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Birkhoff 系统的一类新型绝热不变量*

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研究 Birkhoff 系统对称性的摄动与绝热不变量.给出了未受扰 Birkhoff 系统的 Lie 对称性导致的 Hojman 型精确不变量.基于力学系统的高阶绝热不变量的定义,研究在小扰动作用下 Birkhoff 系统 Lie 对称性的摄动,得到了系统的一类新型绝热不变量.举例说明结果的应用.

关键词:Birkhoff 系统,Lie 对称性,摄动,绝热不变量

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1. 引 言

1927 年 Birkhoff[1] 给出了比 Hamilton 方程更普遍的一类新型运动微分方程.1978 年 Santilli 将其推广并建议方程命名为 Birkhoff 方程[2].1989 年 Galiullan[3] 认为,对 Birkhoff 方程的研究是近代分析力学的一个重要发展方向.1992 年以来,对 Birkhoff 系统动力学进行了较全面深入的研究,并取得了一系列重要成果[4—30].本文基于力学系统的高阶绝热不变量的概念[31],进一步研究 Birkhoff 系统 Lie 对称性的摄动,给出了 Birkhoff 系统的一类新型绝热不变量.

2. 系统的 Lie 对称性与精确不变量

Birkhoff 系统的运动微分方程的一般形式为[4]

$$\left(\frac{\partial R_\nu}{\partial a^\mu}-\frac{\partial R_\mu}{\partial a^\nu}\right)\dot{a}^\nu-\frac{\partial B}{\partial a^\mu}-\frac{\partial R_\mu}{\partial t}=0$$
$$(\mu,\nu=1,\cdots,2n),\tag{1}$$

式中,$B=B(t,\boldsymbol{a})$称为 Birkhoff 函数,$R_\mu=R_\mu(t,\boldsymbol{a})$称为 Birkhoff 函数组.设系统的 Birkhoff 变量 a^μ $(\mu=1,\cdots,2n)$彼此独立,而

$$\Omega_{\mu\nu}=\frac{\partial R_\nu}{\partial a^\mu}-\frac{\partial R_\mu}{\partial a^\nu}\tag{2}$$

称为 Birkhoff 张量.假设系统非奇异,则由方程(1)可解出所有 $\dot{a}^\mu$,有

$$\dot{a}^\mu=\Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^\nu}+\frac{\partial R_\nu}{\partial t}\right),\tag{3}$$

式中,

$$\det(\Omega_{\mu\nu})\neq 0,\qquad \Omega^{\mu\nu}\Omega_{\nu\tau}=\delta_{\mu\tau},\tag{4}$$

而 $\Omega^{\mu\nu}$ 称为 Birkhoff 逆变张量.展开方程(3),有

$$\dot{a}^\mu=h_\mu(t,\boldsymbol{a})\qquad(\mu=1,\cdots,2n).\tag{5}$$

取时间和 Birkhoff 变量 a^μ 的无限小变换

$$t^*=t+\Delta t,$$
$$a^{\mu*}(t^*)=a^\mu(t)+\Delta a^\mu\qquad(\mu=1,\cdots,2n)\tag{6}$$

或其展开式

$$t^*=t+\varepsilon\tau^0(t,\boldsymbol{a}),$$
$$a^{\mu*}(t^*)=a^\mu(t)+\varepsilon\xi_\mu^0(t,\boldsymbol{a})\tag{7}$$
$$(\mu=1,\cdots,2n),$$

式中,ε 为无限小参数,τ^0,ξ_μ^0 为无限小生成元.取无限小生成元向量

$$X_0^{(0)}=\tau^0\frac{\partial}{\partial t}+\xi_\mu^0\frac{\partial}{\partial a^\mu},\tag{8}$$

方程(5)在无限小变换(7)式下的不变性归为如下的 Lie 对称性确定方程:

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_\mu^0-h_\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0=X_0^{(0)}(h_\mu),\tag{9}$$

式中

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}=\frac{\partial}{\partial t}+h_\mu\frac{\partial}{\partial a^\mu}.\tag{10}$$

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定理 1 在无限小变换(7)式下,如果生成元 τ^0,ξ_μ^0 满足确定方程(9),且存在函数 $\lambda_0=\lambda_0(t,\boldsymbol{a})$ 满足条件

$$\frac{\partial h_\mu}{\partial a^\mu}+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda_0=0, \tag{11}$$

则未受扰 Birkhoff 系统的 Lie 对称性直接导致 Hojman 形式的守恒量,形如

$$I_0=\frac{1}{\lambda_0}\frac{\partial}{\partial t}(\lambda_0\tau^0)+\frac{1}{\lambda_0}\frac{\partial}{\partial a^\mu}(\lambda_0\xi_\mu^0)-\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0=\mathrm{const}. \tag{12}$$

证明 由确定方程(9)和条件(11)式,我们有

$$\begin{aligned}\frac{\bar{\mathrm{d}}}{\mathrm{d}t}I_0&=\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\frac{\partial\xi_\mu^0}{\partial a^\mu}-\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(h_\mu\frac{\partial\tau^0}{\partial a^\mu}\right)\\&\quad+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}X_0^{(0)}(\ln\lambda_0)\\&=\frac{\partial}{\partial a^\mu}\left[\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_\mu^0-h_\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0-X_0^{(0)}(h_\mu)\right]\\&\quad+X_0^{(0)}\left(\frac{\partial h_\mu}{\partial a^\mu}+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda_0\right)\\&\quad+\left(\frac{\partial h_\mu}{\partial a^\mu}+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda_0\right)\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0\\&=0.\end{aligned}$$

于是,系统存在守恒量(12)式.证毕.

如果 $\tau^0=0$,则定理 1 给出文献[26—28]的结果.守恒量(12)式是一个精确不变量,它揭示了未受扰动的 Birkhoff 系统的 Lie 对称性与不变量之间的关系.

3. 系统的一类新型绝热不变量

定义[31] 若 $I_z(t,\boldsymbol{a},\varepsilon)$ 是力学系统的一个含有小参量 ε 的最高次幂为 z 的物理量,其对时间 t 的一阶导数正比于 ε^{z+1},则称 I_z 为力学系统的 z 阶绝热不变量.

假设 Birkhoff 系统(1)受到小扰动 $\varepsilon\Omega_\mu$ 的作用,则系统的运动微分方程变为

$$\Omega_{\mu\nu}\dot{a}^\nu-\frac{\partial B}{\partial a^\mu}-\frac{\partial R_\mu}{\partial t}=\varepsilon Q_\mu. \tag{13}$$

由方程(13)解得

$$\dot{a}^\mu=h_\mu(t,\boldsymbol{a})+\varepsilon\Omega^{\mu\nu}Q_\nu. \tag{14}$$

在小扰动 εQ_μ 的作用下,系统原有的对称性与不变量相应地会发生改变.假设扰动后的无限小生成元 τ,ξ_μ 是在系统未扰动的对称性变换生成元基础上发生的小摄动,有

$$\tau=\tau^0+\varepsilon\tau^1+\varepsilon^2\tau^2+\cdots, \tag{15}$$

$$\xi_\mu=\xi_\mu^0+\varepsilon\xi_\mu^1+\varepsilon^2\xi_\mu^2+\cdots, \tag{16}$$

无限小生成元向量为

$$X^{(0)}=\tau\frac{\partial}{\partial t}+\xi_\mu\frac{\partial}{\partial a^\mu}. \tag{17}$$

将(15),(16)式代入(17)式,有

$$X^{(0)}=\varepsilon^m X_m^{(0)}\qquad(m=0,1,2,\cdots), \tag{18}$$

式中

$$X_m^{(0)}=\tau^m\frac{\partial}{\partial t}+\xi_\mu^m\frac{\partial}{\partial a^\mu}. \tag{19}$$

扰动后的运动方程(14)在无限小变换(6)式下的不变性归为如下的 Lie 对称性确定方程:

$$\begin{aligned}&\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_\mu-h_\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau-\varepsilon\Omega^{\mu\nu}Q_\nu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau\\&=X^{(0)}(h_\mu)+\varepsilon X^{(0)}(\Omega^{\mu\nu}Q_\nu),\end{aligned} \tag{20}$$

式中

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}=\frac{\partial}{\partial t}+(h_\mu+\varepsilon\Omega^{\mu\nu}Q_\nu)\frac{\partial}{\partial a^\mu}. \tag{21}$$

将(15),(16)式代入(20)式,并比较等号两端 ε^m 的系数,有

$$\begin{aligned}&\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_\mu^m-h_\mu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^m-\Omega^{\mu\nu}Q_\nu\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^{m-1}\\&=X_m^{(0)}(h_\mu)+X_{m-1}^{(0)}(\Omega^{\mu\nu}Q_\nu).\end{aligned} \tag{22}$$

式中,当 $m=0$ 时,约定 $\tau^{-1}=0,\xi_\mu^{-1}=0$.

定理 2 对于受到小扰动 εQ_μ 作用的 Birkhoff 系统,如果生成元 τ^m,ξ_μ^m 满足确定方程(22),且存在函数 $\lambda=\lambda(t,\boldsymbol{a})$ 满足条件

$$\frac{\partial h_\mu}{\partial a^\mu}+\varepsilon\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda=0, \tag{23}$$

则 Birkhoff 系统存在一类新型高阶绝热不变量,形如

$$\begin{aligned}I_z=\sum_{m=0}^{z}\varepsilon^m\Big[&\frac{1}{\lambda}\frac{\partial}{\partial t}(\lambda\tau^m)\\&+\frac{1}{\lambda}\frac{\partial}{\partial a^\mu}(\lambda\xi_\mu^m)-\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^m\Big],\end{aligned} \tag{24}$$

式中,当 $m=0$ 时,约定 $\lambda=\lambda_0$.

证明

$$\begin{aligned}\frac{\bar{\mathrm{d}}}{\mathrm{d}t}I_z=\sum_{m=0}^{z}\varepsilon^m\Big[&\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\frac{\partial\xi_\mu^m}{\partial a^\mu}+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\frac{\partial}{\partial t}\tau^m\\&-\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^m+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}X_m^{(0)}(\ln\lambda)\Big].\end{aligned} \tag{25}$$

容易验证,对于任意函数 $\phi(t,\boldsymbol{a})$,如果无限小生成元 τ,ξ_μ 满足 Lie 对称性确定方程(20),则有下列关系:

$$\frac{\bar{d}}{dt}X^{(0)}(\phi) = X^{(0)}\left(\frac{\bar{d}}{dt}\phi\right) + \frac{\bar{d}}{dt}\tau\,\frac{\bar{d}}{dt}\phi. \tag{26}$$

由(18)和(26)式,得到

$$\begin{aligned}&\frac{\bar{d}}{dt}X_m^{(0)}(\ln\lambda)\\&= X_m^{(0)}\left(\frac{\bar{d}}{dt}\ln\lambda\right) + \frac{\bar{d}}{dt}\tau^m\,\frac{\bar{d}}{dt}\ln\lambda.\end{aligned} \tag{27}$$

经过直接的运算,我们得到

$$\begin{aligned}&\frac{\partial}{\partial a^\mu}\Big[\frac{\bar{d}}{dt}\xi_\mu - (h_\mu + \varepsilon\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau\\&\quad - X^{(0)}(h_\mu + \varepsilon\Omega^{\mu\nu}Q_\nu)\Big]\\&= \frac{\bar{d}}{dt}\frac{\partial\xi_\mu}{\partial a^\mu} + \frac{\bar{d}}{dt}\frac{\partial\tau}{\partial t} - \frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\tau\\&\quad - \frac{\partial}{\partial a^\mu}(h_\mu + \varepsilon\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau\\&\quad - X^{(0)}\Big[\frac{\partial}{\partial a^\mu}(h_\mu + \varepsilon\Omega^{\mu\nu}Q_\nu)\Big].\end{aligned} \tag{28}$$

将(15),(16)和(18)式代入(28)式,并令等号两端 ε^m 的系数分别相等,我们有

$$\begin{aligned}&\frac{\partial}{\partial a^\mu}\Big[\frac{\bar{d}}{dt}\xi_\mu^m - h_\mu\frac{\bar{d}}{dt}\tau^m - \Omega^{\mu\nu}Q_\nu\frac{\bar{d}}{dt}\tau^{m-1}\\&\quad - X_m^{(0)}(h_\mu) - X_{m-1}^{(0)}(\Omega^{\mu\nu}Q_\nu)\Big]\\&= \frac{\bar{d}}{dt}\frac{\partial\xi_\mu^m}{\partial a^\mu} + \frac{\bar{d}}{dt}\frac{\partial\tau^m}{\partial t} - \frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\tau^m\\&\quad - \frac{\partial h_\mu}{\partial a^\mu}\frac{\bar{d}}{dt}\tau^m - \frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau^{m-1}\\&\quad - X_m^{(0)}\left(\frac{\partial h_\mu}{\partial a^\mu}\right) - X_{m-1}^{(0)}\Big[\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\Big].\end{aligned} \tag{29}$$

将(27),(29)式代入(25)式,并利用(23)式和方程(22),我们得到

$$\begin{aligned}\frac{\bar{d}}{dt}I_z &= \sum_{m=0}^{z}\varepsilon^m\Big\{\frac{\partial}{\partial a^\mu}\Big[\frac{\bar{d}}{dt}\xi_\mu^m - h_\mu\frac{\bar{d}}{dt}\tau^m - \Omega^{\mu\nu}Q_\nu\frac{\bar{d}}{dt}\tau^{m-1}\\&\quad - X_m^{(0)}(h_\mu) - X_{m-1}^{(0)}(\Omega^{\mu\nu}Q_\nu)\Big]\\&\quad + \frac{\partial h_\mu}{\partial a^\mu}\frac{\bar{d}}{dt}\tau^m + \frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau^{m-1}\\&\quad + X_m^{(0)}(h_\mu) + X_{m-1}^{(0)}\Big[\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\Big]\\&\quad + X_m^{(0)}\left(\frac{\bar{d}}{dt}\ln\lambda\right) + \frac{\bar{d}}{dt}\tau^m\,\frac{\bar{d}}{dt}\ln\lambda\Big\}\\&= \sum_{m=0}^{z}\varepsilon^m\Big\{X_{m-1}^{(0)}\Big[\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\Big]\\&\quad - \varepsilon X_m^{(0)}\Big[\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\Big]\\&\quad + \frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau^{m-1}\\&\quad - \varepsilon\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau^m\Big\}\\&= -\varepsilon^{z+1}\Big\{X_z^{(0)}\Big[\frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\Big]\\&\quad + \frac{\partial}{\partial a^\mu}(\Omega^{\mu\nu}Q_\nu)\frac{\bar{d}}{dt}\tau^z\Big\}.\end{aligned} \tag{30}$$

因此,I_z 为 Birkhoff 系统的 z 阶绝热不变量.证毕.

4.算　　例

例　已知四阶 Birkhoff 系统的 Birkhoff 函数组和 Birkhoff 函数分别为

$$\begin{aligned}&R_1 = a^3,\\&R_2 = a^4,\\&R_3 = R_4 = 0,\end{aligned} \tag{31}$$

$$\begin{aligned}B &= \frac{1}{2}(a^3 - \arctan bt)^2\\&\quad + \frac{1}{2}\Big[a^4 - \frac{1}{2b}\ln(1 + b^2t^2)\Big]^2,\end{aligned} \tag{32}$$

试研究系统 Lie 对称性的摄动与绝热不变量.

系统的运动微分方程(3)给出

$$\begin{aligned}&\dot{a}^1 = a^3 - \frac{1}{b}\arctan bt,\\&\dot{a}^2 = a^4 - \frac{1}{2b}\ln(1 + b^2t^2),\\&\dot{a}^3 = 0,\\&\dot{a}^4 = 0.\end{aligned} \tag{33}$$

Lie 对称性确定方程(9)给出

$$\begin{aligned}&\frac{\bar{d}}{dt}\xi_1^0 - \left(a^3 - \frac{1}{b}\arctan bt\right)\frac{\bar{d}}{dt}\tau^0\\&= \xi_3^0 - \frac{1}{1 + b^2t^2}\tau^0,\\&\frac{\bar{d}}{dt}\xi_2^0 - \Big[a^4 - \frac{1}{2b}\ln(1 + b^2t^2)\Big]\frac{\bar{d}}{dt}\tau^0\\&= \xi_4^0 - \frac{bt}{1 + b^2t^2}\tau^0,\\&\frac{\bar{d}}{dt}\xi_3^0 = 0,\\&\frac{\bar{d}}{dt}\xi_4^0 = 0.\end{aligned} \tag{34}$$

方程(34)有如下解:

$$\begin{aligned}&\tau^0 = 1,\\&\xi_1^0 = -\frac{1}{b}\arctan bt,\end{aligned}$$

$$\xi_2^0 = -\frac{1}{2b}\ln(1+b^2t^2),$$
$$\xi_3^0 = 0,$$
$$\xi_4^0 = 0. \tag{35}$$

由方程(33),(11)式给出

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda_0 = 0. \tag{36}$$

它有解

$$\lambda_0 = a^4 t - a^2 - \frac{1}{2b}\int\ln(1+b^2t^2)\mathrm{d}t. \tag{37}$$

由(35)和(37)式,根据定理1,系统存在如下Hojman形式的精确不变量:

$$I_0 = a^4\left[a^4 t - a^2 - \frac{1}{2b}\int\ln(1+b^2t^2)\mathrm{d}t\right]^{-1} = \mathrm{const}. \tag{38}$$

下面给出系统的一阶绝热不变量.假设系统受到的小扰动为

$$\varepsilon Q_1 = -\frac{\varepsilon}{1+b^2t^2},$$
$$\varepsilon Q_2 = -\frac{\varepsilon bt}{1+b^2t^2}, \tag{39}$$
$$\varepsilon Q_3 = 0,$$
$$\varepsilon Q_4 = 0,$$

确定方程(22)给出

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_1^1 - \left(a^3 - \frac{1}{b}\arctan bt\right)\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^1 = \xi_3^1 - \frac{1}{1+b^2t^2}\tau^1,$$
$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_2^1 - \left[a^4 - \frac{1}{2b}\ln(1+b^2t^2)\right]\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^1 = \xi_4^1 - \frac{bt}{1+b^2t^2}\tau^1,$$
$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_3^1 - \frac{1}{1+b^2t^2}\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0 = -\frac{2b^2t}{(1+b^2t^2)^2}\tau^0,$$
$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_4^1 - \frac{bt}{1+b^2t^2}\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\tau^0 = \frac{b(1-b^2t^2)}{(1+b^2t^2)^2}\tau^0. \tag{40}$$

方程(40)有解

$$\tau^0 = 1,\quad \tau^1 = 1,$$
$$\xi_1^1 = 0,\quad \xi_2^1 = 0,$$
$$\xi_3^1 = \frac{1}{1+b^2t^2}, \tag{41}$$
$$\xi_4^1 = \frac{bt}{1+b^2t^2}.$$

(23)式给出

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\lambda = 0. \tag{42}$$

它有解

$$\lambda = bta^3 - a^4 - ba^1 - \int\arctan bt\,\mathrm{d}t. \tag{43}$$

由生成元(35),(41)及(37),(43)式,根据定理2,系统存在一阶绝热不变量,形如

$$I_1 = a^4\left[a^4 t - a^2 - \frac{1}{2b}\int\ln(1+b^2t^2)\mathrm{d}t\right]^{-1} + \left(a^3 - \frac{1}{b}\arctan bt\right)\left(ta^3 - \frac{1}{b}a^4 - a^1 - \frac{1}{b}\int\arctan bt\,\mathrm{d}t\right)^{-1}\varepsilon. \tag{44}$$

进一步可求得系统的更高阶绝热不变量.

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A new type of adiabatic invariants for Birkhoffian system*

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Abstract

The perturbation of symmetries and adiabatic invariants for Birkhoffian system are studied. The exact invariants in the form of Hojman conserved quantities introduced by the Lie symmetries of Birkhoffian system without perturbations are given. Based on the definition of high-order adiabatic invariants of a mechanical system, the perturbation of Lie symmetries for Birkhoffian system under the action of small disturbance is investigated, and a new type of adiabatic invariants of the system are obtained. An example is given to illustrate the application of the results.

Keywords: Birkhoffian system, Lie symmetry, perturbation, adiabatic invariant

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