\Lambda^2}-3\Sigma_kX_k^2\Big],$$

$$C_9=\int\mathrm{d}\tilde{k}\Big[\Big(\frac{1}{3}k^2\Sigma_k'\Sigma_k''-\frac{1}{3}k^2\Sigma_k\Sigma_k''\Big)X_k$$

$$+(-C_k+D_k)\frac{X_k}{\Lambda^2}-(C_k-D_k)X_k^2+2E_kX_k^3$$

$$+2E_k\frac{X_k^2}{\Lambda^2}+E_k\frac{X_k}{\Lambda^4}\Big],$$

$$C_{10}=4\mathrm{i}\int\mathrm{d}\tilde{k}\Big[2F_kX_k^3+2F_k\frac{X_k^2}{\Lambda^2}+F_k\frac{X_k}{\Lambda^4}$$

$$+\frac{k^2}{2}\Sigma_k'^2\frac{X_k}{\Lambda^2}+\frac{k^2}{2}\Sigma_k'^2X_K^2\Big],$$

$$C_{11}=4\int\mathrm{d}\tilde{k}\Big[\Big(\Sigma_k-\frac{1}{2}k^2\Sigma_k'\Big)\frac{X_k}{\Lambda^2}$$

$$+\Big(\Sigma_k-\frac{1}{2}k^2\Sigma_k'\Big)X_k^2\Big],\tag{34}$$

其中

$$\int\mathrm{d}\tilde{k}\equiv\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\frac{k^2+\Sigma^2(k^2)}{\Lambda^2}},\tag{35}$$

$$\Sigma_k\equiv\Sigma(k^2),\quad X_k\equiv\frac{1}{k^2+\Sigma^2(k^2)},\tag{36}$$

A_k,B_k,C_k,D_k,E_k,F_k 对 $\Sigma(k^2)$ 的依赖形式在附录A中给出. 可以看出根据 $\Sigma(k^2)$ 的渐近行为(29)式, $\mathrm{e}^{-\frac{k^2+\Sigma^2(k^2)}{\Lambda^2}}$ 在紫外保证了动量积分的收敛性.

(33)和(34)式就是含有动力学夸克自能和外规范场的费米子行列式实部的最终结果. 我们这里只给出了至 p^4 阶的结果,如果需要还可以扩展到更高阶. 作为理论的自检,取 $\Sigma(k^2)$ 为常数项 m,在 $\Lambda^2\to\infty$ 的情况下,动量积分(34)式很容易完成. 完成动量积分后,将各参数 C_i 在此极限下的结果代入(33)式,就重新得到了(12)式的结果,即我们的推广公式可以很容易地重复出含有硬费米子质量项的结果.

5. 含自能和外规范场的费米子凝聚

含有动量相关的费米子自能及外规范场的费米子凝聚可由直接计算(28)式给出,下面分别列出的这个凝聚的标量、赝标量、轴矢量和矢量部分.

标量部分:

$$(1)^{\xi\zeta}\big[[D+\Sigma(-\bar{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x,x)$$
$$+[D^++\Sigma(-\bar{\nabla}^{+2})]^{-1,(b\zeta)(a\xi)}(x,x)\big]$$
$$=-2[C_0+2C_6s+C_8a^2]^{ba}+O(p^4);\tag{37}$$

赝标量部分:

$$(\gamma_5)^{\xi\zeta}\big[[D+\Sigma(-\bar{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x,x)$$
$$-[D^++\Sigma(-\bar{\nabla}^{+2})]^{-1,(b\zeta)(a\xi)}(x,x)\big]$$
$$=-2\mathrm{i}[2C_7p+C_{11}d^\mu a_\mu]^{ba}+O(p^4);\tag{38}$$

轴矢量部分:

$$(\gamma_5\gamma_\mu)^{\xi\zeta}\big[[D+\Sigma(-\bar{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x,x)$$
$$+[D^++\Sigma(-\bar{\nabla}^{+2})]^{-1,(b\zeta)(a\xi)}(x,x)\big]$$
$$=-2\mathrm{i}[2C_1a^\mu-2C_2[d^\mu d^\nu a_\nu]$$
$$+2C_3d^\nu(d^\mu a_\nu-d_\nu a^\mu)+2C_4(a^2a^\mu+a^\mu a^2)$$
$$+4C_5a_\nu a^\mu a^\nu+C_8(sa^\mu+a^\mu s)$$
$$+C_{10}(aV^{\mu\nu}-V^{\mu\nu}a_\nu)-C_{11}d^\mu p]^{ba}+O(p^5);\tag{39}$$

矢量部分:

$$(\gamma_\mu)^{\xi\zeta}\big[[D+\Sigma(-\bar{\nabla}^2)]^{-1,(b\zeta)(a\xi)}(x,x)$$
$$-[D^++\Sigma(-\bar{\nabla}^{+2})]^{-1,(b\zeta)(a\xi)}(x,x)\big]$$
$$=2\mathrm{i}[2\mathrm{i}\bar{C}_1[d_\nu a^\nu,a^\mu]+4\mathrm{i}\bar{C}_2[d^\mu a^\nu-d^\nu a^\mu,a_\nu]$$
$$+4\bar{C}_3d^\nu V^{\mu\nu}+\bar{C}_4d_\nu[a^\mu,a^\nu]$$
$$+\mathrm{i}\bar{C}_5[p,a^\mu]]^{ba}+O(p^5),\tag{40}$$

系数 C_i 的值由上一部分给出,系数 $\bar{C}_i$ 的解析表达式为

$$\bar{C}_1 = 2\int \mathrm{d}\tilde{k}\ \bar{A}_k\left[2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}\right],$$

$$\bar{C}_2 = \int \mathrm{d}\tilde{k}\ \left[\bar{B}_k\left(2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}\right] + X_k^2 + \frac{X_k}{\Lambda^2}\right],$$

$$\bar{C}_3 = -\int \mathrm{d}\tilde{k}\ \bar{C}_k\left[\frac{X_k}{\Lambda^2} + X_k^2\right],$$

$$\bar{C}_4 = 4\mathrm{i}\int \mathrm{d}\tilde{k}\ \bar{F}_k\left[2X_k^3 + 2\frac{X_k^2}{\Lambda^2} + \frac{X_k}{\Lambda^4}\right],$$

$$\bar{C}_5 = 4\int \mathrm{d}\tilde{k}\ \Sigma_k\left[\frac{X_k}{\Lambda^2} + X_k^2\right]. \tag{41}$$

其中

$$\bar{A}_k = \frac{1}{6}[2\Sigma_k^2 - k^2\Sigma_k\Sigma_k'],$$

$$\bar{B}_k = \frac{1}{3}[k^2 + \Sigma_k^2 + k^2\Sigma_k\Sigma_k'],$$

$$\bar{C}_k = \frac{1}{3}\left[1 - \Sigma_k\Sigma_k' + \frac{3}{2}k^2\Sigma_k'^2\right],$$

$$\bar{F}_k = \frac{1}{6}[-4\Sigma_k^2 + 5k^2\Sigma_k\Sigma_k' + k^2]. \tag{42}$$

同样,如果取 $\Sigma(k^2)$ 为常数项 m,将各参数 C_i,$\bar{C}_i$ 在 $\Lambda^2\to\infty$ 的极限下的结果代入,就重新得到了(16)式的结果,这也说明这些推广是自洽的.

6. 总结和讨论

本文将含有硬费米子质量的狄拉克算符推广为含有动力学费米子自能的情况,它使得我们在理论中引入了可以表征系统动力学手征对称性破缺的项.为了计算推广后的费米子行列式和凝聚,我们又将传统的 Schwinger proper-time 方法进行了推广.这一推广在数学上表现为 Schwinger proper-time 公式中起关键作用的矩阵元指数上的算符从原来的 2 阶椭圆微分算符 $E-\nabla^2$((11)式)推广到可能含有任意高阶的微分算符 $\bar{E}-\nabla^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-\not{d}\Sigma(-\bar{\nabla}^2)$((30)式)的情况,高阶微分算符来源于表征动力学手征对称性破缺效应的费米子自能项 $\Sigma(-\bar{\nabla}^2)$.

推广以后的公式自动保持了(17)式所表示的 $U_{\mathrm{V}}(N_{\mathrm{f}})$ 定域对称性,通过非线性实现,该公式还可以推广为保持手征对称性 $U_{\mathrm{L}}(N_{\mathrm{f}})\otimes U_{\mathrm{R}}(N_{\mathrm{f}})$ 的理论,为此,需要引入在(3)式变换下非线性变换的定域场 $\Omega(x)$,

$$\Omega(x)\to\Omega'(x) = h^+(x)\Omega(x)V_{\mathrm{L}}^+(x)$$

$$= V_{\mathrm{R}}(x)\Omega(x)h(x), \tag{43}$$

然后将所有的外场进行旋转:

$$J_\Omega(x) = [\Omega(x)P_{\mathrm{R}} + \Omega^+(x)P_{\mathrm{L}}]$$
$$\times[J(x) + \mathrm{i}\not{\partial}][\Omega(x)P_{\mathrm{R}} + \Omega^+(x)P_{\mathrm{L}}]$$
$$\equiv -s_\Omega(x) + \mathrm{i}p_\Omega(x)\gamma_5 + \not{v}_\Omega(x)$$
$$+ \not{a}_\Omega(x)\gamma_5. \tag{44}$$

式中的 $h(x)$ 为引入的隐藏定域对称性 $U_{\mathrm{V}}(N_{\mathrm{f}})$ 的变换矩阵.将(26)式和(28)式中 $D+\Sigma(-\bar{\nabla}^2)$ 项中的所有外场都用旋转后的外场代替,即变为 $D_\Omega+\Sigma(-\bar{\nabla}_\Omega^2)$,$\bar{\nabla}_\Omega^\mu\equiv\partial^\mu-\mathrm{i}v_\Omega^\mu(x)$,其中

$$D_\Omega \equiv \not{\nabla}_\Omega - s_\Omega + \mathrm{i}p_\Omega\gamma_5,$$
$$\nabla_\Omega^\mu \equiv \partial^\mu - \mathrm{i}v_\Omega^\mu - \mathrm{i}a_\Omega^\mu\gamma_5,$$
$$\bar{\nabla}_\Omega^\mu \equiv \partial^\mu - \mathrm{i}v_\Omega^\mu(x). \tag{45}$$

在定域变换(3)和(43)下,$D_\Omega+\Omega(-\bar{\nabla}_\Omega^2)$ 的变换式为

$$J_\Omega(x)\to J_\Omega'(x)$$
$$= h^+(x)[J_\Omega(x) + \mathrm{i}\not{\partial}_x]h(x), \tag{46}$$

$$D_\Omega + \Sigma(-\bar{\nabla}_\Omega^2)\to D_\Omega' + \Sigma(-\bar{\nabla}_\Omega'^2)$$
$$= h^+(x)[D_\Omega + \Sigma(-\bar{\nabla}_\Omega^2)]h(x). \tag{47}$$

即 $J_\Omega(x)$ 和 $D_\Omega+\Sigma(-\bar{\nabla}_\Omega^2)$ 都为协变量.由此可知,通过隐藏对称性 $U_{\mathrm{V}}(N_{\mathrm{f}})$,定域手征对称性 $U_{\mathrm{L}}(N_{\mathrm{f}})\otimes U_{\mathrm{R}}(N_{\mathrm{f}})$ 得以非线性实现了,即我们推广的公式是可以通过非线性实现而保持定域手征对称性 $U_{\mathrm{L}}(N_{\mathrm{f}})\otimes U_{\mathrm{R}}(N_{\mathrm{f}})$ 的.为了简便,在前面的推广中,我们采用了未作手征旋转的外场,同时在理论推导中保持了 $U_{\mathrm{V}}(N_{\mathrm{f}})$ 对称性.

附录 A　系数定义

$$A_k = \frac{2}{3}k^2\Sigma_k\Sigma_k'(-1-2\Sigma_k\Sigma_k')$$
$$-\frac{1}{3}\Sigma_k^2(-1-2\Sigma_k\Sigma_k') + \frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2 + \Sigma_k\Sigma_k'')$$
$$+\frac{1}{6}k^4(\Sigma_k'^2 + \Sigma_k\Sigma_k''),$$

$$B_k = \frac{2}{3}k^2\Sigma_k\Sigma_k'(-1-2\Sigma_k\Sigma_k')$$
$$-\frac{1}{3}\Sigma_k^2(-1-2\Sigma_k\Sigma_k') + \frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2 + \Sigma_k\Sigma_k'')$$
$$+\frac{1}{18}k^4(\Sigma_k'^2 + \Sigma_k\Sigma_k'') + \frac{1}{6}k^2(-1-2\Sigma_k\Sigma_k'),$$

$$C_k = \frac{1}{3} - \frac{1}{3}\Sigma_k\Sigma_k' + \frac{1}{2}k^2\Sigma_k'^2,$$

$$D_k = -\frac{1}{2}k^2\Sigma_k'^2 + \frac{1}{6}k^2\Sigma_k\Sigma_k''(-1-2\Sigma_k\Sigma_k')$$
$$+\frac{2}{9}k^4\Sigma_k'\Sigma_k''(1+2\Sigma_k\Sigma_k')$$

$$-\frac{2}{9}k^4\Sigma_k'^2(-\Sigma_k'^2-\Sigma_k\Sigma_k'')$$
$$+\frac{1}{3}k^2\Sigma_k\Sigma_k'(-\Sigma_k'^2-\Sigma_k\Sigma_k''),$$
$$E_k=\frac{1}{6}k^2\Sigma_k\Sigma_k'(-1-2\Sigma_k\Sigma_k')^2$$
$$-\frac{1}{9}k^4\Sigma_k'^2(1+2\Sigma_k\Sigma_k')^2,$$
$$F_k=\frac{4}{3}k^2\Sigma_k\Sigma_k'-\frac{4}{3}k^2(\Sigma_k\Sigma_k')^2$$
$$-\frac{2}{3}\Sigma_k^2+\frac{2}{3}\Sigma_k^3\Sigma_k'$$
$$+\frac{1}{3}k^2\Sigma_k^2(\Sigma_k'^2+\Sigma_k\Sigma_k'')$$
$$+\frac{1}{9}k^4(\Sigma_k'^2+\Sigma_k\Sigma_k'')$$
$$+\frac{1}{3}k^2(-1-2\Sigma_k\Sigma_k')+\frac{1}{2}k^2.$$

附录 B：SEELY-DEWITT 展开的推广式

矩阵元$\langle x|\mathrm{e}^{-\tau[\bar{E}-\bar{\nabla}^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-d\Sigma(-\bar{\nabla}^2)]}|x\rangle$**的完整解析式**

$$\langle x|\mathrm{e}^{-\tau[\bar{E}-\bar{\nabla}^2+\Sigma^2(-\bar{\nabla}^2)+Jg(\bar{\nabla}^2)+\tilde{g}(\bar{\nabla}^2)K-d\Sigma(-\bar{\nabla}^2)]}|x\rangle$$
$$=\int\frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\tau f}\Big\{Y_1+k^2Y_2+k^4Y_3$$
$$+\frac{1}{8}k^2\left(-\frac{\tau}{3}f''+\tau^2f'f''-\frac{\tau^3}{3}f'^3\right)[\bar{\nabla}^\mu,\bar{\nabla}^\nu][\bar{\nabla}_\mu,\bar{\nabla}_\nu]$$
$$-\mathrm{i}\frac{\tau^2}{36}f'''k^4\gamma_5([\bar{\nabla}^\mu,[\bar{\nabla}_\mu,\bar{\nabla}^\nu]]a_\nu+a_\nu[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,\bar{\nabla}^\nu]])$$
$$+\mathrm{i}\frac{\tau^3}{36}f'f''k^4\gamma_5(-2[a^\nu[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,\bar{\nabla}_\nu]]]$$
$$+[\bar{\nabla}^\mu,\bar{\nabla}^\nu][\bar{\nabla}_\nu,a_\mu]+[\bar{\nabla}_\nu,a_\mu][\bar{\nabla}^\mu,\bar{\nabla}^\nu]$$
$$+[\bar{\nabla}_\nu,[\bar{\nabla}^\mu,\bar{\nabla}^\nu]a_\mu]+[\bar{\nabla}_\nu,a_\mu[\bar{\nabla}^\mu,\bar{\nabla}^\nu]])$$
$$-\mathrm{i}\frac{\tau^4}{36}f'^3\gamma_5([\bar{\nabla}_\nu,a][\bar{\nabla}^\mu,\bar{\nabla}^\nu]+[\bar{\nabla}^\mu,\bar{\nabla}^\nu][\bar{\nabla}_\nu,a_\mu])$$
$$-\frac{\tau^3}{12}f'^2k^2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,a^2]])$$
$$-\frac{\tau^3}{36}f''k^4([\bar{\nabla}^\mu,a_\mu][\bar{\nabla}^\nu,a_\nu]+2[\bar{\nabla}^\nu,a^\mu][\bar{\nabla}_\mu,a_\nu]$$
$$-2[\bar{\nabla}^\mu,[\bar{\nabla}^\nu,a_\mu a_\nu]]-2a_\mu a_\nu[\bar{\nabla}^\mu,\bar{\nabla}^\nu])$$
$$+\frac{\tau^4}{36}f'^2k^4(a^\nu[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,a_\nu]]+[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,a_\nu]]a^\nu$$
$$+[\bar{\nabla}^\mu,a^\nu][\bar{\nabla}_\mu,a_\nu]+[\bar{\nabla}^\nu,a^\mu][\bar{\nabla}_\mu,a_\nu]$$
$$-a_\mu a_\nu[\bar{\nabla}^\mu,\bar{\nabla}^\nu]-[\bar{\nabla}^\mu,\bar{\nabla}^\nu]a_\mu a_\nu)$$
$$-\frac{\tau^3}{12}f'k^2\gamma_5([a^\mu,[\bar{\nabla}^\mu,a^2]]+[\bar{\nabla}^\mu,[a_\mu,a^2]])$$
$$+\frac{\mathrm{i}\tau^4}{36}f'k^4\gamma_5([\bar{\nabla}^\mu,a^\nu][a_\nu,a_\mu]+[a_\nu,a_\mu][\bar{\nabla}^\mu,a^\nu]$$
$$+[\bar{\nabla}^\mu,[a_\mu,a^2]]+[a^\mu,[\bar{\nabla}^\mu,a^2]])$$
$$-\frac{\tau^3}{12}f'^2k^2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]+\frac{\tau^2}{4}f'k^2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F']]$$
$$+\frac{\tau^2}{4}k^2(g'^2J[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,J]]$$
$$+\tilde{g}'^2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,K]]K+g'\tilde{g}'[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,KJ]])$$
$$-\frac{\tau^3}{12}f'k^2(2F'[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]$$
$$+2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F']]F+2[\bar{\nabla}^\mu,F'][\bar{\nabla}_\mu,F]$$
$$+2F[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F']]+2[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]F'$$
$$+2[\bar{\nabla}^\mu,F][\bar{\nabla}_\mu,F']-[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,Jg'F+F\tilde{g}'K]]$$
$$-[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]Jg'-\tilde{g}'K[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]])$$
$$+\frac{\tau^4}{24}f'^2k^2(F[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]$$
$$+[\bar{\nabla}^\mu,[\bar{\nabla}_\mu,F]]F+[\bar{\nabla}^\mu,F][\bar{\nabla}_\mu,F])$$
$$+\frac{\tau^4}{4}k^2\gamma_5[g'(a_\mu[\bar{\nabla}^\mu,J]+[\bar{\nabla}^\mu,a_\mu J]$$
$$-J[\bar{\nabla}^\mu,a_\mu]+\tilde{g}'([\bar{\nabla}^\mu,a_\mu]K-[\bar{\nabla}^\mu,Ka_\mu]$$
$$-[\bar{\nabla}^\mu,K]a_\mu)]-\frac{\mathrm{i}\tau^2}{12}f'k^2\gamma_5([\bar{\nabla}^\mu,[a_\mu,F]]$$
$$+[a_\mu+[\bar{\nabla}^\mu,F]])-\frac{\tau^3}{12}k^2\gamma_5(F'[\bar{\nabla}^\mu,[a_\mu,F]]$$
$$+[a_\mu,[\bar{\nabla}^\mu,F']]F+[a_\mu,F'][\bar{\nabla}^\mu,F]$$
$$+F'[\bar{\nabla}^\mu,[a_\mu,F]]+[\bar{\nabla}^\mu,[a_\mu,F']]F$$
$$+[\bar{\nabla}^\mu,F'][a_\mu,F])-\frac{\mathrm{i}\tau^3}{12}g'k^2\gamma_5[g([\bar{\nabla}^\mu,J]a_\mu F$$
$$-a_\mu J[\bar{\nabla}^\mu,F]-J[\bar{\nabla}^\mu,a_\mu F]+[\bar{\nabla}^\mu,J]Fa_\mu$$
$$+F[\bar{\nabla}^\mu,J]a_\mu+[\bar{\nabla}^\mu,Fa_\mu J]+a_\mu F[\bar{\nabla}^\mu,J]$$
$$+[\bar{\nabla}^\mu,FJ]a_\mu)+[\tilde{g}(-a_\mu[\bar{\nabla}^\mu,KF]$$
$$-[\bar{\nabla}^\mu,K]Fa_\mu-[\bar{\nabla}^\mu,Ka_\mu F]-a_\mu[\bar{\nabla}^\mu,K]F$$
$$-a_\mu F[\bar{\nabla}^\mu,K]+[\bar{\nabla}^\mu,Fa_\mu]K-Fa_\mu[\bar{\nabla}^\mu,K]$$
$$+[\bar{\nabla}^\mu,F]Ka_\mu)]+\frac{\mathrm{i}\tau^4}{24}f'k^2\gamma_5(F[\bar{\nabla}^\mu,[a_\mu,F]]$$
$$+[\bar{\nabla}^\mu,[a_\mu\gamma_5,F]]F+[\bar{\nabla}^\mu,F][a_\mu,F]$$
$$+F[a_\mu,[\bar{\nabla}^\mu,F]]$$
$$+[a_\mu,[\bar{\nabla}^\mu,F]]F+[a_\mu,F][\bar{\nabla}^\mu,F])$$
$$-\frac{\tau^2}{4}\Sigma'k^2(F'[\bar{\nabla}_\mu,[-\mathrm{i}\nabla,\bar{\nabla}^\mu]]-[\bar{\nabla}^\mu,[-\mathrm{i}\nabla,\bar{\nabla}^\mu]]F')$$
$$+\mathrm{i}\frac{\tau^2}{2}\Sigma'k^2((\tilde{g}[\bar{\nabla}^\mu,K][\nabla,\bar{\nabla}^\mu]$$
$$+g[\nabla,\bar{\nabla}^\mu]][J,\bar{\nabla}^\mu])+\frac{\tau^2}{2}\Sigma'k^2[\nabla,\bar{\nabla}^\mu]^2$$
$$+\mathrm{i}\frac{\tau^3}{12}f'\Sigma'k^2([\bar{\nabla}^\mu,[\nabla,\bar{\nabla}^\mu]F]+[\bar{\nabla}_\mu,F[\nabla,\bar{\nabla}^\mu]]$$
$$+[[\nabla,\bar{\nabla}_\mu],[\bar{\nabla}_\mu,F]])-\frac{\tau^2}{2}\Sigma'k^2\gamma_5([\nabla,\bar{\nabla}^\mu]a_\mu$$
$$+a_\mu[\nabla,\bar{\nabla}^\mu])+\frac{\tau^3}{6}\Sigma'\gamma_5k^2(a_\mu[\nabla,\bar{\nabla}^\mu]F$$
$$+a^\mu F[\nabla,\bar{\nabla}^\mu]+[\nabla,\bar{\nabla}^\mu]Fa^\mu+[\nabla,\bar{\nabla}^\mu]a^\mu F$$
$$+Fa^\mu[\nabla,\bar{\nabla}^\mu]+F[\nabla,\bar{\nabla}^\mu]a^\mu)\Big\}$$
$$+\text{全微分项}.\qquad\text{(B1)}$$

上式的全微商项为

$$
\begin{aligned}
&\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\partial}{\partial k^\mu} k^\mu \Big\{ \mathrm{e}^{-\tau f} \Big[\frac{\tau}{72} k^2 f''' + \frac{\tau^2}{8} f'' \\
&- \frac{\tau^2}{24} k^2 f' f'' + \frac{\tau^3}{72} k^2 f'^3 + \frac{\tau^2}{8} f'^2 \Big] \nabla_x^4 \\
&- k^2 \mathrm{e}^{-\tau f} \Big[- \frac{\tau}{72} f''' + \frac{\tau^2}{24} f' f'' - \frac{\tau^3}{72} f'^3 \Big] \\
&\times \Big[\bar{\nabla}^\mu \bar{\nabla}^2 \bar{\nabla}_\mu + \bar{\nabla}^\mu \bar{\nabla}^\nu \bar{\nabla}_\mu \bar{\nabla}_\nu \Big] \\
&- \frac{\mathrm{i}\tau}{4} k^2 \gamma_5 \mathrm{e}^{-\tau f} (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \\
&+ \frac{\mathrm{i}\tau^2}{8} k^2 \gamma_5 \mathrm{e}^{-\tau f} f' [\bar{\nabla}^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \\
&+ (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}^2] \\
&- \frac{\mathrm{i}\tau^2 \gamma_5}{24} k^2 \mathrm{e}^{-\tau f} f'' [\bar{\nabla}_\mu \bar{\nabla}^2 a^\mu + \bar{\nabla}^2 \bar{\nabla}^\mu a_\mu \\
&+ a^\mu \bar{\nabla}_\mu \bar{\nabla}^2 + a_\mu \bar{\nabla}^2 \bar{\nabla}^\mu] \\
&+ \frac{\mathrm{i}\tau^3 \gamma_5}{72} k^2 \mathrm{e}^{-\tau f} f'^2 [\bar{\nabla}^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \\
&+ (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}^2 + 2 \bar{\nabla}^\mu (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \bar{\nabla}_\mu \\
&+ 2 \bar{\nabla}_\mu \bar{\nabla}^2 + 2 a_\mu \bar{\nabla}^2 \bar{\nabla}^\mu] \\
&+ \frac{\tau^2}{8} k^2 \mathrm{e}^{-\tau f(-k^2)} f'(-k^2) [\bar{\nabla}^2 a^2 + a^2 \bar{\nabla}^2] \\
&+ \frac{\mathrm{i}\tau^2 \gamma_5}{8} \mathrm{e}^{-\tau f} [a^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) \\
&+ (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) a^2] - \frac{\tau^2}{8} \mathrm{e}^{-\tau f} (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu)^2 \\
&- \frac{\tau^3}{72} k^2 \mathrm{e}^{-\tau f} f' [\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu)^2 \\
&+ 2 \bar{\nabla}^\nu (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu) a_\nu + 2 a^\nu (\bar{\nabla}^\mu a_\mu + a^\mu \bar{\nabla}_\mu) \bar{\nabla}_\nu \\
&+ 2 a^2 \bar{\nabla}^2 + 2 \bar{\nabla}^2 a^2 + 2 a^\mu \bar{\nabla}^2 a_\mu] \\
&- \frac{\mathrm{i}\tau^3}{36} \mathrm{e}^{-\tau f} [a^2 (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) + (a^\mu \bar{\nabla}_\mu + \bar{\nabla}^\mu a_\mu) a^2 \\
&+ a^\nu (a^\mu \bar{\nabla}_\mu + a^\mu a_\mu) a_\nu] - \frac{\tau}{4} \mathrm{e}^{-\tau f} (g' J \bar{\nabla}^2 + \tilde{g} \ \bar{\nabla}^2 K) \\
&+ \frac{\tau^2}{8} \mathrm{e}^{-\tau f} f' (F \bar{\nabla}^2 + \bar{\nabla}^2 F) \\
&+ \frac{\tau^2}{8} \mathrm{e}^{-\tau f} \{ g' \{ J \bar{\nabla}^2 F + F J \bar{\nabla}^2) \\
&+ \tilde{g} \ (\bar{\nabla}^2 K F + F \bar{\nabla}^2 K)] \\
&- \frac{\tau^3}{24} \mathrm{e}^{-\tau f} (\bar{\nabla}^2 F^2 + F^2 \bar{\nabla}^2 + F \bar{\nabla}^2 F) \\
&+ \frac{\mathrm{i}\tau^2 \gamma_5}{8} \mathrm{e}^{-\tau f} ([a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu] F + F [a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu]) \\
&- \frac{\mathrm{i}\tau^3 \gamma_5}{24} \mathrm{e}^{-\tau f} ([a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu] F^2 \\
&+ F^2 [a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu] + F [a^\mu \bar{\nabla}_\mu + \bar{\nabla}_\mu a^\mu] F) \\
&+ \mathrm{i} \frac{\tau}{4} \mathrm{e}^{-\tau f} \Sigma' [\not\nabla, \bar{\nabla}^2] \\
&- \mathrm{i} \frac{\tau^2}{8} \mathrm{e}^{-\tau f} \Sigma' ([\not\nabla, \bar{\nabla}^2] F + F [\not\nabla, \bar{\nabla}^2]) \Big\},
\end{aligned} \tag{B2}
$$

其中

$$
\begin{aligned}
f &\equiv k^2 + \Sigma^2(k^2), \\
f' &\equiv - \frac{\partial f}{\partial k^2}, \\
f'' &\equiv \frac{\partial^2 f}{\partial (k^2)^2}, \\
f''' &\equiv \frac{\partial^3 f}{\partial (k^2)^3}, \\
F &= \bar{E}(x) + J(x) g(-k^2) + \tilde{g} \ (-k^2) K(x), \\
F' &= J(x) g'(-k^2) + \tilde{g} \ '(-k^2) K(x), \\
Y_1(x) &= 1 - \tau [a^2(x) + F(x)] + \frac{\tau^2}{2} [a^2(x) + F(x)]^2 \\
&\quad - \frac{\tau^3}{6} [a^2(x) F^2(x) + F^2(x) a^2(x) \\
&\quad \times F(x) a^2(x) F(x) + F^3(x)] + \frac{\tau^4}{4!} F^4(x), \\
Y_2(x) &= \frac{\tau^2}{2} a^2(x) - \frac{\tau^3}{6} [2 a^4(x) + a^\mu(x) a^2(x) a_\mu(x) \\
&\quad + F(x) a^2(x) + a^2(x) F(x) + a^\mu F(x) a_\mu(x)] \\
&\quad - \frac{\tau}{4} [F^2(x) a^2(x) + a^2(x) F^2(x) \\
&\quad + a^\mu(x) F^2(x) a_\mu(x) + F(x) a^2(x) F(x) \\
&\quad + F(x) a^\mu(x) F(x) a_\mu(x) + a^\mu(x) F(x) a_\mu(x) F(x)], \\
Y_3(x) &= \frac{\tau^4}{4!} [a^4(x) + a^\mu(x) a^\nu(x) a_\mu(x) a_\nu(x) \\
&\quad + a^\mu(x) a^2(x) a_\mu(x)].
\end{aligned}
$$

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Fermion condensate and Dirac operator determinant with external gauge fields and dynamical fermion self energy*

Yang Hua[1)2)] Liu Zeng-Ping[2)] Wang Xue-Lei[2)] Zhang Sheng-Hai[1)]

1)(*Department of Mathematics and Physics*, *College of Science*, *Information Engineering University*, *Zhengzhou* 450001, *China*)

2)(*Department of Physics*, *Henan Normal University*, *Xinxiang* 453002, *China*)

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Abstract

The calculation of Dirac operator determinant in non-Abelian gauge theory is generalized from only containing hard fermion mass to that containing momentum dependent fermion self energy, and the calculation of determiant and fermion condensate is generalized to the case with arbitrary external gauge fields.

Keywords: fermion self energy, external field, Dirac operator determinant, fermion condensate

PACC: 1130R, 1000

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