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磁各向异性色散介质散射的 Padé 时域有限差分方法分析*

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根据矩阵 Padé 逼近理论,把磁化色散介质的相对磁导率张量表示成以 jω 为自变量的矩阵函数形式,用 $\partial/\partial t$ 代替 jω 后过渡到时域,再引入离散时域移位算子代替时间微分算子.进而导出磁化色散介质中的磁感应强度 $\boldsymbol{B}$ 和磁场强度 $\boldsymbol{H}$ 在离散时域的色散关系,并将其具体应用于旋磁介质,得到了这种磁化色散介质的 Padé 时域有限差分方法的递推表达式.作为验证,用这种方法计算了磁化铁氧体球的后向雷达散射截面,所得结果与文献结果一致.理论推导及算例表明,该方法是正确和有效的.

关键词:各向异性介质,色散介质,矩阵 Padé 逼近,时域有限差分方法
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1. 引　言

铁氧体材料为色散介质,在有外加磁场时为各向异性.近年来,对其物理特性的研究已成为热点之一,如文献[1,2]分别研究了磁化铁氧体材料的自旋电流及自旋极化电流的影响和作用.本文主要研究应用于磁各向异性色散介质的电磁散射的时域有限差分(FDTD)算法.对于色散介质的 FDTD 方法研究现在有递推卷积法[3]、辅助差分方程法[4—7]、Z 变换法[8]、移位算子法[9]等,但它们都是用来解决各向同性介质的电磁问题.本文讨论磁各向异性色散介质的 FDTD 方法.根据矩阵 Padé 逼近理论[10],把色散介质的相对磁导率张量写成以 jω 为自变量的矩阵形式,用 $\partial/\partial t$ 代替 jω 过渡到时域,再引入离散时域移位算子代替时间微分算子来处理矩阵函数形式的磁导率张量,进而导出磁化色散介质中磁感应强度 $\boldsymbol{B}$ 和磁场强度 $\boldsymbol{H}$ 之间的色散关系在离散时域的表达式,并且将其具体应用于旋磁介质,得到了这种介质的 Padé-FDTD 算法表达式.

作为验证,计算了磁化铁氧体球的后向雷达散射截面(RCS),所得结果与文献结果一致.理论推导及算例表明,该方法是正确有效的.

2. 磁各向异性色散介质中的 FDTD 迭代式

磁各向异性介质中无源麦克斯韦旋度方程为

$$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1}$$

$$\nabla\times\boldsymbol{H} = \varepsilon_0\varepsilon_{\mathrm{r}}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{2}$$

$$\boldsymbol{B} = \mu_0\boldsymbol{\mu}_{\mathrm{r}}\cdot\boldsymbol{H}. \tag{3}$$

设介质的介电系数 ε_{r} 为各向同性且与频率无关,但介质磁性质为色散且各向异性,即 $\boldsymbol{\mu}_{\mathrm{r}} = \boldsymbol{\mu}_{\mathrm{r}}(\omega)$ 且为张量.所以在用 FDTD 方法处理这种介质的电磁散射时,关于电场的 FDTD 迭代式与常规 FDTD 相同,即

$$E_x^{n+1}\left(i+\frac{1}{2},j,k\right)$$
$$=E_x^{n}\left(i+\frac{1}{2},j,k\right)$$

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$$+\frac{\Delta t}{\varepsilon}\left[\frac{H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)-H_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y}\right.$$
$$\left.-\frac{H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-H_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\right]. \quad (4)$$

而磁场分量的 FDTD 迭代式则要按如下方式进行处理(以 x 分量为例):

$$B_x^{n+\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)$$
$$=B_x^{n-\frac{1}{2}}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)$$
$$-\Delta t\left[\frac{E_z^n\left(i,j+1,k+\frac{1}{2}\right)-E_z^n\left(i,j,k+\frac{1}{2}\right)}{\Delta y}\right.$$
$$\left.-\frac{E_y^n\left(i,j+\frac{1}{2},k+1\right)-E_y^n\left(i,j+\frac{1}{2},k\right)}{\Delta z}\right]. \quad (5)$$

然后利用方程(3)求出 H 与 B 之间的关系式,进而将磁场强度 H 与电场强度 E 相关联,即按照 $H\to E\to B\to H$ 的方式进行时域计算.计算的难点在于求出 H 与 B 之间色散的时域递推关系式.下面根据矩阵 Padé 逼近理论,采用移位算子法来处理这一问题.

3. 离散时域的本构关系

3.1. 矩阵 Padé 逼近原理

设某一关于复变量 z 的矩阵函数 $f(z)$,有矩阵多项式展式

$$f(z)=\sum_{j=0}^{\infty}f_j z^j, \quad (6)$$

式中 f_j 为矩阵,且彼此不相关.对这种矩阵函数 $f(z)$,可以用如下两种矩阵 Padé 近似来逼近[10]:

$$f(z)-\alpha_M^{-1}(z)\beta_L(z)=O(z^{L+M+1}) \quad (7)$$

和

$$f(z)-P_L(z)Q_M^{-1}(z)=O(z^{L+M+1}), \quad (8)$$

式中 L,M 分别为矩阵多项式 β_L,P_L 和 α_M,Q_M 的最高阶数.尽管 $\beta_L\neq P_L,\alpha_M\neq Q_M$,但可以证明上述两种逼近是一样的[10],且矩阵函数 $f(z)$ 的矩阵 Padé 近似可以唯一地表示为

$$\left[\frac{L}{M}\right]_f\equiv\alpha_M^{-1}(z)\beta_L(z)\equiv P_L(z)Q_M^{-1}(z). \quad (9)$$

(9)式成立的条件是

$$\alpha_M(z)\cdot f(z)-\beta_L(z)=O(z^{L+M+1}), \quad (10a)$$
$$\alpha_M(0)=I, \quad (10b)$$

式中 I 是单位矩阵.(10b)式为标准化条件.

3.2. 离散时域含移位算子本构关系的推导

设频域(时间因子为 $e^{j\omega t}$)中介质的本构关系为

$$B=\mu_0\mu_r(\omega)\cdot H, \quad (11)$$

磁化铁氧体材料的磁导率 $\mu_r(\omega)$ 为一个 3×3 的复矩阵,根据矩阵 Padé 逼近(7)式,可以将它写成如下矩阵函数形式:

$$\mu_r(\omega)=\alpha_M^{-1}(z)\beta_L(z)$$
$$=\left[\sum_{m=0}^{M}\alpha_m(j\omega)^m\right]^{-1}\sum_{n=0}^{N}\beta_n(j\omega)^n, \quad (12a)$$
$$\alpha_0(0)=I, \quad (12b)$$

式中,α_m,β_n 分别为矩阵多项式 α_M,β_L 的系数矩阵,$[\cdot]^{-1}$表示矩阵求逆.

利用频域到时域的算子转换关系 $j\omega\to\frac{\partial}{\partial t}$,代入(11)式得到时域本构关系为

$$B(t)=\mu_0\mu_r\left(\frac{\partial}{\partial t}\right)\cdot H(t), \quad (13)$$

式中 $\mu_r\left(\frac{\partial}{\partial t}\right)$ 为磁导率的时域算子形式,

$$\mu_r\left(\frac{\partial}{\partial t}\right)=\left[\sum_{m=0}^{M}\alpha_m\left(\frac{\partial}{\partial t}\right)^m\right]^{-1}\sum_{n=0}^{N}\beta_n\left(\frac{\partial}{\partial t}\right)^n. \quad (14)$$

将(14)式代入(11)式,整理后得

$$\left[\sum_{m=0}^{M}\alpha_m\left(\frac{\partial}{\partial t}\right)^m\right]\cdot B(t)$$
$$=\mu_0\left[\sum_{n=0}^{N}\beta_n\left(\frac{\partial}{\partial t}\right)^n\right]\cdot H(t). \quad (15)$$

(15)式也是时域中含时间导数算子的本构关系.

为了得到(15)式在时域的递推计算式,下面引入时域的移位算子,讨论时间导数算子在离散时域中的形式.设函数

$$y(t)=\frac{\partial f(t)}{\partial t}, \quad (16)$$

将(16)式在 $\left(n+\frac{1}{2}\right)\Delta t$ 时刻进行中心差分,并将其

左端取平均值近似,可得

$$\frac{y^{n+1}+y^n}{2}=\frac{f^{n+1}-f^n}{\Delta t}. \tag{17}$$

引进离散时域的移位算子 z_t,即

$$z_t f^n = f^{n+1}. \tag{18}$$

联立(17)和(18)式,整理后可得

$$\left(\frac{z_t+1}{2}\right)y^n=\left(\frac{z_t-1}{\Delta t}\right)f^n, \tag{19}$$

或者

$$y^n=\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right)f^n. \tag{20}$$

比较(20)和(16)式,可得

$$\frac{\partial}{\partial t}\sim\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right). \tag{21}$$

(21)式给出的是时间微分算子过渡到离散时域时的移位算子的表示式.

将(21)式代入(15)式,整理后得到离散时域的本构关系

$$\left[\sum_{l=0}^{M}\boldsymbol{\alpha}_l\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right)^l\right]\cdot\boldsymbol{B}=\mu_0\left[\sum_{l=0}^{N}\boldsymbol{\beta}_l\left(\frac{2}{\Delta t}\frac{z_t-1}{z_t+1}\right)^l\right]\cdot\boldsymbol{H}. \tag{22}$$

将(22)式等号两边分别乘以$(z_t+1)^{M+N}$得

$$\left[\sum_{l=0}^{M}\boldsymbol{\alpha}_l\left(\frac{2}{\Delta t}\right)^l(z_t+1)^{M+N-l}(z_t-1)^l\right]\cdot\boldsymbol{B}=\mu_0\left[\sum_{l=0}^{N}\boldsymbol{\beta}_l\left(\frac{2}{\Delta t}\right)^l(z_t+1)^{M+N-l}(z_t-1)^l\right]\cdot\boldsymbol{H}. \tag{23}$$

(23)式给出的是离散时域含移位算子的本构关系.

特别地,当 $M=2$,$N=2$ 时,(23)式变为

$$\begin{aligned}&\left\{\left[\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1\frac{2}{\Delta t}+\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2\right]z_t^2\right.\\&+\left[2\boldsymbol{\alpha}_0-2\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2\right]z_i\\&\left.+\left[\boldsymbol{\alpha}_0-\boldsymbol{\alpha}_1\frac{2}{\Delta t}+\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2\right]\right\}\cdot\boldsymbol{B}\\&=\mu_0\left\{\left[\boldsymbol{\beta}_0+\boldsymbol{\beta}_1\frac{2}{\Delta t}+\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2\right]z_t^2\right.\\&+\left[2\boldsymbol{\beta}_0-2\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2\right]z_t\\&\left.+\left[\boldsymbol{\beta}_0-\boldsymbol{\beta}_1\frac{2}{\Delta t}+\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2\right]\right\}\cdot\boldsymbol{H}.\end{aligned} \tag{24}$$

整理(24)式,可得

$$\boldsymbol{H}^{n+\frac{1}{2}}=\frac{1}{\boldsymbol{b}_0}\left[\boldsymbol{a}_0\cdot\left(\frac{\boldsymbol{B}^{n+\frac{1}{2}}}{\mu_0}\right)+\boldsymbol{a}_1\cdot\left(\frac{\boldsymbol{B}^{n-\frac{1}{2}}}{\mu_0}\right)+\boldsymbol{a}_2\cdot\left(\frac{\boldsymbol{B}^{n-\frac{3}{2}}}{\mu_0}\right)-\boldsymbol{b}_1\cdot\boldsymbol{H}^{n-\frac{1}{2}}-\boldsymbol{b}_2\cdot\boldsymbol{H}^{n-\frac{3}{2}}\right]. \tag{25}$$

(25)式即为从 B 到 H 的时域递推计算公式,其中

$$\begin{aligned}\boldsymbol{a}_0&=\left[\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1\frac{2}{\Delta t}+\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2\right],\\\boldsymbol{a}_1&=2\boldsymbol{\alpha}_0-2\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2,\\\boldsymbol{a}_2&=\boldsymbol{\alpha}_0-\boldsymbol{\alpha}_1\frac{2}{\Delta t}+\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2,\\\boldsymbol{b}_0&=\boldsymbol{\beta}_0+\boldsymbol{\beta}_1\frac{2}{\Delta t}+\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2,\\\boldsymbol{b}_1&=\left[2\boldsymbol{\beta}_0-2\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2\right],\\\boldsymbol{b}_2&=\boldsymbol{\beta}_0-\boldsymbol{\beta}_1\frac{2}{\Delta t}+\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2.\end{aligned} \tag{26}$$

从(25)式可以看出,当色散介质的相对磁导率的矩阵函数的最高次幂 $M=2$,$N=2$ 时,需知道前两个时刻的 B 与 H 值和当前时刻的 B 值才能求出当前时刻的 H.需注意的是,由于方程(25)中的系数都是矩阵形式,所以 H 某一分量的计算不仅与 B 的三个分量有关,而且还与自身其他两个分量有关.

4. 磁化铁氧体材料的离散时域本构关系的推导

当外置磁场平行于 z 轴时,饱和磁化铁氧体的磁导率为

$$\boldsymbol{\mu}=\mu_0(\boldsymbol{I}+\boldsymbol{\chi})=\mu_0\begin{bmatrix}1+\chi_{11}&\chi_{12}&0\\\chi_{21}&1+\chi_{22}&0\\0&0&1\end{bmatrix}, \tag{27}$$

式中,$\boldsymbol{I}$ 为单位矩阵,$\boldsymbol{\chi}$ 为磁化率矩阵,

$$\boldsymbol{\chi}(\omega)=\begin{bmatrix}\chi_{11}(\omega)&\chi_{12}(\omega)&0\\\chi_{21}(\omega)&\chi_{22}(\omega)&0\\0&0&0\end{bmatrix}, \tag{28}$$

$$\begin{aligned}\chi_{11}(\omega)&=\chi_{22}(\omega)=\frac{(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2},\\\chi_{12}(\omega)&=-\chi_{21}(\omega)=\frac{\mathrm{j}\omega\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2}.\end{aligned} \tag{29}$$

这里,$\omega_0=\gamma H_0$,H_0 为外加磁场强度的幅值;γ 为旋磁比($\gamma=1.76\times10^{11}$ Am/kg);$\omega_{\mathrm{m}}=\gamma\cdot4\pi M_{\mathrm{s}}$,$M_{\mathrm{s}}$ 为饱和磁化率;α 为阻尼因子.

根据方程(3),有

$$\boldsymbol{B}=\boldsymbol{\mu}\cdot\boldsymbol{H}=\mu_0\begin{bmatrix}1+\chi_{11} & \chi_{12} & 0\\ \chi_{21} & 1+\chi_{22} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\boldsymbol{H}. \tag{30}$$

将(29)式代入(30)式得

$$\boldsymbol{B}=\boldsymbol{\mu}\cdot\boldsymbol{H}=\mu_0\begin{bmatrix}1+\dfrac{(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2} & \dfrac{j\omega\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2} & 0\\ -\dfrac{\mathrm{j}\omega\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2} & 1+\dfrac{(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}}}{(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2} & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\boldsymbol{H} \tag{31}$$

整理(31)式得

$$\begin{aligned}&\begin{bmatrix}(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2 & 0 & 0\\ 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2 & 0\\ 0 & 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2\end{bmatrix}\cdot\boldsymbol{B}\\ &=\mu_0\begin{bmatrix}(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2+(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}} & \mathrm{j}\omega\omega_{\mathrm{m}} & 0\\ -\mathrm{j}\omega\omega_{\mathrm{m}} & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2+(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}} & 0\\ 0 & 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2\end{bmatrix}\cdot\boldsymbol{H}.\end{aligned} \tag{32}$$

(32)式即为频域的 B 和 H 之间的本构关系.为了求出其离散时域形式,必须求出 (23)式中的相关系数矩阵.从(32)式可知,$M=2$,$N=2$.对比(12a)式可知,只要将(32)式化为以 $\mathrm{j}\omega$ 为变量的二阶矩阵多项式即可求出相关系数矩阵,且要满足标准化条件 $\boldsymbol{\alpha}_0(0)=\boldsymbol{I}$.为此,(32)式的等号左端和右端可分别写为

$$\begin{aligned}&\begin{bmatrix}(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2 & 0 & 0\\ 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2 & 0\\ 0 & 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2\end{bmatrix}\cdot\boldsymbol{B}\\ &=\omega_0^2\left\{(\mathrm{j}\omega)^2\begin{bmatrix}\dfrac{\alpha^2+1}{\omega_0^2} & 0 & 0\\ 0 & \dfrac{\alpha^2+1}{\omega_0^2} & 0\\ 0 & 0 & \dfrac{\alpha^2+1}{\omega_0^2}\end{bmatrix}+\mathrm{j}\omega\begin{bmatrix}\dfrac{2\alpha}{\omega_0} & 0 & 0\\ 0 & \dfrac{2\alpha}{\omega_0} & 0\\ 0 & 0 & \dfrac{2\alpha}{\omega_0}\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right\}\cdot\boldsymbol{B},\end{aligned} \tag{33a}$$

$$\begin{aligned}&\mu_0\begin{bmatrix}(\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2+(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}} & \mathrm{j}\omega\omega_{\mathrm{m}} & 0\\ -\mathrm{j}\omega\omega_{\mathrm{m}} & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2+(\omega_0+\mathrm{j}\omega\alpha)\omega_{\mathrm{m}} & 0\\ 0 & 0 & (\omega_0+\mathrm{j}\omega\alpha)^2-\omega^2\end{bmatrix}\cdot\boldsymbol{H}\\ &=\mu_0\begin{bmatrix}(\mathrm{j}\omega)^2(\alpha^2+1)+\mathrm{j}\omega(\alpha\omega_{\mathrm{m}}+2\alpha\omega_0)+\omega_0^2+\omega_0\omega_{\mathrm{m}} & \mathrm{j}\omega\omega_{\mathrm{m}} & 0\\ -\mathrm{j}\omega\omega_{\mathrm{m}} & (\mathrm{j}\omega)^2(\alpha^2+1)+\mathrm{j}\omega(\alpha\omega_{\mathrm{m}}+2\alpha\omega_0)+\omega_0^2+\omega_0\omega_{\mathrm{m}} & 0\\ 0 & 0 & (\mathrm{j}\omega)^2(\alpha^2+1)+\mathrm{j}\omega2\alpha\omega_0+\omega_0^2\end{bmatrix}\cdot\boldsymbol{H}\end{aligned}$$

$$= \mu_0\left\{(\mathrm{j}\omega)^2\begin{bmatrix}\alpha^2+1 & 0 & 0\\ 0 & \alpha^2+1 & 0\\ 0 & 0 & \alpha^2\end{bmatrix} + \mathrm{j}\omega\begin{bmatrix}\alpha\omega_{\mathrm{m}}+2\alpha\omega_0 & \omega_{\mathrm{m}} & 0\\ -\omega_{\mathrm{m}} & \alpha\omega_{\mathrm{m}}+2\alpha\omega_0 & 0\\ 0 & 0 & 2\alpha\omega_0\end{bmatrix}\right.$$

$$\left.+\begin{bmatrix}\omega_0^2+\omega_0\omega_{\mathrm{m}} & 0 & 0\\ 0 & \omega_0^2+\omega_0\omega_{\mathrm{m}} & 0\\ 0 & 0 & \omega_0^2\end{bmatrix}\right\}\cdot \boldsymbol{H}. \tag{33b}$$

又由方程(3)和(12a)可得

$$\begin{aligned}\boldsymbol{B} &= \mu_0\boldsymbol{\mu}_{\mathrm{r}}\cdot\boldsymbol{H}\\ &= \mu_0\Big[\sum_{m=0}^{M}\boldsymbol{\alpha}_{\mathrm{m}}(\mathrm{j}\omega)^m\Big]^{-1}\sum_{n=0}^{N}\boldsymbol{\beta}_n(\mathrm{j}\omega)^n\cdot\boldsymbol{H}.\end{aligned}\tag{34}$$

当 $M=2$，$N=2$ 时，(34)式变为

$$\begin{aligned}&[\boldsymbol{\alpha}_0+\boldsymbol{\alpha}_1(\mathrm{j}\omega)+\boldsymbol{\alpha}_2(\mathrm{j}\omega)^2]\cdot\boldsymbol{B}\\ &= \mu_0[\boldsymbol{\beta}_0+\boldsymbol{\beta}_1(\mathrm{j}\omega)+\boldsymbol{\beta}_2(\mathrm{j}\omega)^2]\cdot\boldsymbol{H}.\end{aligned}\tag{35}$$

将(35)与(33)式进行比较可得

$$\boldsymbol{\alpha}_0=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},$$

$$\boldsymbol{\alpha}_1=\begin{bmatrix}\frac{2\alpha}{\omega_0}&0&0\\0&\frac{2\alpha}{\omega_0}&0\\0&0&\frac{2\alpha}{\omega_0}\end{bmatrix},$$

$$\boldsymbol{\alpha}_2=\begin{bmatrix}\frac{\alpha^2+1}{\omega_0^2}&0&0\\0&\frac{\alpha^2+1}{\omega_0^2}&0\\0&0&\frac{\alpha^2+1}{\omega_0^2}\end{bmatrix},$$

$$\boldsymbol{\beta}_0=\omega_0^{-2}\begin{bmatrix}\omega_0^2+\omega_0\omega_{\mathrm{m}}&0&0\\0&\omega_0^2+\omega_0\omega_{\mathrm{m}}&0\\0&0&\omega_0^2\end{bmatrix},$$

$$\boldsymbol{\beta}_1=\omega_0^{-2}\begin{bmatrix}\alpha\omega_{\mathrm{m}}+2\alpha\omega_0&\omega_{\mathrm{m}}&0\\-\omega_{\mathrm{m}}&\alpha\omega_{\mathrm{m}}+2\alpha\omega_0&0\\0&0&2\alpha\omega_0\end{bmatrix},$$

$$\boldsymbol{\beta}_2=\omega_0^{-2}\begin{bmatrix}\alpha^2+1&0&0\\0&\alpha^2+1&0\\0&0&\alpha^2\end{bmatrix}.$$

所以，将 $\boldsymbol{\alpha}_0$，$\boldsymbol{\alpha}_1$，$\boldsymbol{\alpha}_2$，$\boldsymbol{\beta}_0$，$\boldsymbol{\beta}_1$ 及 $\boldsymbol{\beta}_2$ 代入(26)式就可求出 $\boldsymbol{a}_0$，$\boldsymbol{a}_1$，$\boldsymbol{a}_2$，$\boldsymbol{b}_0$，$\boldsymbol{b}_1$ 及 $\boldsymbol{b}_2$，详见附录.再将它们代入(25)式就可求出 $\boldsymbol{B}$ 与 $\boldsymbol{H}$ 之间的离散时域递推关系.

下面以磁场 x 分量 $H_x^{n+1/2}\left(i,j+\frac{1}{2},k+\frac{1}{2}\right)$ 的计算为例，给出具体的离散式.由(25)式可得

$$\begin{aligned}\boldsymbol{H}^{n+1/2} &= \boldsymbol{b}_0^{-1}\boldsymbol{a}_0\mu_0^{-1}\cdot\boldsymbol{B}^{n+1/2}+\boldsymbol{b}_0^{-1}\boldsymbol{a}_1\mu_0^{-1}\cdot\boldsymbol{B}^{n-1/2}\\ &+\boldsymbol{b}_0^{-1}\boldsymbol{a}_2\mu_0^{-1}\cdot\boldsymbol{B}^{n-3/2}-\boldsymbol{b}_0^{-1}\boldsymbol{b}_1\cdot\boldsymbol{H}^{n-1/2}\\ &-\boldsymbol{b}_0^{-1}\boldsymbol{b}_2\cdot\boldsymbol{H}^{n-3/2},\end{aligned}\tag{36}$$

即

$$\begin{aligned}\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix}^{n+1/2} &= \boldsymbol{C}\cdot\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}^{n+1/2}\\ &+\boldsymbol{D}\cdot\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}^{n-1/2}+\boldsymbol{E}\cdot\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}^{n-3/2}\\ &-\boldsymbol{F}\cdot\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix}^{n-1/2}-\boldsymbol{G}\cdot\begin{bmatrix}H_x\\H_y\\H_z\end{bmatrix}^{n-3/2},\end{aligned}\tag{37}$$

式中，设

$$\boldsymbol{C}=\boldsymbol{b}_0^{-1}\boldsymbol{a}_0\mu_0^{-1}=\begin{bmatrix}c_{11}&c_{12}&c_{13}\\c_{21}&c_{22}&c_{23}\\c_{31}&c_{32}&c_{33}\end{bmatrix},$$

$$\boldsymbol{D}=\boldsymbol{b}_0^{-1}\boldsymbol{a}_1\mu_0^{-1}=\begin{bmatrix}d_{11}&d_{12}&d_{13}\\d_{21}&d_{22}&d_{23}\\d_{31}&d_{32}&d_{33}\end{bmatrix},$$

$$\boldsymbol{E}=\boldsymbol{b}_0^{-1}\boldsymbol{a}_2\mu_0^{-1}=\begin{bmatrix}e_{11}&e_{12}&e_{13}\\e_{21}&e_{22}&e_{23}\\e_{31}&e_{32}&e_{33}\end{bmatrix},$$

$$\boldsymbol{F}=\boldsymbol{b}_0^{-1}\boldsymbol{b}_1=\begin{bmatrix}f_{11}&f_{12}&f_{13}\\f_{21}&f_{22}&f_{23}\\f_{31}&f_{32}&f_{33}\end{bmatrix},$$

$$\boldsymbol{G}=\boldsymbol{b}_0^{-1}\boldsymbol{b}_2=\begin{bmatrix}g_{11}&g_{12}&g_{13}\\g_{21}&g_{22}&g_{23}\\g_{31}&g_{32}&g_{33}\end{bmatrix}.$$

于是有

$$H_x^{n+1/2}(i,j+1/2,k+1/2)$$

$$
\begin{aligned}
&= c_{11} B_x^{n+1/2}(i,j+1/2,k+1/2)\\
&\quad + c_{12} B_y^{n+1/2}(i,j+1/2,k+1/2)\\
&\quad + c_{13} B_y^{n+1/2}(i,j+1/2,k+1/2)\\
&\quad + d_{11} B_x^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad + d_{12} B_y^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad + d_{13} B_y^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad + e_{11} B_x^{n-3/2}(i,j+1/2,k+1/2)\\
&\quad + e_{12} B_y^{n-3/2}(i,j+1/2,k+1/2)\\
&\quad + e_{13} B_y^{n-3/2}(i,j+1/2,k+1/2)\\
&\quad - f_{11} H_x^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad - f_{12} H_y^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad - f_{13} H_y^{n-1/2}(i,j+1/2,k+1/2)\\
&\quad - g_{11} H_x^{n-3/2}(i,j+1/2,k+1/2)\\
&\quad - g_{12} H_y^{n-3/2}(i,j+1/2,k+1/2)\\
&\quad - g_{13} H_y^{n-3/2}(i,j+1/2,k+1/2). \qquad (38)
\end{aligned}
$$

由(38)式可知,在进行空间离散时 $\boldsymbol{B}$ 与 $\boldsymbol{H}$ 的相同分量在同一节点进行离散,如 $B_x(i,j+1/2,k+1/2)$, $H_x(i,j+1/2,k+1/2)$.但在 $\boldsymbol{B}$ 与 $\boldsymbol{H}$ 之间的离散时域的递推关系中,$H_x^{n+1/2}(i,j+1/2,k+1/2)$的计算不仅与当前时刻同一节点的 $B_x^{n+1/2}(i,j+1/2,k+1/2)$ 有关,而且与同一节点前两个时间步的 $B_x^{n-1/2}(i,j+1/2,k+1/2)$, $B_x^{n-3/2}(i,j+1/2,k+1/2)$, $H_x^{n-1/2}(i,j+1/2,k+1/2)$, $H_x^{n-3/2}(i,j+1/2,k+1/2)$相关.另外还与前两个时刻的 $B_y^{n-1/2}$, $B_z^{n-3/2}$, $H_y^{n-1/2}$, $H_z^{n-3/2}$ 有关,但这四个分量均不在$(i,j+1/2,k+1/2)$节点,因此必须在空间上进行线性插值过渡到离散节点,例如

$$
\begin{aligned}
&B_y\Big|_{i,j+1/2,k+1/2}^{n-1/2}\\
&= \frac{1}{4}\Big[\, B_y\Big|_{i+1/2,j+1,k+1/2}^{n-1/2}\\
&\quad + B_y\Big|_{i+1/2,j,k+1/2}^{n-1/2}\\
&\quad + B_y\Big|_{i-1/2,j,k+1/2}^{n-1/2}\\
&\quad + B_y\Big|_{i-1/2,j+1,k+1/2}^{n-1/2}\Big]. \qquad (39)
\end{aligned}
$$

磁场强度的其他两个分量的计算与此相同.至此已完成磁化铁氧体材料本构关系的时域离散形式推导.

5. 数值结果

作为验证,用上述方法计算了半径为 0.15 m 的磁化铁氧体球的后向 RCS,结果如图 1 所示.FDTD 计算中 $\delta = 1$ cm, $\Delta t = \delta/(2c)$,其中 c 为光速,入射波为高斯脉冲 $E_{\mathrm{i}}(t) = \exp\left[-\frac{4\pi(t-t_0)^2}{\tau^2}\right]$ 沿着 z 轴入射,其中 $\tau = 60\Delta t$, $t_0 = 0.8\tau$.外加磁场平行于 z 轴,$\omega_0 = 2\pi \times 20$ GHz, $\omega_{\mathrm{m}} = 2\pi \times 10$ GHz, $\alpha = 0.1$, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ L/m.图 1 中三角形表示本文计算值,作为对比,图 1 中还给出了文献[11]的计算值(图中圆圈所示),可见两者符合得非常好.

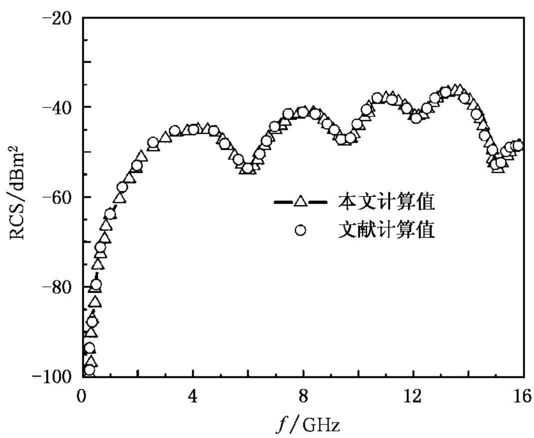

图 1 铁氧体球(半径为 0.15 m)的后向 RCS

6. 结 论

铁氧体材料为色散介质,在外加磁场的条件下又呈现出磁各向异性.根据矩阵 Padé 逼近理论将色散介质的相对磁导率张量写成以 $\mathrm{j}\omega$ 为自变量的矩阵函数,用 $\partial/\partial t$ 代替 $\mathrm{j}\omega$,过渡到时域,再引入离散时域移位算子代替时间微分算子来处理矩阵函数形式的磁导率张量.进而导出磁化色散介质中的磁感应强度 $\boldsymbol{B}$ 和磁场强度 $\boldsymbol{H}$ 之间的离散时域表达式,并将其具体应用于磁旋介质,得到了这种磁化色散介质的 Padé-FDTD 递推表达式.作为验证,计算了磁化铁氧体球的后向 RCS,所得结果与文献一致.理论推导及算例表明,该方法正确可行,且推导简单、概念简明.

附　录

将 $\boldsymbol{\alpha}_0$，$\boldsymbol{\alpha}_1$，$\boldsymbol{\alpha}_2$，$\boldsymbol{\beta}_0$，$\boldsymbol{\beta}_1$ 及 $\boldsymbol{\beta}_2$ 代入(26)式，可得(25)式中的系数矩阵 $\boldsymbol{a}_0$，$\boldsymbol{a}_1$，$\boldsymbol{a}_2$，$\boldsymbol{b}_0$，$\boldsymbol{b}_1$ 及 $\boldsymbol{b}_2$，即

$$\boldsymbol{a}_0 = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 \frac{2}{\Delta t} + \boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{2}{\Delta t}\begin{bmatrix} \frac{2\alpha}{\omega_0} & 0 & 0 \\ 0 & \frac{2\alpha}{\omega_0} & 0 \\ 0 & 0 & \frac{2\alpha}{\omega_0} \end{bmatrix} + \left(\frac{2}{\Delta t}\right)^2 \begin{bmatrix} \frac{\alpha^2+1}{\omega_0^2} & 0 & 0 \\ 0 & \frac{\alpha^2+1}{\omega_0^2} & 0 \\ 0 & 0 & \frac{\alpha^2+1}{\omega_0^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 & 0 \\ 0 & 1 + \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 \\ 0 & 0 & 1 + \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) \end{bmatrix},$$

$$\boldsymbol{a}_1 = 2\boldsymbol{\alpha}_0 - 2\boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2$$

$$= \begin{bmatrix} 2 - 2\left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 & 0 \\ 0 & 2 - 2\left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 \\ 0 & 0 & 2 - 2\left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) \end{bmatrix},$$

$$\boldsymbol{a}_2 = \boldsymbol{\alpha}_0 - \boldsymbol{\alpha}_1 \frac{2}{\Delta t} + \boldsymbol{\alpha}_2\left(\frac{2}{\Delta t}\right)^2$$

$$= \begin{bmatrix} 1 - \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 & 0 \\ 0 & 1 - \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) & 0 \\ 0 & 0 & 1 - \frac{2}{\Delta t}\frac{2\alpha}{\omega_0} + \left(\frac{2}{\Delta t}\right)^2\left(\frac{\alpha^2+1}{\omega_0^2}\right) \end{bmatrix},$$

$$\boldsymbol{b}_0 = \boldsymbol{\beta}_0 + \boldsymbol{\beta}_1 \frac{2}{\Delta t} + \boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2$$

$$= \begin{bmatrix} \omega_0^2 + \omega_0\omega_{\mathrm{m}} + \frac{2}{\Delta t}(\alpha\omega_{\mathrm{m}} + 2\alpha\omega_0) + \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & \frac{2}{\Delta t}\omega_{\mathrm{m}} & 0 \\ -\frac{2}{\Delta t}\omega_{\mathrm{m}} & \omega_0^2 + \omega_0\omega_{\mathrm{m}} + \frac{2}{\Delta t}(\alpha\omega_{\mathrm{m}} + 2\alpha\omega_0) + \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & 0 \\ 0 & 0 & \omega_0^2 + \frac{2}{\Delta t}2\alpha\omega_0 + \left(\frac{2}{\Delta t}\right)^2\alpha^2 \end{bmatrix},$$

$$\boldsymbol{b}_1 = 2\boldsymbol{\beta}_0 - 2\boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2$$

$$= 2\begin{bmatrix} \omega_0^2 + \omega_0\omega_{\mathrm{m}} - \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & 0 & 0 \\ 0 & \omega_0^2 + \omega_0\omega_{\mathrm{m}} - \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & 0 \\ 0 & 0 & \omega_0^2 - \left(\frac{2}{\Delta t}\right)^2\alpha^2 \end{bmatrix},$$

$$\boldsymbol{b}_2 = \boldsymbol{\beta}_0 - \boldsymbol{\beta}_1 \frac{2}{\Delta t} + \boldsymbol{\beta}_2\left(\frac{2}{\Delta t}\right)^2$$

$$
= \begin{bmatrix} \omega_0^2 + \omega_0\omega_m - \frac{2}{\Delta t}(\alpha\omega_m + 2\alpha\omega_0) + \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & -\frac{2}{\Delta t}\omega_m & 0 \\ \frac{2}{\Delta t}\omega_m & \omega_0^2 + \omega_0\omega_m - \frac{2}{\Delta t}(\alpha\omega_m + 2\alpha\omega_0) + \left(\frac{2}{\Delta t}\right)^2(\alpha^2+1) & 0 \\ 0 & 0 & \omega_0^2 - \frac{2}{\Delta t}2\alpha\omega_0 + \left(\frac{2}{\Delta t}\right)^2\alpha^2 \end{bmatrix}.
$$

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Padé-finite-difference time-domain analysis of electromagnetic scattering in magnetic anisotropic medium*

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Abstract

Based on the matrix Padé approximation theory, the complex relative permeability tensor of magnetized dispersive media is described by a matrix function expansion with respect to jω in frequency domain. By substituting the operator of $\partial/\partial t$ into jω, the expansion is then transferred into the time-domain. In order to derive the formulation of the matrix expansion of complex relative permeability tensor in discretised time domain, a shifted operator in discretised time domain is introduced as a replacement of time differential operator $\partial/\partial t$. Therefore the dispersion relation between $\boldsymbol{B}$ and $\boldsymbol{H}$ in discretised time domain can be obtained, which is then implemented to the gyrotropic medium, yielding the time iterative formulation for finite-difference time-domain computation. To verify the feasibility of the presented scheme, we apply the above-mentioned method to the electromagnetic scattering by a magnetized ferrite sphere. The computed result is in good agreement with the one obtained by recursive convolution technique. The analysis and example show the feasibility of the proposed scheme.

Keywords: anisotropic medium, dispersive medium, matrix Padé approximants, finite-difference time-domain method

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