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# 耗散介观电容耦合电路的量子效应*

邱深玉[1)†]　蔡绍洪[1)2)]

1)(贵州大学物理系,贵阳　550025)

2)(贵阳财经学院,贵阳　550004)

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对介观耗散电容耦合电路作阻尼谐振子处理,将其量子化,在此基础上研究电荷和电流在能量本征态下的量子涨落,并对其进行讨论.结果表明,每个回路的电荷、电流都存在量子涨落,且两回路的量子噪声是相互关联的.

**关键词**:介观耗散电路,电容耦合,量子涨落

**PACC**:7335

## 1. 引　　言

随着纳米技术和纳米电子学的飞速发展,电路及电路器件小型化、高集成度的趋势越来越明显[1,2].关于电路及器件量子效应已成为研究介观物理的热点问题之一,大量的文献分别对 LC 串联电路、RLC 串联电路、电容耦合电路和电感耦合电路进行了研究[3—14],但是在研究耦合电路中涉及到耗散耦合的文献不多,由于实际电路总是存在耗散的,因此有必要进行研究讨论.本文借鉴关于阻尼谐振子量子化[15]的研究思想,运用线性变换,研究耗散介观电容耦合电路的电荷、电流在能量本征态下的量子涨落.

## 2. 耗散介观电容耦合电路的量子化

对于图一所示的耗散电容耦合电路,根据 Kirchhoff 定律,其经典运动方程为

$$L_1\ddot{q}_1 + R_1\dot{q}_1 + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C} = \varepsilon(t), \tag{1}$$

$$L_2\ddot{q}_2 + R_2\dot{q}_2 + \frac{q_2}{C_2} + \frac{q_2 - q_1}{C} = 0. \tag{2}$$

令 $i_1 = \dot{q}_1$ 和 $i_2 = \dot{q}_2$,由(1)式和(2)式得

$$\dot{q}_1 = i_1,$$

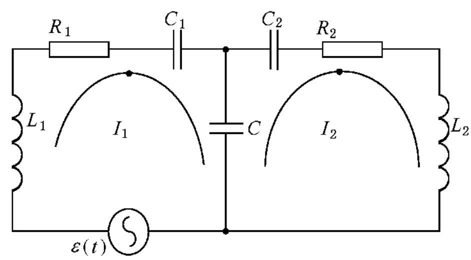


图 1　耗散电容耦合电路

$$\dot{i}_1 = -\lambda_1 i_1 - \frac{q_1}{L_1 C_1} - \frac{q_1 - q_2}{L_1 C} + \frac{\varepsilon(t)}{L_1}$$

$$\left(\lambda_1 = \frac{R_1}{L_1}\right), \tag{3}$$

$$\dot{q}_2 = i_2,$$

$$\dot{i}_2 = -\lambda_2 i_2 - \frac{q_2}{L_2 C_2} - \frac{q_2 - q_1}{L_2 C}$$

$$\left(\lambda_2 = \frac{R_2}{L_2}\right). \tag{4}$$

我们将 $q$ 和 $i$ 分别视为坐标算符和动量算符,但它们不是对易量,满足下列条件:

$$\frac{\partial \dot{q}_s}{\partial q_s} + \frac{\partial \dot{i}_s}{\partial i_s} = -\frac{R_s}{L_s}, \tag{5}$$

$$[\hat{q}_s, \hat{i}_s] = \frac{\mathrm{j}\hbar}{L_s}\mathrm{e}^{\frac{R_s}{L_s}t} \neq \frac{\mathrm{j}\hbar}{L_s}$$

$$(s = 1,2,\quad \mathrm{j}^2 = -1). \tag{6}$$

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† 通信联系人. E-mail:qiushenyu@163.com

这表明在有耗散情况下 $q_s$，$i_s$ 不能构成一对正则共轭变量. 引入正则化变换

$$\begin{aligned} Q_s &= q_s \exp(\lambda_s t/2), \\ I_s &= (L_s i_s + R_s q_s/2)\exp(\lambda_s t/2). \end{aligned} \tag{7}$$

由(7)式容易证明：$[Q_s, I_s] = \mathrm{j}\hbar\ (s = 1,2)$，这时 $Q_s$，$I_s$ 分别为复正则电荷和电流. 这时(1)式和(2)式可写为

$$\ddot{Q}_1 + \Omega_1^2 Q_1 + \omega_1^2(Q_1 - Q_2) = \frac{1}{L_1}\varepsilon(t)\mathrm{e}^{\frac{R_1}{2L_1}t}, \tag{8}$$

$$\ddot{Q}_2 + \Omega_2^2 Q_2 + \omega_2^2(Q_2 - Q_1) = 0, \tag{9}$$

其中 $\Omega_1^2 = \frac{1}{L_1 C_1} - \left(\frac{R_1}{2L_1}\right)^2$，$\omega_1^2 = \frac{1}{L_1 C}$和 $\Omega_2^2 = \frac{1}{L_2 C_2} - \left(\frac{R_2}{2L_2}\right)^2$，$\omega_2^2 = \frac{1}{L_2 C}$，因为复正则电荷和电流是一对共轭量，当 $\varepsilon(t) = 0$ 时，根据哈密顿正则方程可得体系的哈密顿量为

$$\begin{aligned} H &= \frac{I_1^2}{2L_1} + \frac{I_2^2}{2L_2} + \frac{1}{2}L_1\Omega_1^2 Q_1^2 + \frac{1}{2}L_2\Omega_2^2 Q_2^2 \\ &\quad + \frac{1}{2}L_1\omega_1^2(Q_1 - Q_2)^2 + \frac{1}{2}L_2\omega_2^2(Q_2 - Q_1)^2. \end{aligned} \tag{10}$$

对(10)式进行如下的电荷、电流的线性变换：

$$Q_1' = \left(\frac{L_1}{L_2}\right)^{1/4} Q_1\cos\frac{\phi}{2} - \left(\frac{L_2}{L_1}\right)^{1/4} Q_2\sin\frac{\phi}{2}, \tag{11}$$

$$Q_2' = \left(\frac{L_1}{L_2}\right)^{1/4} Q_1\sin\frac{\phi}{2} + \left(\frac{L_2}{L_1}\right)^{1/4} Q_2\cos\frac{\phi}{2}, \tag{12}$$

$$I_1' = \left(\frac{L_2}{L_1}\right)^{1/4} I_1\cos\frac{\phi}{2} - \left(\frac{L_1}{L_2}\right)^{1/4} I_2\sin\frac{\phi}{2}, \tag{13}$$

$$I_2' = \left(\frac{L_2}{L_1}\right)^{1/4} I_1\sin\frac{\phi}{2} + \left(\frac{L_1}{L_2}\right)^{1/4} I_2\cos\frac{\phi}{2}, \tag{14}$$

并取

$$\mathrm{tg}\phi = \frac{2\sqrt{L_1 L_2}}{[L_1 L_2\Omega_1^2 - L_1 L_2\Omega_2^2 + 2C(L_1 + L_2)]}, \tag{15}$$

则量子化后体系的哈密顿量可以写为

$$H = \frac{I_1'^2}{2\sqrt{L_1 L_2}} + \frac{I_2'^2}{2\sqrt{L_1 L_2}} + \frac{\alpha}{2}Q_1'^2 + \frac{\beta}{2}Q_2'^2, \tag{16}$$

式中

$$\begin{aligned} \alpha &= \left[\frac{\sqrt{L_1 L_2}}{2}\Omega_1^2 + C\left(\frac{L_2}{L_1}\right)^{1/2}\right]\cos^2\frac{\phi}{2} \\ &\quad + \left[\frac{\sqrt{L_1 L_2}}{2}\Omega_2^2 + C\left(\frac{L_1}{L_2}\right)^{1/2}\right]\sin^2\frac{\phi}{2} + C\sin\phi, \end{aligned}$$

$$\begin{aligned} \beta &= \left[\frac{\sqrt{L_1 L_2}}{2}\Omega_1^2 + C\left(\frac{L_2}{L_1}\right)^{1/2}\right]\sin^2\frac{\phi}{2} \\ &\quad + \left[\frac{\sqrt{L_1 L_2}}{2}\Omega_2^2 + C\left(\frac{L_1}{L_2}\right)^{1/2}\right]\cos^2\frac{\phi}{2} - C\sin\phi. \end{aligned}$$

由(16)式可见，经过线性变换后，哈密顿量的耦合项被消除，化为两个彼此独立的线性谐振子的哈密顿代数之和，其等效质量和等效频率为

$$\begin{aligned} M_1 &= M_2 = \sqrt{L_1 L_2}, \\ \gamma_1^2 &= \frac{\alpha}{\sqrt{L_1 L_2}}, \\ \gamma_2^2 &= \frac{\beta}{\sqrt{L_1 L_2}}. \end{aligned} \tag{17}$$

由量子力学理论可得耗散介观电容耦合的能谱和体系的本征矢量可分别写为

$$E_{n_1,n_2} = \left(n_1 + \frac{1}{2}\right)\hbar\gamma_1 + \left(n_2 + \frac{1}{2}\right)\hbar\gamma_2 \quad (n_1, n_2 = 0,1,2,3,\cdots) \tag{18}$$

$$|\psi_{n_1,n_2}\rangle = |n_1\rangle \otimes |n_2\rangle, \quad (n_1, n_2 = 0,1,2,3,\cdots) \tag{19}$$

式中 $|n_1\rangle$和 $|n_2\rangle$分别表示频率为 $\gamma_1$，$\gamma_2$ 的单个谐振子的本征矢量.

## 3. 能量本征态下耗散介观电容耦合电路的量子涨落

为求体系处在任意的本征态下耗散介观电容耦合电路中电荷、电流的量子涨落，对于上述两个独立的谐振子，引入推广的消灭和产生算符：

$$a_s = \left(\frac{M_s\gamma_s}{2\hbar}\right)^{1/2}\left(Q_s' + \frac{\mathrm{j}}{M_s\gamma_s}I_s'\right), \tag{20}$$

$$a_s^+ = \left(\frac{M_s\gamma_s}{2\hbar}\right)^{1/2}\left(Q_s' - \frac{\mathrm{j}}{M_s\gamma_s}I_s'\right), (s = 1,2) \tag{21}$$

由 $[\hat{Q}_s', \hat{I}_s'] = \mathrm{j}\hbar$ 可得 $[a_s, a_s^+] = 1$. 由(20)式和(21)式可得

$$\begin{aligned} Q_s' &= \left(\frac{\hbar}{2M_s\gamma_s}\right)^{1/2}(a_s^+ + a_s), \\ I_s' &= \mathrm{j}\left(\frac{\hbar M_s\gamma_s}{2}\right)^{1/2}(a_s^+ - a_s), \end{aligned} \tag{22}$$

式中 $s = 1,2$，利用(22)式则可求得在态式(19)下 $Q_s'$ 和 $I_s'(s = 1,2)$ 的涨落平均值和方均值

$$\begin{aligned} \langle Q_s'\rangle &= \langle I_s'\rangle = 0, \\ \langle Q_s'^2\rangle &= \hbar(2n + 1)/2M_s\gamma_s, \\ \langle I_s'^2\rangle &= \hbar M_s\gamma_s(2n + 1)/2, \end{aligned} \tag{23}$$

$$\langle(\Delta Q_s')^2\rangle=\langle Q_s'^2\rangle,$$
$$\langle(\Delta I_s')^2\rangle=\langle I_s'^2\rangle,$$
$$\langle(\Delta Q_s')^2\rangle\langle(\Delta I_s')^2\rangle=\frac{\hbar^2}{4}(2n+1)^2. \quad (24)$$

由(23)式和(11)式—(14)式可得在态式(19)下,耗散介观电容上电路中的 $Q_s$,$I_s$ 量子涨落的平均值和方均值为

$$\langle Q_s\rangle=\langle I_s\rangle=0,\quad (s=1,2) \tag{25}$$

$$\langle Q_1^2\rangle=\frac{\hbar}{2L_1}\left[(2n_1+1)\frac{1}{\gamma_1}\cos^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\sin^2\frac{\phi}{2}\right], \tag{26}$$

$$\langle Q_2^2\rangle=\frac{\hbar}{2L_2}\left[(2n_1+1)\frac{1}{\gamma_1}\sin^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\cos^2\frac{\phi}{2}\right], \tag{27}$$

$$\langle I_1^2\rangle=\frac{L_1\hbar}{2}\left[(2n_1+1)\gamma_1\cos^2\frac{\phi}{2}+(2n_2+1)\gamma_2\sin^2\frac{\phi}{2}\right], \tag{28}$$

$$\langle I_2^2\rangle=\frac{L_2\hbar}{2}\left[(2n_1+1)\gamma_1\cos^2\frac{\phi}{2}+(2n_2+1)\gamma_2\sin^2\frac{\phi}{2}\right],(n_1,n_2=1,2,3,\cdots). \tag{29}$$

由正则变换式(7)及(26)式—(29)式可知变换前耗散介观电容耦合电路的电荷、电流的平均值和均方值:

$$\langle q_s\rangle=\langle i_s\rangle=0,(s=1,2) \tag{30}$$

$$\langle q_1^2\rangle=\langle Q_1^2\exp(-\lambda_1 t)\rangle=\frac{\hbar}{2L_1}\left[(2n_1+1)\frac{1}{\gamma_1}\cos^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\sin^2\frac{\phi}{2}\right]\exp(-\lambda_1 t), \tag{31}$$

$$\langle q_2^2\rangle=\langle Q_2^2\exp(-\lambda_2 t)\rangle=\frac{\hbar}{2L_2}\left[(2n_1+1)\frac{1}{\gamma_2}\sin^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\cos^2\frac{\phi}{2}\right]\exp(-\lambda_2 t), \tag{32}$$

$$\begin{aligned}\langle i_1^2\rangle&=\frac{1}{L_1^2}\exp(-\lambda_1 t)\langle I_1\rangle^2+\frac{L_1^2R_1^2}{4}\exp(-\lambda_1 t)\langle Q_1\rangle^2\\&=\frac{\hbar}{2L_1}\left[(2n_1+1)\gamma_1\cos^2\frac{\phi}{2}+(2n_2+1)\gamma_2\sin^2\frac{\phi}{2}\right]\exp(-\lambda_1 t)\\&\quad+\frac{L_1R_1^2\hbar}{8}\left[(2n_1+1)\frac{1}{\gamma_1}\cos^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\sin^2\frac{\phi}{2}\right]\exp(-\lambda_1 t),\end{aligned} \tag{33}$$

$$\begin{aligned}\langle i_2^2\rangle&=\frac{1}{L_2^2}\exp(-\lambda_2 t)\langle I_2\rangle^2+\frac{L_2^2R_2^2}{4}\exp(-\lambda_2 t)\langle Q_2\rangle^2\\&=\frac{\hbar}{2L_2}\left[(2n_1+1)\gamma_1\cos^2\frac{\phi}{2}+(2n_2+1)\gamma_2\sin^2\frac{\phi}{2}\right]\exp(-\lambda_2 t)\\&\quad+\frac{L_2R_2^2\hbar}{8}\left[(2n_1+1)\frac{1}{\gamma_1}\sin^2\frac{\phi}{2}+(2n_2+1)\frac{1}{\gamma_2}\cos^2\frac{\phi}{2}\right]\exp(-\lambda_2 t).\end{aligned} \tag{34}$$

## 4. 讨 论

本文讨论了耗散介观电容电路在任意能量本征态下的量子涨落,(26)式—(29)式与文献[10]中的(23)式—(26)式相似,所不同的是个别参数(频率)的不同,这是因为在本文中的电路是耗散电路,而文献[10]中电路是非耗散的;并且选择的参量不同,从(25)式—(34)式计算结果表明:每个回路的电荷和电流的量子涨落不仅与其所处的量子态有关,还和回路自身的器件参数有关,而且还与另一个回路的器件有关,特别是从(34)式可以看出,在回路中电流的量子涨落受本回路耗散电阻影响较大,而受另一个回路耗散电阻影响相对较少.同时涨落作为自组织有序结构的触发器和系统稳定性的干扰[16],会对介观自组织结构的形成和消失产生重要影响,会给介观电路的运行带来不利因素.通过研究耗散介观电容耦合电路的量子涨落,对于进一步设计微小电路,降低量子噪声有一定的指导意义,该结果也对我们进一步深入地研究介观物质系统中自组织有序结

构的形成及临界涨落机理具有参考价值.


[1] Srivastava Y, Widom A 1987 *Phys. Rep.* **148** 1

[2] Buot F A 1993 *Phys. Rep.* **224** 73

[3] Chen B, Li Y Q, Sha J, Zhang Q R 1996 *Chin. Sci. Bull.* **41** 1275 (in Chinese)[陈 斌、李有泉、沙 键、张其瑞 1996 科学通报 **41** 1275]

[4] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: John wiley)

[5] Dong C H 1997 *Acta Phys. Sin.* **46** 467 (in Chinese)[董传华 1997 物理学报 **46** 467]

[6] Ji Y H, Lei M S, Xie F S 1999 *Chin. J. Quantum Electron* **16** 526 (in Chinese)[嵇英华、雷敏生、谢芳森 1999 量子电子学报 **16** 526]

[7] Li H Q 2004 *Chin. J. Quantum Optic* **10** 143(in Chinese)[李洪奇 2004 量子光学学报 **10** 143]

[8] Cui Y S 1998 *Chin. J. Quantum Electron* **15** 542(in Chinese)[崔元顺 1998 量子电子学报 **15** 542]

[9] Wang J S, Liu T K, Zhan M S 2000 *Acta Phys. Sin.* **49** 2271(in Chinese)[王继锁、刘堂昆、詹明生 2000 物理学报 **49** 2271]

[10] Wang J S, Han B C, Sun C Y 1998 *Acta Phys. Sin.* **47** 1187(in Chinese)[王继锁、韩保存、孙长勇 1998 物理学报 **47** 1187]

[11] Wang J S, Feng J, Zhan M S 2001 *Acta Phys. Sin.* **50** 299(in Chinese)[王继锁、冯 健、詹明生 2001 物理学报 **50** 299]

[12] Wang J S, Feng J, Zhan M S 2001 *Phys. Lett.* A **281** 341

[13] Ji Y H, Lei M S, Ouyang C Y 2002 *Chin. Phys.* **11** 163

[14] Song T Q 2004 *Acta Phys. Sin.* **53** 1352(in Chinese)[宋同强 2004 物理学报 **53** 1352]

[15] Peng H W 1980 *Acta Phys. Sin.* **29** 1084(in Chinese)[彭恒武 1980 物理学报 **29** 1084]

[16] Cai S H, Hu L, Dai L J *et al* 2000 *Chin. Phys.* **9** 450


# Quantum effect of dissipative mesoscopic capacitance coupled circuit*


Qiu Shen-Yu[1)†] Cai Shao-Hong[1)2)]

1)(*Department of Physics, Guizhou University, Guiyang* 550025, *China*)

2)(*Guizhou College of Finance and Economics, Guiyang* 550004, *China*)


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Abstract

The dissipative mesoscopic coupled circuits are obtained by the method of damped harmonic oscillator quantization. The quantum fluctuations of charge and current in an arbitrary eigenstate of the system have also been given. The result shows that the quantum fluctuations of the charge and current exist in all states, and the fluctuations in the component circuits are corelated.



**Keywords**: mesoscopic dissipative circuit, capacitance coupling, quantum fluctuation

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† Corresponding author. E-mail: quishenyu@163.com