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相空间中非完整可控力学系统的对称性摄动与绝热不变量*

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研究相空间中非完整可控力学系统的对称性摄动与绝热不变量．列出相空间中未受扰非完整可控力学系统的形式不变性导致的 Noether 守恒量．基于力学系统高阶绝热不变量的定义，研究小扰动作用下相空间中非完整可控力学系统的形式不变性摄动与绝热不变量，给出了精确不变量与绝热不变量存在的条件与形式，并举例说明结果的应用．

关键词：相空间，对称性，摄动，绝热不变量

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1．引 言

1917 年，Burgers 针对一类特殊的 Hamilton 系统提出绝热不变量概念[1]，绝热不变量又称缓慢不变量或浸渐不变量[2]，它是指当参数在缓慢变化时几乎不变的量，实际上参数的缓慢变化等同于一个小扰动作用，小扰动作用下对称性的改变及其不变量与力学系统的近可积性之间有密切关系，有必要进行研究[3]，本文以此为模型进行研究．

力学系统的运动依赖于作用力以及所加力的约束，对约束力学系统的研究在物理学中具有重要的意义．可控力学系统作为约束力学系统的扩充，长期以来，人们对其研究并不是很多，但是随着近代科学的进步，控制理论在社会中发挥着越来越重要的作用，这在很大程度上刺激了可控力学的发展，使可控力学的研究具有重要的理论和现实意义[4—13]．

文献[14—28]研究各种力学系统的对称性摄动与绝热不变量问题，得到了 Noether 形式的绝热不变量，最近，Zhang 等研究了 Lagrange 系统在小扰动作用下 Lie 对称性摄动，得到 Lagrange 系统的一类 Hojman 形式的高阶绝热不变量[29]．本文基于力学系统高阶绝热不变量的定义，研究相空间中非完整可控力学系统在小扰动作用下的形式不变性摄动和绝热不变量，证明了精确不变量与绝热不变量存在的条件与形式，并举例说明结果的应用．

2．系统的运动微分方程

研究一质点系，系统的位形由 n 个广义坐标 q_s $(s=1,\cdots,n)$来确定．系统的运动受 g 个 Chetaev 型非完整可控约束，

$$\varphi_\beta(t,q_s,\dot{q}_s,\mu_r,\dot{\mu}_r)=0$$

$$(\beta=1,\cdots,g;r=1,\cdots,b;s=1,\cdots,n), \quad (1)$$

$$\sum_{\beta=1}^{g}\frac{\partial\varphi_\beta}{\partial\dot{q}_s}\delta q_s=0. \quad (2)$$

一般情况下方程(1)包含控制参数 μ_r，则系统的运动微分方程可表示为

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{q}_s}-\frac{\partial L}{\partial q_s}=Q_s+\sum_{\beta=1}^{g}\lambda_\beta\frac{\partial\varphi_\beta}{\partial\dot{q}_s}$$

$$(s=1,\cdots,n;\beta=1,\cdots,g), \quad (3)$$

其中 $L=L(t,q_s,\dot{q}_s)$为系统的 Lagrange 函数，$Q_s=Q_s(t,q_s,\dot{q}_s)$为广义力，$\Lambda_s=\Lambda_s(t,q_s,\dot{q}_s,\mu_r,\dot{\mu}_r)$广义约束反力．$\lambda_\beta=\lambda_\beta(t,q_s,\dot{q}_s,\mu_r,\dot{\mu}_r)$为约束乘子．

引入广义动量和 Hamilton 函数

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$$p_s = \frac{\partial L}{\partial \dot{q}_s}, H = p_s \dot{q}_s - L, \tag{4}$$

方程(3)可表示为正则形式

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \dot{p}_s = -\frac{\partial H}{\partial q_s} + \widetilde{Q}_s + \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial \varphi_\beta}{\partial \dot{q}_s}. \tag{5}$$

设系统非奇异,可得相空间中与非完整系统(1),(3)相应完整系统的运动方程

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \dot{p}_s = -\frac{\partial H}{\partial q_s} + \widetilde{Q}_s + \widetilde{\Lambda}_s, \tag{6}$$

$$\widetilde{\Lambda}_s = \sum_{\beta=1}^{g} \lambda_\beta \frac{\partial \varphi_\beta}{\partial \dot{q}_s}, \tag{7}$$

其中的 H 和 $\widetilde{Q}_s$ 均表示为 t, q_s, p_s 的函数. $\widetilde{\Lambda}_s$ 表示为 $t, q_s, p_s, \mu_r, \dot{\mu}_r$ 的函数.

3. 无限小变换与精确不变量

在增广相空间中,取群的无限小变换为

$$\begin{aligned} t^* &= t + \varepsilon\tau(t, q_s, p_s), \\ q_s^*(t^*) &= q_s(t) + \varepsilon\xi_s(t, q_s, p_s), \\ p_s^*(t^*) &= p_s(t) + \varepsilon\eta_s(t, q_s, p_s), \end{aligned} \tag{8}$$

式中 ε 为无限小参数, τ, ξ_s, η_s 为无限小生成元.

取无限小生成元向量

$$X^{(0)} = \tau\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q_s} + \eta_s\frac{\partial}{\partial p_s}, \tag{9}$$

约束方程(1)和方程(6)在无限小变换下保持形式不变,即

$$\varphi_\beta^* = 0 \quad (\beta = 1, \cdots, g; r = 1, \cdots, b; s = 1, \cdots, n), \tag{10}$$

$$\dot{q}_s = \frac{\partial H^*}{\partial p_s}, \dot{p}_s = -\frac{\partial H^*}{\partial q_s} + \widetilde{Q}_s^* + \widetilde{\Lambda}_s^*. \tag{11}$$

展开 $\varphi_\beta^*, H^*, \widetilde{Q}_s^*$ 和 $\widetilde{\Lambda}_s^*$,可得形式不变性确定方程

$$X^{(0)}(\varphi_\beta) = 0, \tag{12}$$

$$\frac{\partial}{\partial p_s}\{X^{(0)}(H)\} = 0,$$

$$\frac{\partial}{\partial q_s}\{X^{(0)}(H)\} = X^{(0)}(\widetilde{Q}_s + \widetilde{\Lambda}_s). \tag{13}$$

定理 1 如果无限小生成元 $\tau^0, \xi_s^0, \eta_s^0$ 满足方程(12)和方程(13),且存在规范函数 $G^0 = G^0(t, q_s, p_s, \mu_r, \dot{\mu}_r)$满足

$$p_s\dot{\xi}_s^0 - \frac{\partial H}{\partial t}\tau^0 - \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^0 - \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^0 - \frac{\partial H}{\partial q_s}\xi_s^0 - H\dot{\tau}^0 + (\widetilde{Q}_s + \widetilde{\Lambda}_s)(\xi_s^0 - \dot{q}_s\tau^0) + \dot{G}^0 = 0, \tag{14}$$

则相空间中未受扰非完整可控力学系统存在如下精确不变量

$$I_0 = p_s\xi_s^0 - H\tau^0 + G^0 = \text{const}. \tag{15}$$

证明 将(15)式两边对 t 求导,联合(6)式,有

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}I_0 &= \dot{p}_s\xi_s^0 + p_s\dot{\xi}_s^0 - \frac{\partial H}{\partial t}\tau^0 - \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^0 - \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^0 \\ &\quad - \frac{\partial H}{\partial p_s}\dot{p}_s\tau^0 - \frac{\partial H}{\partial q_s}\dot{q}_s\tau^0 - H\dot{\tau}^0 - p_s\dot{\xi}_s^0 \\ &\quad + \frac{\partial H}{\partial t}\tau^0 + \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^0 + \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^0 \\ &\quad + \frac{\partial H}{\partial q_s}\xi_s^0 + H\dot{\tau}^0 - (\widetilde{Q}_s + \widetilde{\Lambda}_s)(\xi_s^0 - \dot{q}_s\tau^0) \\ &= \dot{p}_s\left(\xi_s^0 - \frac{\partial H}{\partial p_s}\tau^0\right) + \left(\frac{\partial H}{\partial q_s} - \widetilde{Q}_s - \widetilde{\Lambda}_s\right)(\xi_s^0 - \dot{q}_s\tau^0) \\ &= \left(\dot{p}_s + \frac{\partial H}{\partial q_s} - \widetilde{Q}_s - \widetilde{\Lambda}_s\right)(\xi_s^0 - \dot{q}_s\tau^0) \\ &= 0. \end{aligned} \tag{16}$$

定理 1 得证.

4. 对称性摄动与绝热不变量

首先,我们提出相空间中非完整可控力学系统的高阶不变量的定义.

定义 1 若 $I_z(t, q_s, p_s, \mu_r, \dot{\mu}_r, \varepsilon)$是力学系统的一个含有小参数 ε 的最高次幂为 z 的物理量,其时间 t 的一阶导数正比于 ε^{z+1},则称 I_z 为力学系统的 z 阶绝热不变量.

假设非完整可控力学系统(6)受到了一个小扰动 εW_s 的作用,则系统的运动微分方程变为

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \dot{p}_s = -\frac{\partial H}{\partial q_s} + \widetilde{Q}_s + \widetilde{\Lambda}_s + \varepsilon W_s, \tag{17}$$

在 εW_s 的作用下,系统原有的对称性和不变量相应地发生改变,假设这种改变是在系统无扰动的对称变换基础上发生的小摄动,如果 $\tau(t, q_s, p_s)$, $\xi_s(t, q_s, p_s)$和 $\eta_s(t, q_s, p_s)$表示扰动后时间和空间对应的生成函数,则

$$\begin{aligned} \tau &= \tau^0 + \varepsilon\tau^1 + \varepsilon^2\tau^2 + \cdots \\ \xi_s &= \xi_s^0 + \varepsilon\xi_s^1 + \varepsilon^2\xi_s^2 + \cdots \\ \eta_s &= \eta_s^0 + \varepsilon\eta_s^1 + \varepsilon^2\eta_s^2 + \cdots \end{aligned} \tag{18}$$

且满足

$$\begin{aligned} &p_s\dot{\xi}_s - \frac{\partial H}{\partial t}\tau - \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau - \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau - \frac{\partial H}{\partial q_s}\xi_s \\ &- H\dot{\tau} + (\widetilde{Q}_s + \widetilde{\Lambda}_s)(\xi_s - \dot{q}_s\tau) \\ &+ \varepsilon W_s(\xi_s - \dot{q}_s\tau) + \dot{G} = 0, \end{aligned} \tag{19}$$

式中 G 为规范函数，若记

$$G = G^0 + \varepsilon G^1 + \varepsilon^2 G^2 + \cdots \tag{20}$$

$$X^{(0)k} = \tau^k \frac{\partial}{\partial t} + \xi_s^k \frac{\partial}{\partial q_s} + \eta_s^k \frac{\partial}{\partial p_s}, \tag{21}$$

将(18)，(20)式代入(12)，(13)，(19)式，有

$$X^{(0)k}(\varphi_\beta) = 0, \tag{22}$$

$$\frac{\partial}{\partial p_s}\{X^{(0)k}(H)\} = 0,$$

$$\frac{\partial}{\partial q_s}\{X^{(0)k}(H)\} = X^{(0)k}(\widetilde{Q}_s + \widetilde{\Lambda}_s) + X^{(0)k}(\varepsilon W_s), \tag{23}$$

$$p_s\dot{\xi}_s^k - \frac{\partial H}{\partial t}\tau^k - \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^k - \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^k - \frac{\partial H}{\partial q_s}\xi_s^k$$
$$- H\dot{\tau}^k + (\widetilde{Q}_s + \widetilde{\Lambda}_s)(\xi_s^k - \dot{q}_s\tau^k)$$
$$+ W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1}) + \dot{G}^k = 0, \tag{24}$$

式中 $k=0$ 时，$W_s=0$.

定理 2　对于受到小扰动 εW_s 作用的非完整可控力学系统，若无穷小变换生成元 $\tau^k, \xi_s^k, \eta_s^k$ 满足方程(24)，$G^k(t, q_s, p_s, \mu_r, \dot{\mu}_r)$ 为规范函数，则

$$I_z = \varepsilon^k(p_s\xi_s^k - H\tau^k + G^k) = \text{const.} \tag{25}$$

是力学系统的一个 z 阶绝热不变量.

证明　(25)式两边对 t 求导，联合(24)式，有

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}I_z &= \varepsilon^k\Big\{\dot{p}_s\xi_s^k + p_s\dot{\xi}_s^k - \frac{\partial H}{\partial t}\tau^k - \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^k \\
&\quad - \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^k - \frac{\partial H}{\partial p_s}\dot{p}_s\tau^k - \frac{\partial H}{\partial q_s}\dot{q}_s\tau^k \\
&\quad - H\dot{\tau}^k - p_s\dot{\xi}_s^k + \frac{\partial H}{\partial t}\tau^k + \frac{\partial H}{\partial \mu_r}\dot{\mu}_r\tau^k + \frac{\partial H}{\partial \dot{\mu}_r}\ddot{\mu}_r\tau^k \\
&\quad + \frac{\partial H}{\partial q_s}\xi_s^k + H\dot{\tau}^k - (\widetilde{Q}_s + \widetilde{\Lambda}_s)(\xi_s^k - \dot{q}_s\tau^k) \\
&\quad - W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1})\Big\} \\
&= \varepsilon^k\Big\{\dot{p}_s\Big(\xi_s^k - \frac{\partial H}{\partial p_s}\tau^k\Big) + \Big(\frac{\partial H}{\partial q_s} - \widetilde{Q}_s - \widetilde{\Lambda}_s\Big) \\
&\quad \times(\xi_s^k - \dot{q}_s\tau^k) - W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1})\Big\} \\
&= \varepsilon^k\Big\{\Big(\dot{p}_s + \frac{\partial H}{\partial q_s} - \widetilde{Q}_s - \widetilde{\Lambda}_s\Big)(\xi_s^k - \dot{q}_s\tau^k) \\
&\quad - W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1})\Big\} \\
&= \varepsilon^k\{\varepsilon W_s(\xi_s^k - \dot{q}_s\tau^k) - W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1})\} \\
&= \varepsilon^{k+1}W_s(\xi_s^k - \dot{q}_s\tau^k) - \varepsilon^k W_s(\xi_s^{k-1} - \dot{q}_s\tau^{k-1}) \\
&= \varepsilon W_s(\xi_s^0 - \dot{q}_s\tau^0) + \varepsilon^2 W_s(\xi_s^1 - \dot{q}_s\tau^1) \\
&\quad + \varepsilon^3 W_s(\xi_s^2 - \dot{q}_s\tau^2) + \cdots \\
&\quad + \varepsilon^{z-1}W_s(\xi_s^{z-2} - \dot{q}_s\tau^{z-2}) \\
&\quad + \varepsilon^z W_s(\xi_s^{z-1} - \dot{q}_s\tau^{z-1}) + \varepsilon^{z+1}W_s(\xi_s^z - \dot{q}_s\tau^z) \\
&\quad - \varepsilon^0 W_s(\xi_s^{-1} - \dot{q}_s\tau^{-1}) - \varepsilon W_s(\xi_s^0 - \dot{q}_s\tau^0) \\
&\quad - \varepsilon^2 W_s(\xi_s^1 - \dot{q}_s\tau^1) - \varepsilon^3 W_s(\xi_s^2 - \dot{q}_s\tau^2) \\
&\quad - \cdots - \varepsilon^{z-1}W_s(\xi_s^{z-2} - \dot{q}_s\tau^{z-2}) \\
&\quad - \varepsilon^z W_s(\xi_s^{z-1} - \dot{q}_s\tau^{z-1}) \\
&= \varepsilon^{z+1}W_s(\xi_s^z - \dot{q}_s\tau^z) - \varepsilon^0 W_s(\xi_s^{-1} - \dot{q}_s\tau^{-1}) \\
&= \varepsilon^{z+1}W_s(\xi_s^z - \dot{q}_s\tau^2),
\end{aligned}$$

$k=0$ 时，约定 $W_s=0$.

由上式可得，$\frac{\mathrm{d}I_z}{\mathrm{d}t}$ 正比于 ε^{z+1}，即 I_z 是该力学系统的一个 z 阶绝热不变量.

5. 算　　例

假设系统的 Lagrange 函数为

$$L = \frac{1}{2}(\dot{q}_1^2 + \dot{q}_2^2), \tag{26}$$

非势广义力

$$Q_1 = Q_2 = 0, \tag{27}$$

系统所受的非完整约束力为

$$\varphi_\beta = \dot{q}_2 - \mu(t)q_2 = 0, \tag{28}$$

其中 $\mu(t)$ 为控制参数，$\mu(t)$ 是 t 的函数.

研究相空间中系统对称性摄动与绝热不变量.令

$$p_1 = \dot{q}_1, p_2 = \dot{q}_2,$$
$$H = \frac{1}{2}(p_1^2 + p_2^2), \tag{29}$$

系统的运动微分方程可表示为

$$\dot{q}_1 = p_1, \dot{q}_2 = p_2,$$
$$\dot{p}_1 = 0, \dot{p}_2 = \dot{\mu}q_2 + \mu p_2, \tag{30}$$

由限制方程(12)得

$$-\tau^0 q_2\dot{\mu} - \xi_2^0\mu + \eta_2^0 = 0. \tag{31}$$

由确定方程(13)得

$$\eta_1^0 + \eta_2^0 = 0,$$
$$\tau^0 q_2\ddot{\mu} + \tau^0 p_2\dot{\mu} + \xi_2^0\dot{\mu} + \eta_2^0\mu = 0, \tag{32}$$

由结构方程(14)得

$$p_1\dot{\xi}_1^0 + p_2\dot{\xi}_2^0 - \frac{1}{2}(p_1^2 + p_2^2)\dot{\tau}^0$$
$$+ (\dot{\mu}q_2 + \mu p_2)(\xi_2^0 - \dot{q}_2\tau^0) + \dot{G}^0 = 0, \tag{33}$$

(31)，(32)和(33)式有如下解：

$$\tau^0 = 0, \xi_1^0 = 1, \xi_2^0 = 0,$$
$$\eta_1^0 = \eta_2^0 = 0, G^0 = 0. \tag{34}$$

由定理 1 可得系统的精确不变量

$$I_0 = p_1 = \text{const}. \tag{35}$$

下面研究系统的绝热不变量,假设系统受到小扰动为

$$\varepsilon W_1 = \varepsilon \dot{p}_2, \varepsilon W_2 = \varepsilon p_2, \tag{36}$$

方程(22)给出

$$-\tau^1 q_2 \dot{\mu} - \xi_2^1 \mu + \eta_2^1 = 0, \tag{37}$$

方程(23)给出

$$\eta_1^1 + \eta_2^1 = 0,$$
$$\tau^1 q_2 \ddot{\mu} + \tau^1 p_2 \dot{\mu} + \xi_2^1 \dot{\mu} + \eta_2^1 \mu + \varepsilon \tau^1 \frac{\partial W_1}{\partial t}$$
$$+ \varepsilon \xi_1^1 \frac{\partial W_1}{\partial q_1} + \varepsilon \eta_1^1 \frac{\partial W_1}{\partial p_1} + \varepsilon \tau^1 \frac{\partial W_2}{\partial t}$$
$$+ \varepsilon \xi_2^1 \frac{\partial W_2}{\partial q_2} + \varepsilon \eta_2^1 \frac{\partial W_2}{\partial p_2} = 0, \tag{38}$$

方程(24)给出

$$p_1 \dot{\xi}_1^1 + p_2 \dot{\xi}_2^1 - \frac{1}{2}(p_1^2 + p_2^2)\dot{\tau}^1$$
$$+ (\dot{\mu} q_2 + \mu p_2)(\xi_2^1 - \dot{q}_2 \tau^1)$$
$$+ W_1(\xi_1^0 - \dot{q}_1 \tau^0)$$
$$+ W_2(\xi_2^0 - \dot{q}_2 \tau^0) + \dot{G}^1 = 0, \tag{39}$$

方程(37),(38)和(39)有如下解:

$$\tau^1 = 0, \xi_1^1 = 1, \xi_2^1 = 0,$$
$$\eta_1^1 = \eta_2^1 = 0, G^1 = -p_2, \tag{40}$$

由定理 2 可得系统的一阶绝热不变量

$$I_1 = p_1 + \varepsilon(p_1 - p_2) = \text{const}. \tag{41}$$

进一步可求得系统的更高阶绝热不变量.

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Perturbation to symmetries and adiabatic invariant for nonholonomic controllable mechanical system in phase place *

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Abstract

This paper studies the perturbation to symmetries and the adiabatic invariant for nonholonomic controllable mechanical system in the phase space. The exact invariants introduced by the form invariance of the nonholonomic controllable mechanical system in phase space without perturbation are given. Based on the definition of high-order adiabatic invariants of the mechanical system, the perturbation to symmetries and the adiabatic invariant for nonholonomic controllable mechanical system in phase space under the action of small disturbance is investigated, then the form of high-order adiabatic invariants and the conditions for their existence are presented. An example is finally given to illustrate the application of the results.

Keywords: phase place, symmetry, perturbation, adiabatic invariant

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