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动态广义球对称含荷黑洞 Dirac 场的熵

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利用改进后的薄膜 brick-wall 模型，计算了动态广义球对称含荷黑洞 Dirac 场的熵. 按薄层模型的观点，在视界附近薄场上的熵就是黑洞的熵. 计算结果表明所得到的黑洞熵与其视界面积成正比.

关键词：黑洞，薄膜 brick-wall 模型，熵，Dirac 场
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1. 引 言

自从 Bekenstein 和 Hawking 提出黑洞的熵与其视界面积成正比以来[1—3]，人们对求黑洞的熵做了大量的工作. 1985 年，'t Hooft 提出了计算黑洞熵的 brick-wall 模型，这一方法被用来计算各种黑洞的标量场的熵，并取得了令人满意的结果[4—10]. 但研究发现 brick-wall 模型只适用于处于热平衡状态的黑洞，而对处于非热平衡状态的黑洞就无能为力. 针对这一问题，2000 年，Zhao 等提出的改进型薄膜 brick-wall 模型[11,12]，便可克服这些困难. 利用改进型薄膜 brick-wall 模型，对黑洞的研究取得了许多有价值的成果[13—17].

文献[18,19]分别计算了动态广义球对称含荷黑洞标量场的统计熵和量子熵. 本文尝试用改进后的薄膜 brick-wall 模型计算动态广义球对称含荷黑洞 Dirac 场的熵. 在适当选择截断因子后，得到熵的表达式. 从表达式可以看出，所得到的黑洞熵与其视界面积成正比.

2. 度规与联络

文献[11,12]给出动态广义球对称含荷黑洞周围的时空线元为

$$\mathrm{d}s^2 = -\mathrm{e}^{2\psi}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}v^2+2\mathrm{e}^{\psi}\mathrm{d}\nu\mathrm{d}r + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \tag{1}$$

式中 $M=M(r,\nu)$，$Q=Q(\nu)$ 分别为黑洞的质量和电荷. $\psi=\psi(r,\nu)$，ν 为超前爱丁顿坐标，将(1)式给出的线元用号差为(+，−，−，−)重新表示为

$$\mathrm{d}s^2 = \mathrm{e}^{2\psi}\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}v^2-2\mathrm{e}^{\psi}\mathrm{d}\nu\mathrm{d}r - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2). \tag{2}$$

由(2)式可以求出度规行列式和度规张量的逆变分量

$$g = -\mathrm{e}^{2\psi}r^2\sin^2\theta, \tag{3}$$

$$g^{01}=g^{10}=-\mathrm{e}^{-\psi},$$
$$g^{11}=-\frac{\Delta}{r^2},$$
$$g^{22}=-\frac{1}{r^2},$$
$$g^{33}=-\frac{1}{r^2\sin^2\theta}, \tag{4}$$

其中 $\Delta=r^2-2Mr+Q^2$，由此可以算出不为零的联络分量为

$$\Gamma^0_{00}=\dot{\psi}+\mathrm{e}^{\psi}\psi'\frac{\Delta}{r^2}+\mathrm{e}^{\psi}\left(\frac{M}{r^2}-\frac{M'}{r^2}-\frac{Q^2}{r^3}\right),$$

$$\Gamma^1_{00}=-\mathrm{e}^{\psi}\left(-\frac{\dot{M}}{r}+\frac{Q\dot{Q}}{r^2}\right)+\mathrm{e}^{2\psi}\psi'\frac{\Delta^2}{r^4} + \mathrm{e}^{2\psi}\frac{\Delta}{r^2}\left(\frac{M}{r^2}-\frac{M'}{r}-\frac{Q^2}{r^3}\right),$$

$$\Gamma^1_{01}=\Gamma^1_{10}=-\mathrm{e}^{\psi}\psi'\frac{\Delta}{r^2}-\mathrm{e}^{\psi}\left(\frac{M}{r^2}-\frac{M'}{r}-\frac{2Q^2}{r^3}\right),$$

$$\Gamma^1_{11}=\psi',\Gamma^1_{12}=\Gamma^1_{21}=\frac{1}{r},$$

$$\Gamma^3_{13}=\Gamma^3_{31}=\frac{1}{r},\Gamma^0_{22}=-r\mathrm{e}^{-\psi},$$

$$\Gamma^1_{22}=-\frac{\Delta}{r},\Gamma^3_{23}=\Gamma^3_{32}=\cot\theta,$$

$$\Gamma^0_{33}=-r\mathrm{e}^{-\psi}\sin^2\theta,\Gamma^1_{33}=-\frac{\Delta}{r}\sin^2\theta,$$

$$\Gamma^2_{33} = -\cos\theta\sin\theta, \tag{5}$$

式中 $\dot{\psi} = \mathrm{d}\psi/\mathrm{d}v$, $\psi' = \mathrm{d}\psi/\mathrm{d}r$, $\dot{M} = \mathrm{d}M/\mathrm{d}r$, $M' = \mathrm{d}M/\mathrm{d}v$, $\dot{Q} = \mathrm{d}Q/\mathrm{d}v$.

3. 零标架及 Penrose 旋系数的计算

根据零标架满足的条件

$$\begin{aligned}
g_{\mu\nu} &= l_\mu n_\nu + n_\mu l_\nu - m_\mu \bar{m}_\nu - m_\nu \bar{m}_\mu,\\
l_\mu l^\mu &= n_\mu n^\mu = m_\mu m^\mu = \bar{m}_\mu \bar{m}^\mu = 0,\\
l^\mu n_\mu &= -m_\mu \bar{m}^\mu = 1,\\
l_\mu m^\mu &= l_\mu \bar{m}^\mu = n_\mu m^\mu = n_\mu \bar{m}^\mu = 0.
\end{aligned} \tag{6}$$

可以选取如下的零标架形式：

$$\begin{aligned}
l_\mu &= \left(\frac{1}{2\Delta}\right)^{1/2}\left(\mathrm{e}^{\psi}\frac{\Delta}{r},0,0,0\right),\\
n_\mu &= \left(\frac{1}{2\Delta}\right)^{1/2}\left(\mathrm{e}^{\psi}\frac{\Delta}{r},-2r,0,0\right),\\
m_\mu &= \frac{r}{\sqrt{2}}(0,0,1,\mathrm{i}\sin\theta),\\
\bar{m}_\mu &= \frac{r}{\sqrt{2}}(0,0,1,-\mathrm{i}\sin\theta).
\end{aligned} \tag{7}$$

根据零标架的协变分量和度规可算出其逆变分量为

$$\begin{aligned}
l^\mu &= \left(\frac{1}{2\Delta}\right)^{1/2}\left(0,-\frac{\Delta}{r},0,0,0\right),\\
n^\mu &= \left(\frac{1}{2\Delta}\right)^{1/2}\left(2r\mathrm{e}^{-\psi},\frac{\Delta}{r},0,0\right),\\
m^\mu &= \frac{r}{\sqrt{2}}\left(0,0,-1,-\frac{\mathrm{i}}{\sin\theta}\right),\\
\bar{m}^\mu &= \frac{r}{\sqrt{2}}\left(0,0,-1,\frac{\mathrm{i}}{\sin\theta}\right).
\end{aligned} \tag{8}$$

由(7),(8)式可算出不为零的 Penrose 旋系数为

$$\begin{aligned}
\varepsilon &= \frac{1}{2}(l_{\mu;\nu}n^\mu l^\nu - m_{\mu,\nu}\bar{m}^\mu l^\nu)\\
&= -\frac{1}{2\sqrt{2\Delta}r^2}[r^3\psi' + rM - r^2M' - Q^2],\\
\rho &= l_{\mu;\nu}m^\mu\bar{m}^\nu = \frac{\sqrt{\Delta}}{\sqrt{2}r^2},\\
\gamma &= \frac{1}{2}(l_{\mu;\nu}n^\mu n^\nu - m_{\mu;\nu}\bar{m}^\mu n^\nu)\\
&= \frac{1}{\sqrt{2\Delta}}\Big[\frac{r\mathrm{e}^{-\psi}}{2}(-\dot{M}\gamma + Q\dot{Q}) - \frac{\Delta}{2r}\psi'\\
&\quad - \frac{1}{2r^2}(rM - r^2M' - Q^2)\Big],\\
\alpha &= \frac{1}{2}(l_{\mu;\nu}n^\mu\bar{m}^\nu - m_{\mu;\nu}\bar{m}^\mu\bar{m}^\nu)\\
&= \frac{1}{2\sqrt{2}r}\cot\theta,\\
\mu &= -n_{\mu;\nu}\bar{m}^\mu m^\nu = \frac{\sqrt{\Delta}}{\sqrt{2}r^2},\\
\beta &= \frac{1}{2}(l_{\mu;\nu}n^\mu m^\nu - m_{\mu;\nu}\bar{m}^\mu m^\nu)\\
&= -\frac{1}{2\sqrt{2}r}\cot\theta.
\end{aligned} \tag{9}$$

4. 弯曲时空中的 Dirac 方程

弯曲时空中带电粒子的 Dirac 方程表示为

$$\begin{aligned}
&(D + \varepsilon - \rho + \mathrm{i}eA_\mu l^\mu)F_1\\
&+ (\bar{\delta} + \pi - \alpha + \mathrm{i}eA_\mu\bar{m}^\mu)F_2\\
&- \frac{1}{\sqrt{2}}\mathrm{i}\mu_0 G_1 = 0,\\
&(\Delta' + \mu - \gamma + \mathrm{i}eA_\mu n^\mu)F_2\\
&+ (\delta + \beta - \tau + \mathrm{i}eA_\mu m^\mu)F_1\\
&- \frac{1}{\sqrt{2}}\mathrm{i}\mu_0 G_2 = 0,\\
&(D + \varepsilon^* - \rho^* + \mathrm{i}eA_\mu l^\mu)G_2\\
&- (\delta + \pi^* - \alpha^* + \mathrm{i}eA_\mu m^\mu)G_1\\
&- \frac{1}{\sqrt{2}}\mathrm{i}\mu_0 F_2 = 0,\\
&(\Delta' + \mu^* - \gamma^* + \mathrm{i}eA_\mu n^\mu)G_1\\
&- (\bar{\delta} + \beta^* - \tau^* + \mathrm{i}eA_\mu\bar{m}^\mu)G_2\\
&- \frac{1}{\sqrt{2}}\mathrm{i}\mu_0 F_1 = 0.
\end{aligned} \tag{10}$$

电磁四矢为 $A_\mu = \left(-\frac{Q}{r},0,0,0\right)$，于是

$$\begin{aligned}
A_\mu l^\mu &= 0,\\
A_\mu n^\mu &= \frac{2Q}{\sqrt{2\Delta}}\mathrm{e}^{-\psi},\\
A_\mu m^\mu &= A_\mu\bar{m}^\mu = 0.
\end{aligned} \tag{11}$$

四个微分算子表示为

$$\begin{aligned}
D &= l^\mu\partial_\mu = -\left(\frac{1}{2\Delta}\right)^{1/2}\frac{\Delta}{r}\frac{\partial}{\partial r},\\
\Delta' &= n^\mu\partial_\mu = \left(\frac{1}{2\Delta}\right)^{1/2}2r\mathrm{e}^{-\psi}\frac{\partial}{\partial\nu} + \left(\frac{1}{2\Delta}\right)^{1/2}\frac{\Delta}{r}\frac{\partial}{\partial r},\\
\delta &= m^\mu\partial_\mu = -\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} + \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right),\\
\bar{\delta} &= \bar{m}^\mu\partial_\mu = -\frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} - \frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}\right).
\end{aligned} \tag{12}$$

将(9),(11),(12)式代入(10)式，并整理可得

$$\sqrt{\Delta}\left[\frac{\partial}{\partial r} + \frac{1}{2r\Delta}(r^3\psi' + 2r^2 - 3Mr - M'r^2 + Q^2)\right]F_1$$

$$+\left(\frac{\partial}{\partial\theta}-\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}+\frac{1}{2}\cot\theta\right)F_2+\mathrm{i}\mu_0 rG_1=0,$$

$$\sqrt{\Delta}\Big[\frac{2r^2}{\Delta}\mathrm{e}^{-\psi}\frac{\partial}{\partial\nu}+\frac{\partial}{\partial r}+\frac{1}{2r\Delta}(2r^2-3Mr-M'r^2+Q^2)$$

$$-\frac{r^2}{\Delta^2}\mathrm{e}^{-\psi}(-\dot{M}r+Q\dot{Q})+\psi'+\frac{2\mathrm{i}erQ\mathrm{e}^{-\psi}}{\Delta}\Big]F_2$$

$$-\left(\frac{\partial}{\partial\theta}+\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}+\frac{1}{2}\cot\theta\right)F_1-\mathrm{i}\mu_0 rG_2=0,$$

$$\sqrt{\Delta}\Big[\frac{\partial}{\partial r}+\frac{1}{2r\Delta}(r^3\psi'+2r^2-3Mr-M'r^2+Q^2)\Big]G_2$$

$$+\left(\frac{\partial}{\partial\theta}-\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}+\frac{1}{2}\cot\theta\right)G_1+\mathrm{i}\mu_0 rF_2=0,$$

$$\sqrt{\Delta}\Big[\frac{2r^2}{\Delta}\mathrm{e}^{-\psi}\frac{\partial}{\partial\nu}+\frac{\partial}{\partial r}+\frac{1}{2r\Delta}(2r^2-3Mr-M'r^2+Q^2)$$

$$-\frac{r^2}{\Delta^2}\mathrm{e}^{-\psi}(-\dot{M}r+Q\dot{Q})+\psi'+\frac{2\mathrm{i}erQ\mathrm{e}^{-\psi}}{\Delta}\Big]G_1$$

$$-\left(\frac{\partial}{\partial\theta}+\frac{\mathrm{i}}{\sin\theta}\frac{\partial}{\partial\varphi}+\frac{1}{2}\cot\theta\right)G_2-\mathrm{i}\mu_0 rF_1=0. \tag{13}$$

利用改进后的 brick-wall 模型来计算黑洞的熵，这种模型认为，对黑洞的熵有贡献的是黑洞视界附近一个薄层中的粒子，所以在求黑洞熵时，只在视界附近的一个薄层内求解，即在 $r_{\mathrm{H}}+\varepsilon<r<r_{\mathrm{H}}+\varepsilon+\delta$ 范围内计算黑洞的熵，其中 ε，δ 远远小于 r_{H}，所以虽然整个时空是动态时空，但可以认为在薄层内，系统处于平衡状态. 因此可以按下式分离变量：

$$F_1=\mathrm{e}^{-\mathrm{i}E\nu}R_-(r)Y_-(\theta,\varphi),$$
$$F_2=\mathrm{e}^{-\mathrm{i}E\nu}R_+(r)Y_+(\theta,\varphi),$$
$$G_1=\mathrm{e}^{-\mathrm{i}E\nu}R_+(r)Y_-(\theta,\varphi),$$
$$G_2=\mathrm{e}^{-\mathrm{i}E\nu}R_-(r)Y_+(\theta,\varphi), \tag{14}$$

代入(13)式，并采用小质量近似，整理后可得四个独立方程，其中角向方程为

$$\Big[\frac{\partial^2}{\partial\theta^2}+\cot\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\Big(\frac{1}{4}\cos^2\theta\mp\mathrm{i}\cos\theta\frac{\partial}{\partial\varphi}$$

$$+\frac{\partial^2}{\partial\varphi^2}-\frac{1}{2}\Big)+\lambda^2\Big]Y_\pm=0, \tag{15}$$

方程的解为权是 1/2 的球谐函数，分离变量常数为 $\lambda=l+1/2$ 其中 $l>1/2$. 两个径向方程为

$$\sqrt{\Delta}\Big[\frac{\partial}{\partial r}+\frac{1}{2r\Delta}(r^3\psi'+2r^2-3Mr$$

$$-M'r^2+Q^2)\Big]\sqrt{\Delta}\Big[\frac{2r^2}{\Delta}\mathrm{e}^{-\psi}\frac{\partial}{\partial\nu}+\frac{\partial}{\partial r}$$

$$+\frac{1}{2r\Delta}(2r^2-3Mr-M'r^2+Q^2)$$

$$-\frac{r^2}{\Delta^2}\mathrm{e}^{-\psi}(-\dot{M}r+Q\dot{Q})$$

$$+\psi'+\frac{2\mathrm{i}erQ\mathrm{e}^{-\psi}}{\Delta}\Big]R_+-\lambda^2R_+=0, \tag{16}$$

$$\sqrt{\Delta}\Big[\frac{2r^2}{\Delta}\mathrm{e}^{-\psi}\frac{\partial}{\partial\nu}+\frac{\partial}{\partial r}+\frac{1}{2r\Delta}$$

$$\times(2r^2-3Mr-M'r^2+Q^2)$$

$$-\frac{r^2}{\Delta^2}\mathrm{e}^{-\psi}\times(-\dot{M}r+Q\dot{Q})+\psi'+\frac{2\mathrm{i}erQ\mathrm{e}^{-\psi}}{\Delta}\Big]$$

$$\times\sqrt{\Delta}\Big[\frac{\partial}{\partial r}+\frac{1}{2r\Delta}(r^3\psi'+2r^2$$

$$-3Mr-M'r^2+Q^2)\Big]R_--\lambda^2R_-=0. \tag{17}$$

5. Dirac 场的熵

采用 Wengel-Kramers-Brillouin 近似，将 $R_-=\mathrm{e}^{\mathrm{i}s(r)}$ 代入(17)式，取方程的实部函数，整理可得

$$\Delta(s')^2-2r\mathrm{e}^{-\psi}(rE-eQ)s'+[A-B+\lambda^2]=0, \tag{18}$$

其中

$$A=\frac{1}{2r\Delta}(r-rM'-M)(r^3\psi+2r^2$$
$$-3Mr-M'r^2+Q^2)$$
$$-\frac{1}{2}\Big(2r\psi'+r^2\psi''+2-4M'-rM''-\frac{Q^2}{r^2}\Big), \tag{19}$$

$$B=\frac{1}{2}\Big(r^2\psi'+2r-3M-rM'+\frac{Q^2}{r}\Big)$$
$$\times\Big[\frac{1}{2r}(2r^2-3Mr-M'r^2+Q^2)$$
$$-\mathrm{e}^{-\psi}\frac{r^2}{\Delta^2}(-\dot{M}r+Q\dot{Q})+\psi'\Big], \tag{20}$$

于是可以得到

$$s'_\pm=\{r\mathrm{e}^{-\psi}(rE-eQ)\pm[r^2\mathrm{e}^{-2\psi}(rE-eQ)^2$$
$$-\Delta(A-B+\lambda^2)]^{1/2}\}/\Delta. \tag{21}$$

采用薄膜 brick-wall 模型，在 $r_{\mathrm{H}}+\varepsilon<r<r_{\mathrm{H}}+\varepsilon+\delta$ 范围内计算黑洞的熵. 根据半经典量子化条件[11,20—22]

$$2n\pi=\int_{\varepsilon}^{\varepsilon+\delta}s_+\,\mathrm{d}r+\int_{\varepsilon+\delta}^{\varepsilon}s_-\,\mathrm{d}r=2\int_{\varepsilon}^{\varepsilon+\delta}s_r\mathrm{d}r, \tag{22}$$

式中

$$s_r=[r^2\mathrm{e}^{-2\psi}(rE-eQ)^2$$
$$-\Delta(A-B+\lambda^2)]^{1/2}/\Delta. \tag{23}$$

设

$$E'=E-\frac{eQ}{r}, \tag{24}$$

则(22)式可以化为

$$n = \frac{1}{\pi}\int_{\varepsilon}^{\varepsilon+\delta} \mathrm{d}r \frac{\left[E'\mathrm{e}^{-2\psi} - \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{A-B+\lambda^2}{r^2}\right]^{1/2}}{1-\frac{2M}{r}+\frac{Q^2}{r^2}}. \tag{25}$$

根据统计理论,系统的自由能表示为

$$\beta F = \sum_{E'} \ln\left(1+\mathrm{e}^{-\beta E'}\right). \tag{26}$$

作半经典近似,视能量为连续分布.则上式求和可以化为求积分

$$\begin{aligned}\beta F &= \int_0^{\infty} \mathrm{d}E' g\left(E'\right)\ln\left(1+\mathrm{e}^{-\beta E'}\right)\\ &= \int_{E'} \mathrm{d}\Gamma\left(E'\right)\ln\left(1+\mathrm{e}^{-\beta E'}\right)\\ &= -\beta\int_0^{\infty}\mathrm{d}E'\Gamma\left(E'\right)/\left(\mathrm{e}^{\beta E'}+1\right),\end{aligned} \tag{27}$$

式中 $\Gamma\left(E'\right)$ 为系统的能量小于微观状态数的, $g\left(E'\right)=\mathrm{d}\Gamma\left(E'\right)/\mathrm{d}E'$ 为态密度,

$$\Gamma\left(E'\right) = \frac{1}{\pi}\int_l (2l+1)\mathrm{d}l\int_r s_r \mathrm{d}r. \tag{28}$$

把(28)式和 $\lambda = l + 1/2$ 代入(27)式可得

$$\begin{aligned}F =& -\frac{1}{\pi}\int_0^{\infty}\mathrm{d}E'\int_r \mathrm{d}r\int_l \mathrm{d}l(2l+1)\frac{1}{\mathrm{e}^{\beta E'}+1}\\ &\times\frac{\left[E'\mathrm{e}^{-2\psi}-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{A-B+\lambda^2}{r^2}\right]^{1/2}}{1-\frac{2M}{r}+\frac{Q^2}{r^2}}.\end{aligned} \tag{29}$$

对 l 积分时,积分下限为 0,上限为使上式根号下有意义时 l 的取值,于是积分化为

$$\begin{aligned}F =& -\frac{2}{3\pi}\int_0^{\infty}\frac{\mathrm{d}E'}{\mathrm{e}^{\beta E'}+1}\int_{\varepsilon}^{\varepsilon+\delta}\mathrm{d}r r^2\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-2}\\ &\times\left[E'^2\mathrm{e}^{-2\psi}-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\frac{A-B}{r^2}\right]^{3/2}.\end{aligned} \tag{30}$$

上式对 E' 积分时,略去高阶小量,且在事件视界外的一个很薄的薄层上对 r 积分,于是得到

$$\begin{aligned}F &= -\int_{\varepsilon}^{\varepsilon+\delta}\frac{7}{180}\frac{\pi^3\mathrm{e}^{-3\psi}}{\beta^4}\frac{r^2}{\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^2}\mathrm{d}r\\ &= -\frac{7}{180}\frac{\pi^3\mathrm{e}^{-3\psi}}{\beta^4}\frac{r_{\mathrm{H}}^6}{4\left(r_{\mathrm{H}}-M\right)^2}\frac{\delta}{\varepsilon(\varepsilon+\delta)}.\end{aligned} \tag{31}$$

于是系统的波函数第一分量贡献的熵为

$$s_1 = \beta^2\frac{\partial F}{\partial\beta} = \frac{7}{180}\frac{\pi^3\mathrm{e}^{-3\psi}}{\beta^3}\frac{r_{\mathrm{H}}^6}{\left(r_{\mathrm{H}}-M\right)^2}\frac{\delta}{\varepsilon(\varepsilon+\delta)}. \tag{32}$$

同理计算可知,在取一级近似的情况下,另外的三个波函数分量贡献的熵与 s_1 相同,根据熵的可加性可以得到系统的熵为

$$\begin{aligned}S &= s_1+s_2+s_3+s_4 = 4s_1\\ &= \frac{7}{45}\frac{\pi^3\mathrm{e}^{-3\psi}}{\beta^3}\frac{r_{\mathrm{H}}^6}{\left(r_{\mathrm{H}}-M\right)^2}\frac{\delta}{\varepsilon(\varepsilon+\delta)}.\end{aligned} \tag{33}$$

取

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta, \tag{34}$$

则(33)式可以化为

$$S = \frac{7}{2}\frac{\pi^3\mathrm{e}^{-3\psi}}{\beta^2}\frac{r_{\mathrm{H}}^6}{\left(r_{\mathrm{H}}-M\right)^2}. \tag{35}$$

由文献[23]所示算法可以算出

$$\kappa = \frac{r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}\psi'_{\mathrm{H}}-\left(r_{\mathrm{H}}-M'r_{\mathrm{H}}-M\right)\mathrm{e}^{\psi}-2r_{\mathrm{H}}\dot{r}_{\mathrm{H}}}{r_{\mathrm{H}}^2\left(1-2\dot{r}_{\mathrm{H}}\right)}. \tag{36}$$

于是可以得到

$$\begin{aligned}\frac{1}{\beta} &= T^2 = \left(\frac{\kappa}{2\pi}\right)^2\\ &= \frac{1}{4\pi^2}\frac{\left[r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}\psi'_{\mathrm{H}}-\left(r_{\mathrm{H}}-M'r_{\mathrm{H}}-M\right)\mathrm{e}^{\psi}-2r_{\mathrm{H}}\dot{r}_{\mathrm{H}}\right]^2}{r_{\mathrm{H}}^4\left(1-2\dot{r}_{\mathrm{H}}\right)^2}.\end{aligned} \tag{37}$$

代入(35)式可以得到

$$\begin{aligned}S =& \frac{7}{2}\frac{A_{\mathrm{H}}}{4}\frac{\mathrm{e}^{-3\psi}}{\left(r_{\mathrm{H}}-M\right)^2}\\ &\times\frac{\left[r_{\mathrm{H}}^2\dot{r}_{\mathrm{H}}\psi'_{\mathrm{H}}-\left(r_{\mathrm{H}}-M'r_{\mathrm{H}}-M\right)\mathrm{e}^{\psi}-2r_{\mathrm{H}}\dot{r}_{\mathrm{H}}\right]^2}{\left(1-2\dot{r}_{\mathrm{H}}\right)^2},\end{aligned} \tag{38}$$

式中 $A_{\mathrm{H}} = \iint\begin{vmatrix} g_{22} & g_{23}\\ g_{32} & g_{33}\end{vmatrix}\mathrm{d}\theta\mathrm{d}\varphi = 4\pi r_{\mathrm{H}}^2$ 为黑洞的视界面积.

6. 结　　论

本文运用了薄膜 brick-wall 模型计算了动态广义球对称含荷黑洞 Dirac 场的熵,从所得到的表达式可以看出,要得到熵与其视界面积成正比的结论,其截断因子是时间依赖的.同时,波函数四个分量只有在取一级近似下才有相同的表达式,这是由于分离变量常数 λ 满足关系式 $\lambda^2=(l-s)(l+s+1)$,其 s 有两个取值 ±1/2,对应自旋在空间上的两个投影.因此计算量子态数目时,有微小的差别.但对于大质量的黑洞,这种差别很小,可以忽略不计,所以波函数四个分量在取一级近似下,贡献的熵相等.

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Entropy of Dirac field in a generalized non-stationary spherically symmetric black hole with charge

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Abstract

Using the improved brick-wall film model, we calculated the entropy of Dirac field in a generalized non-stationary spherically symmetric black hole with charge. From the point of view of the model, these entropies are nothing but the entropy of the black hole. The result showed that this entropy is proportion to the area of event horizon.

Keywords: black hole, thin film brick-wall model, entropy, Dirac field

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