第 56 卷 第 12 期 2007 年 12 月
1000-3290/2007/56(12)/6937-08

物 理 学 报
ACTA PHYSICA SINICA

Vol.56, No.12, December, 2007

磁化铁氧体材料电磁散射递推卷积-时域有限差分方法分析*

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(2007 年 4 月 15 日收到;2007 年 6 月 25 日收到修改稿)

根据递推卷积原理,将磁化铁氧体材料的频域磁导率过渡到时域,通过引入时域复数磁化率张量和时域复数磁感应强度矢量,提出了磁化铁氧体材料电磁散射的三维时域有限差分方法.为了验证该方法,用它计算了磁化铁氧体球的后向雷达散射截面,与文献结果符合很好.计算结果表明,该方法是分析磁化铁氧体材料电磁散射一种可行的方法.

关键词:递推卷积,磁化铁氧体,电磁散射,时域有限差分方法

PACC:4110H,5170

1. 引 言

铁氧体材料为色散介质,在有外加磁场时为各向异性.近年来,对其物理特性的研究成为热点之一,如文献[1,2]分别研究了磁化铁氧体材料的自旋电流及自旋极化电流的影响和作用.本文主要研究了应用于磁各向异性色散介质的电磁散射的时域有限差分(FDTD)算法.根据文献,对于色散介质的FDTD方法研究现在有递推卷积[3](RC)法、辅助差分方程法[4—7]、Z 变换法[8]、移位算子法[9]等,但它们都是用色散介质FDTD方法来解决各向同性介质的电磁特性问题.作者最近提出了一种分析磁各向异性色散介质电磁散射的Padé-FDTD方法[10].文献[11]将文献[3]的方法推广应用于磁化铁氧体介质的电磁散射问题,本文在文献[11]的基础上研究并实现了分析磁各向异性色散介质的三维RC-FDTD方法.与Padé-FDTD方法相比,该方法能节约内存.

作为验证,计算了磁化铁氧体球的后向雷达散射截面(RCS),所得结果与文献[11]一致.理论推导及算例表明,这是一个正确有效的方法.

2. 磁化铁氧体材料电磁散射的FDTD迭代式

无源麦克斯韦方程组为

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},\tag{1}$$

$$\nabla\times\boldsymbol{H}=\varepsilon_0\varepsilon_{\mathrm r}\frac{\partial\boldsymbol{E}}{\partial t},\tag{2}$$

$$\boldsymbol{B}=\mu_0\boldsymbol{\mu}_{\mathrm r}\cdot\boldsymbol{H},\tag{3}$$

式中相对介电系数 $\varepsilon_{\mathrm r}$ 与频率无关,而 $\boldsymbol{\mu}_{\mathrm r}=\boldsymbol{\mu}_{\mathrm r}(\omega)$ 为一并矢,即介质电参数为各向同性,但介质磁性质为色散且各向异性.所以,用FDTD方法处理这种介质的电磁散射时,电场三个分量的FDTD迭代式与常规FDTD的迭代式相同,以 x 方向分量为例,即

$$E_x^{n+1}\left(i+\frac12,j,k\right)=E_x^n\left(i+\frac12,j,k\right)+\frac{\Delta t}{\varepsilon}\left[\frac{H_z^{n+1/2}\left(i+\frac12,j+\frac12,k\right)-H_z^{n+1/2}\left(i+\frac12,j-\frac12,k\right)}{\Delta y}\right.$$
$$\left.-\frac{H_y^{n+1/2}\left(i+\frac12,j,k+\frac12\right)-H_y^{n+1/2}\left(i+\frac12,j,k-\frac12\right)}{\Delta z}\right].$$

* 国家自然科学基金(批准号:40674060)资助的课题.

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磁场三个分量的 FDTD 迭代式,则要作出特殊处理.下面结合分析磁化铁氧体材料的本构关系进行详细分析.

3. 磁化铁氧体材料的离散时域本构关系的推导

3.1. 时域复数形式的磁化铁氧体的磁导率张量的引入

当外置磁场平行于 z 轴时,饱和磁化铁氧体的磁导率为

$$\boldsymbol{\mu} = \mu_0(\boldsymbol{I} + \boldsymbol{\chi}) = \mu_0\begin{bmatrix} 1+\chi_{11} & \chi_{12} & 0 \\ \chi_{21} & 1+\chi_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

式中 $\boldsymbol{I}$ 为单位矩阵,$\boldsymbol{\chi}$ 为磁化率矩阵.

$$\boldsymbol{\chi}(\omega) = \begin{bmatrix} \chi_{11}(\omega) & \mathrm{j}\chi_{12}(\omega) & 0 \\ -\mathrm{j}\chi_{21}(\omega) & \chi_{22}(\omega) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

$$\begin{aligned} \chi_{11}(\omega) &= \chi_{22}(\omega) = \frac{(\omega_0 + \mathrm{j}\omega\alpha)\omega_{\mathrm{m}}}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2}, \\ \chi_{12}(\omega) &= \chi_{21}(\omega) = \frac{\omega\omega_{\mathrm{m}}}{(\omega_0 + \mathrm{j}\omega\alpha)^2 - \omega^2}, \end{aligned} \tag{6}$$

式中

$$\omega_0 = \gamma H_0,$$

其中 H_0 为外加磁场强度的幅值,γ 为旋磁比,$\gamma = 1.76\times10^{11}$ Am/kg;

$$\omega_{\mathrm{m}} = \gamma 4\pi M_{\mathrm{s}},$$

其中 M_{s} 为饱和磁化率;α 为阻尼因子.

将(6)式进行逆快速傅里叶变换(IFFT)到时域,然后写成复数形式,有

$$\widehat{\chi}_{xx}(t) = \widehat{\chi}_{yy}(t) = \frac{\omega_m}{1+\alpha^2}(\alpha - \mathrm{j})\exp\left[-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})t\right], \tag{7}$$

$$\widehat{\chi}_{xy}(t) = \widehat{\chi}_{yx}(t) = \frac{\omega_m}{1+\alpha^2}(1 + \mathrm{j}\alpha)\exp\left[-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})t\right], \tag{8}$$

$$\chi_{xx}(t) = \chi_{yy}(t) = \mathrm{Re}(\widehat{\chi}_{xx}(t)) = \mathrm{Re}(\widehat{\chi}_{yy}(t)), \tag{9}$$

$$\chi_{xy}(t) = \chi_{yx}(t) = \mathrm{Re}(\widehat{\chi}_{xy}(t)) = \mathrm{Re}(\widehat{\chi}_{yx}(t)). \tag{10}$$

3.2. 离散时域的 FDTD 迭代式的推导

在频域,根据 $\boldsymbol{B}(\omega)$ 和 $\boldsymbol{H}(\omega)$ 之间的本构关系,将(4)式代入(3)式得

$$\begin{aligned} B_x(\omega) &= \mu_0(1 + \chi_{xx}(\omega))H_x(\omega) \\ &\quad + \mathrm{j}\chi_{xy}(\omega)\mu_0 H_y(\omega), \end{aligned} \tag{11}$$

$$\begin{aligned} B_y(\omega) &= \mu_0(1 + \chi_{yy}(\omega))H_y(\omega) \\ &\quad - \mathrm{j}\chi_{yx}(\omega)\mu_0 H_x(\omega), \end{aligned} \tag{12}$$

$$B_z(\omega) = \varepsilon_0 H_z(\omega). \tag{13}$$

在时域,(11),(12)和(13)式分别过渡为卷积关系

$$\begin{aligned} \frac{B_x(t)}{\mu_0} &= H_x(t) + \chi_{xx}(t) * H_x(t) \\ &\quad + \chi_{xy}(t) * H_y(t), \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{B_y(t)}{\mu_0} &= H_y(t) + \chi_{yy}(t) * H_y(t) \\ &\quad - \chi_{yx}(t) * H_x(t), \end{aligned} \tag{15}$$

$$\frac{B_z(t)}{\mu_0} = H_z(t). \tag{16}$$

根据卷积积分,(14),(15)和(16)式可变为

$$\begin{aligned} \frac{B_x(t)}{\mu_0} &= H_x(t) + \int_0^t H_x(t-\tau)\chi_{xx}(\tau)\mathrm{d}\tau \\ &\quad + \int_0^t H_y(t-\tau)\chi_{xy}(\tau)\mathrm{d}\tau, \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{B_y(t)}{\mu_0} &= H_y(t) + \int_0^t H_y(t-\tau)\chi_{yy}(\tau)\mathrm{d}\tau \\ &\quad - \int_0^t H_x(t-\tau)\chi_{yx}(\tau)\mathrm{d}\tau, \end{aligned} \tag{18}$$

$$\frac{B_z(t)}{\mu_0} = H_z(t). \tag{19}$$

将(17)—(19)三式在 $n+1/2$ 时刻进行离散,并考虑到指数函数的卷积积分,运用时域复数磁化率(7),(8)式,得到(17)—(19)式的时域复数形式如下:

$$\begin{aligned} \frac{\widehat{B}_x^{n+1/2}}{\mu_0} &= H_x^{n+1/2} + \int_0^{(n+1/2)\Delta t} H_x(n\Delta t - \tau)\widehat{\chi}_{xx}(\tau)\mathrm{d}\tau \\ &\quad + \int_0^{(n+1/2)\Delta t} H_y(n\Delta t - \tau)\widehat{\chi}_{xy}(\tau)\mathrm{d}\tau, \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\widehat{B}_y^{n+1/2}}{\varepsilon_0} &= H_y^{n} + \int_0^{(n+1/2)\Delta t} H_y(n\Delta t - \tau)\widehat{\chi}_{yy}(\tau)\mathrm{d}\tau \\ &\quad + \int_0^{(n+1/2)\Delta t} H_x(n\Delta t - \tau)\widehat{\chi}_{yx}(\tau)\mathrm{d}\tau, \end{aligned} \tag{21}$$

$$\frac{B_z^{n+1/2}}{\mu_0} = H_z^{n+1/2}. \tag{22}$$

应当注意的是，在电磁场的迭代更新过程中只用到时域复数磁场强度矢量的实部．因为卷积积分在离散情况下可写成卷积和的形式，在求和过程中复函数的实部与实部求和，虚部与虚部求和，所以在电磁场的迭代更新过程中只用到时域复数电位移矢量的实部．考虑到书写方便，下面将 $n+1/2$ 时间步简记为 n 时间步．

在 FDTD 迭代计算过程中，每个时间步 Δt 内 H_x 和 H_y 可近似看作是常量，并把卷积积分写成卷积和的形式，于是(20)式可变为

$$\frac{\widehat{B}_x^n}{\mu_0} = H_x^n + \widehat{\chi}_{xx}^0 H_x^n + \sum_{m=1}^{n-1} H_x^{n-m}\widehat{\chi}_{xx}^m + \widehat{\chi}_{xy}^0 H_y^n + \sum_{m=1}^{n-1} H_y^{n-m}\widehat{\chi}_{xy}^m, \tag{23}$$

式中

$$\begin{aligned}
\widehat{\chi}_{xx}^0 &= \int_0^{\Delta t/2} \widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{xy}^0 &= \int_0^{\Delta t/2} \widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{xx}^m &= \int_{(2m-1)\Delta t/2}^{(2m+1)\Delta t/2} \widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{xy}^m &= \int_{(2m-1)\Delta t/2}^{(2m+1)\Delta t/2} \widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau.
\end{aligned} \tag{24}$$

这样，

$$\frac{\widehat{B}_x^{n+1}}{\mu_0} = H_x^{n+1} + \widehat{\chi}_{xx}^0 H_x^{n+1} + \sum_{m=1}^{n} H_x^{n+1-m}\widehat{\chi}_{xx}^m + \widehat{\chi}_{xy}^0 H_y^{n+1} + \sum_{m=1}^{n} H_y^{n+1-m}\widehat{\chi}_{xy}^m. \tag{25}$$

将(25)式减去(23)式得

$$\begin{aligned}
\frac{\widehat{B}_x^{n+1} - \widehat{B}_x^n}{\mu_0} ={}& H_x^{n+1} - H_x^n + \chi_{xx}^0 H_x^{n+1} + \chi_{xy}^0 H_y^{n+1}\\
&+ \sum_{m=1}^{n} H_x^{n+1-m}\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau\\
&+ \sum_{m=1}^{n} H_y^{n+1-m}\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau\\
&- \sum_{m=0}^{n-1} H_x^{n-m}\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xx}(\tau)\,\mathrm{d}\tau\\
&- \sum_{m=0}^{n-1} H_y^{n-m}\int_{m\Delta t}^{(m+1)\Delta t}\widehat{\chi}_{xy}(\tau)\,\mathrm{d}\tau,
\end{aligned} \tag{26}$$

$$\begin{aligned}
\frac{\widehat{B}_x^{n+1} - \widehat{B}_x^n}{\mu_0} ={}& (1+\widehat{\chi}_{xx}^0)H_x^{n+1} + (\widehat{\chi}_{xx}^1 - 1 - \widehat{\chi}_{xx}^0)H_x^n\\
&+ \widehat{\chi}_{xy}^0 H_y^{n+1} + (\widehat{\chi}_{xy}^1 - \widehat{\chi}_{xy}^0)H_y^n\\
&- \sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^m H_x^{n-m} - \sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^m H_y^{n-m},
\end{aligned} \tag{27}$$

式中

$$\begin{aligned}
\Delta\widehat{\chi}_{xx}^m &= \widehat{\chi}_{xx}^m - \widehat{\chi}_{xx}^{m+1},\\
\Delta\widehat{\chi}_{xy}^m &= \widehat{\chi}_{xy}^m - \widehat{\chi}_{xy}^{m+1}.
\end{aligned} \tag{28}$$

同理，由(18)式有

$$\begin{aligned}
\frac{\widehat{B}_y^{n+1} - \widehat{B}_y^n}{\mu_0} ={}& (1+\widehat{\chi}_{yy}^0)H_x^{n+1} + (\widehat{\chi}_{yy}^1 - 1 - \widehat{\chi}_{yy}^0)H_y^n\\
&- \widehat{\chi}_{yx}^0 H_x^{n+1} - (\widehat{\chi}_{yx}^1 - \widehat{\chi}_{yx}^0)H_x^n\\
&- \sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yy}^m H_y^{n-m} + \sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^m H_x^{n-m},
\end{aligned} \tag{29}$$

式中

$$\begin{aligned}
\widehat{\chi}_{yy}^0 &= \int_0^{\Delta t/2} \widehat{\chi}_{yy}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{yx}^0 &= \int_0^{\Delta t/2} \widehat{\chi}_{yx}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{yy}^m &= \int_{(2m-1)\Delta t/2}^{(2m+1)\Delta t/2} \widehat{\chi}_{yy}(\tau)\,\mathrm{d}\tau,\\
\widehat{\chi}_{yx}^m &= \int_{(2m-1)\Delta t/2}^{(2m+1)\Delta t/2} \widehat{\chi}_{yx}(\tau)\,\mathrm{d}\tau,
\end{aligned} \tag{30}$$

$$\begin{aligned}
\Delta\widehat{\chi}_{yy}^m &= \widehat{\chi}_{yy}^m - \widehat{\chi}_{yy}^{m+1},\\
\Delta\widehat{\chi}_{yx}^m &= \widehat{\chi}_{yx}^m - \widehat{\chi}_{yx}^{m+1}.
\end{aligned} \tag{31}$$

由麦克斯韦旋度方程的(1)式，有

$$\frac{B_x^{n+1} - B_x^n}{\mu_0} = -\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_x, \tag{32}$$

$$\frac{B_y^{n+1} - B_y^n}{\mu_0} = -\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_y. \tag{33}$$

由(27)和(32)式，有

$$\begin{aligned}
&\frac{\mathrm{Re}(\widehat{B}_x^{n+1} - \widehat{B}_x^n)}{\mu_0}\\
&= -\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_x\\
&= (1+\chi_{xx}^0)H_x^{n+1} + (\chi_{xx}^1 - 1 - \chi_{xx}^0)H_x^n\\
&\quad + \chi_{xy}^0 H_y^{n+1} + (\chi_{xy}^1 - \chi_{xy}^0)H_y^n\\
&\quad - \mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^m H_x^{n-m}\Big) - \mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^m H_y^{n-m}\Big).
\end{aligned} \tag{34}$$

于是

$$H_x^{n+1}=\frac{1}{1+\chi_{xx}^0}\Bigg[-(\chi_{xx}^1-1-\chi_{xx}^0)H_x^n-\chi_{xy}^0H_y^{n+1}-(\chi_{xy}^1-\chi_{xy}^0)H_y^n$$
$$+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_x^{n-m}\Big)+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^mH_y^{n-m}\Big)-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_x\Bigg].\tag{35}$$

同理,有

$$H_y^{n+1}=\frac{1}{1+\chi_{yy}^0}\Bigg[-(\chi_{yy}^1-1-\chi_{yy}^0)H_y^n+\chi_{yx}^0H_x^{n+1}+(\chi_{yx}^1-\chi_{yx}^0)H_x^n-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_y$$
$$+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yy}^mH_y^{n-m}\Big)-\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^mH_x^{n-m}\Big)\Bigg].\tag{36}$$

由(35)和(36)式知,H_x^{n+1} 和 H_y^{n+1} 互相耦合,不能直接求出,所以将(35)式代入(36)式,有

$$H_x^{n+1}=\frac{(1+\widehat{\chi}_{xx}^0)}{(1+\widehat{\chi}_{xx}^0)^2+(\widehat{\chi}_{xy}^0)^2}\Bigg[-(\chi_{xx}^1-1-\chi_{xx}^0)H_x^n-(\chi_{xy}^1-\chi_{xy}^0)H_y^n-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_x$$
$$+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_x^{n-m}\Big)+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^mH_y^{n-m}\Big)\Bigg]$$
$$-\frac{\widehat{\chi}_{xy}^0}{(1+\widehat{\chi}_{xx}^0)^2+(\widehat{\chi}_{xy}^0)^2}\Bigg[-(\chi_{yy}^1-1-\chi_{yy}^0)H_y^n+(\chi_{yx}^1-\chi_{yx}^0)H_x^n+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yy}^mH_y^{n-m}\Big)$$
$$-\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^mH_x^{n-m}\Big)-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_y\Bigg].\tag{37}$$

同理,有

$$H_y^{n+1}=\frac{(1+\widehat{\chi}_{xx}^0)}{(1+\widehat{\chi}_{xx}^0)^2+(\widehat{\chi}_{xy}^0)^2}\Bigg[-(\chi_{yy}^1-1-\chi_{yy}^0)H_y^n+(\chi_{yx}^1-\chi_{yx}^0)H_x^n-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_y$$
$$+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yy}^mH_y^{n-m}\Big)-\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^mH_x^{n-m}\Big)\Bigg]$$
$$+\frac{\widehat{\chi}_{xy}^0}{(1+\widehat{\chi}_{xx}^0)^2+(\widehat{\chi}_{xy}^0)^2}\Bigg[-(\chi_{xx}^1-1-\chi_{xx}^0)H_x^n-(\chi_{xy}^1-\chi_{xy}^0)H_y^n+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_x^{n-m}\Big)$$
$$+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^mH_y^{n-m}\Big)-\frac{\Delta t}{\mu_0}(\nabla\times\boldsymbol{E})_x\Bigg].\tag{38}$$

整理(37)和(38)式,有

$$H_x^{n+1}=C_AH_x^n-C_BH_y^n-C_C(\nabla\times\boldsymbol{E})_x+C_D(\nabla\times\boldsymbol{E})_y$$
$$+C_E\Bigg[\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_x^{n-m}\Big)+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^mH_y^{n-m}\Big)\Bigg]$$
$$-C_F\Bigg[\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yy}^mH_y^{n-m}\Big)-\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^mH_x^{n-m}\Big)\Bigg],\tag{39}$$

$$H_y^{n+1}=C_AH_y^n+C_BH_x^n-C_C(\nabla\times\boldsymbol{E})_y-C_D(\nabla\times\boldsymbol{E})_x$$
$$+C_E\Bigg[\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_y^{n-m}\Big)-\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{yx}^mH_x^{n-m}\Big)\Bigg]$$
$$+C_F\Bigg[\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xx}^mH_x^{n-m}\Big)+\mathrm{Re}\Big(\sum_{m=1}^{n-1}\Delta\widehat{\chi}_{xy}^mH_y^{n-m}\Big)\Bigg],\tag{40}$$

式中

$$C_A = \frac{(1+\chi^0_{xx})(1+\chi^0_{xx}-\chi^1_{xx})-\chi^0_{xy}(\chi^1_{yx}-\chi^0_{yx})}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2},$$

$$C_B = \frac{(1+\widehat{\chi}^0_{xx})(\chi^1_{xy}-\chi^0_{xy})+\chi^0_{xy}(1+\chi^0_{xx}-\chi^1_{xx})}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2},$$

$$C_C = \frac{(1+\chi^0_{xx})\dfrac{\Delta t}{\mu_0}}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2},$$

$$C_D = \frac{\chi^0_{xy}\dfrac{\Delta t}{\mu_0}}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2},$$

$$C_E = \frac{1+\chi^0_{xx}}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2},$$

$$C_F = \frac{\chi^0_{xy}}{(1+\chi^0_{xx})^2+(\chi^0_{xy})^2}.$$

3.3. 磁化铁氧体材料电磁散射的 RC-FDTD 公式的推导

引入中间复型变量 $\widehat{\psi}^n_{xx}$，令

$$\widehat{\psi}^n_{xx} = \sum_{m=1}^{n-1} H_x^{n-m}\Delta\widehat{\chi}^m_{xx}. \tag{41}$$

根据(28)式，有

$$\Delta\widehat{\chi}^{m+1}_{xx} = \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha-\mathrm{j})\Delta t\right)\Delta\widehat{\chi}^m_{xx}.$$

于是(41)式又可写为

$$\widehat{\psi}^n_{xx} = \sum_{m=0}^{n} H_x^{n-m}\Delta\widehat{\chi}^m_{xx} = H_x^{n-1}\Delta\widehat{\chi}^1_{xx} + \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha-\mathrm{j})\Delta t\right)\widehat{\psi}^{n-1}_{xx}. \tag{42}$$

具体的推导见附录.于是，(39)和(41)式中的卷积和变成了一个递推迭代式，涉及的只有前一时间步的相应场量和辅助变量.这样也就大大降低了内存的消耗.同理，有

$$\widehat{\psi}^n_{yxy} = \sum_{m=1}^{n-1} H_y^{n-m}\Delta\widehat{\chi}^m_{xy} = H_y^{n-1}\Delta\widehat{\chi}^1_{xy} + \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha-\mathrm{j})\Delta t\right)\widehat{\psi}^{n-1}_{yxy}, \tag{43}$$

$$\widehat{\psi}^n_{yy} = \sum_{m=1}^{n-1} H_y^{n-m}\Delta\widehat{\chi}^m_{yy} = H_y^{n-1}\Delta\widehat{\chi}^1_{yy} + \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha-\mathrm{j})\Delta t\right)\widehat{\psi}^{n-1}_{yy}, \tag{44}$$

$$\widehat{\psi}^n_{xyx} = \sum_{m=1}^{n-1} H_x^{n-m}\Delta\widehat{\chi}^m_{yx} = H_x^{n-1}\Delta\widehat{\chi}^1_{yx} + \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha-\mathrm{j})\Delta t\right)\widehat{\psi}^{n-1}_{xyx}. \tag{45}$$

由以上所述可得，磁场三分量的 RC-FDTD 迭代式可转化为

$$H_x^{n+1} = C_A H_x^n - C_B H_y^n - C_C(\nabla\times\boldsymbol{E})_x + C_D(\nabla\times\boldsymbol{E})_y + C_E(\psi_{xx}+\psi_{xy}) - C_F(\psi_{yy}-\psi_{yx}), \tag{46}$$

$$H_y^{n+1} = C_A H_y^n + C_B H_x^n - C_C(\nabla\times\boldsymbol{E})_y - C_D(\nabla\times\boldsymbol{E})_x + C_E(\psi^n_{yy}-\psi^n_{yx}) + C_F(\psi^n_{xx}+\psi^n_{xy}), \tag{47}$$

式中

$$\psi_{xx} = \mathrm{Re}(\widehat{\psi}_{xx}),$$
$$\psi_{xy} = \mathrm{Re}(\widehat{\psi}_{xy}),$$
$$\psi_{yy} = \mathrm{Re}(\widehat{\psi}_{yy}),$$
$$\psi_{yx} = \mathrm{Re}(\widehat{\psi}_{yx}).$$

对于磁场分量 H_z 的计算与各向同性介质相同.至此，磁场的 RC-FDTD 迭代公式已推导完毕.

4. 数值结果

作为验证，用上述方法计算半径为 1.5 cm 的磁

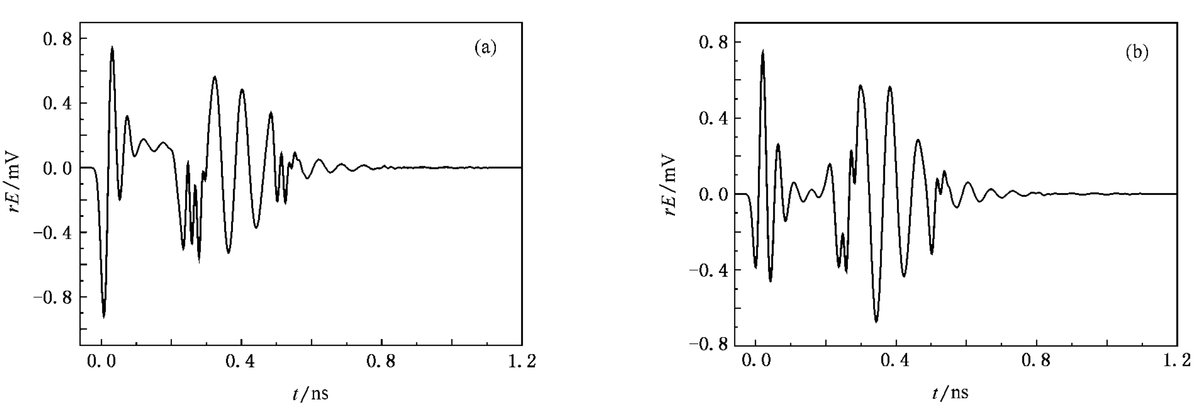

图 1　磁化铁氧体球的后向散射时域波形　(a)同极化，(b)交叉极化

化铁氧体球的后向散射. FDTD 计算中设 $\delta = 0.75$ mm, $\Delta t = \delta/(2c)$, c 为光速,入射波为高斯脉冲

$$E_{\mathrm{i}}(t) = \exp\left[-\frac{4\pi(t-t_0)^2}{\tau^2}\right], \tag{48}$$

沿着 z 轴入射,其中 $\tau = 34\Delta t$ 和 $t_0 = 0.8\tau$. 外加磁场平行于 z 轴, $\omega_0 = 2\pi\times 20$ GHz, $\omega_{\mathrm{m}} = 2\pi\times 10$ GHz, $\alpha = 0.1$. 计算结果如图 1、图 2 所示. 图 1 为磁化铁氧体球后向散射的同极化和交叉极化的后向时域波形,图 1纵坐标的 r 表示观察点到坐标原点的距离. 图 2为傅里叶变换后的磁化铁氧体球的后向 RCS,其中图 2(a)为同极化后向 RCS,图 2(b)为交叉极化后向 RCS,作为对比,图 2 中还给出了文献[11]的计算值. 由图 2 可见,两者符合得非常好.

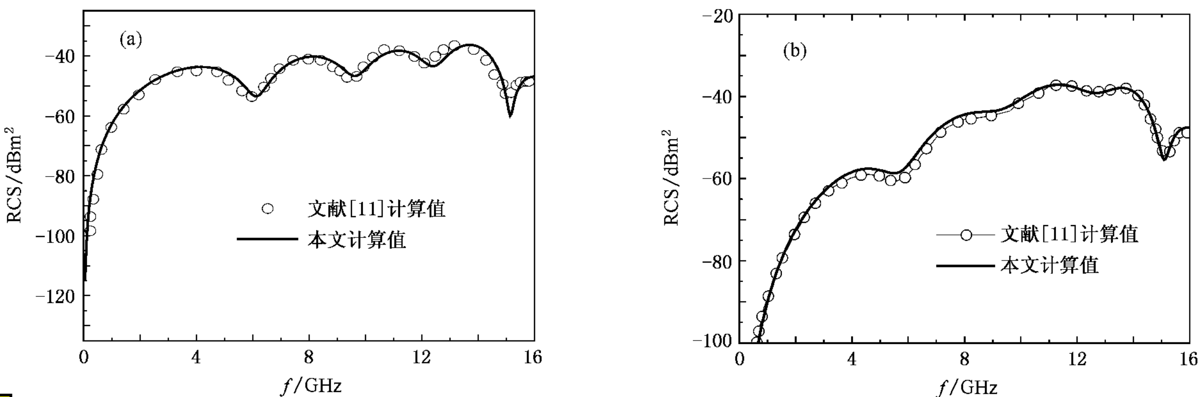

图 2　磁化铁氧体球的后向 RCS　(a)同极化,(b)交叉极化

5. 结　　论

铁氧体材料为色散介质,在外加磁场的条件下又呈现出磁各向异性. 通过 IFFT,将磁化铁氧体的频域介电系数过渡到时域,再根据卷积积分原理,引入时域复数磁化率张量和时域复数磁场强度矢量,得到离散时域带有卷积和的 FDTD 迭代式. 为了解决 FDTD 计算中卷积和的计算困难,引入中间辅助变量,运用 RC 原理,得到了磁化铁氧体材料电磁散射的三维 RC-FDTD 方法. 作为验证,计算了磁化铁氧体球的后向 RCS,所得结果与文献一致. 理论推导及算例表明该方法正确可行.

感谢 Recom 公司的 Joseph W. Schuster 博士对本文研究工作的帮助和建议.

附录　(41)—(45)式中间变量 $\widehat{\psi}_{xx}^{n}$, $\widehat{\psi}_{yxy}^{n}$, $\widehat{\psi}_{yy}^{n}$, $\widehat{\psi}_{xyx}^{n}$ 的推导[11]

正文中的(41)—(45)式中的变量 $\widehat{\psi}_{xx}^{n}$, $\widehat{\psi}_{yxy}^{n}$, $\widehat{\psi}_{yy}^{n}$, $\widehat{\psi}_{xyx}^{n}$ 的计算可用归纳法得到. 由(41)式有

$$\widehat{\psi}_{xx}^{n} = \sum_{m=1}^{n-1} H_x^{n-m}\Delta\widehat{\chi}_{xx}^{m}. \tag{A1}$$

先计算 $\widehat{\psi}_{xx}^{n}$ 的前 4 个时间步. 当 $n = 0, 1$ 时,

$$\begin{aligned}\widehat{\psi}_{xx}^{0} &= 0,\\ \widehat{\psi}_{xx}^{1} &= 0.\end{aligned} \tag{A2}$$

当 $n = 2$ 时,

$$\begin{aligned}\widehat{\psi}_{xx}^{2} &= \sum_{m=1}^{1} H_x^{2-m}\Delta\widehat{\chi}_{xx}^{m}\\ &= H_x^{1}\Delta\widehat{\chi}_{xx}^{1}.\end{aligned} \tag{A3}$$

当 $n = 3$ 时,

$$\begin{aligned}\widehat{\psi}_{xx}^{3} &= \sum_{m=1}^{2} H_x^{3-m}\Delta\widehat{\chi}_{xx}^{m}\\ &= H_x^{2}\Delta\widehat{\chi}_{xx}^{1} + H_x^{1}\Delta\widehat{\chi}_{xx}^{2}.\end{aligned} \tag{A4}$$

由(28)式知,

$$\Delta\widehat{\chi}_{xx}^{m+1} = \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})\Delta t\right)\Delta\widehat{\chi}_{xx}^{m}. \tag{A5}$$

于是

$$\begin{aligned}\widehat{\psi}_{xx}^{3} &= \sum_{m=1}^{2} H_x^{3-m}\Delta\widehat{\chi}_{xx}^{m}\\ &= H_x^{2}\Delta\widehat{\chi}_{xx}^{1} + H_x^{1}\Delta\widehat{\chi}_{xx}^{2}\\ &= H_x^{2}\Delta\widehat{\chi}_{xx}^{1} + \widehat{\psi}_{xx}^{2}\exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})\Delta t\right).\end{aligned} \tag{A6}$$

同理,

$$
\begin{aligned}
\widehat{\psi}^{4}_{xx} &= \sum_{m=1}^{3} H_x^{4-m} \Delta\widehat{\chi}^{m}_{xx} \\
&= H_x^3 \Delta\widehat{\chi}^{1}_{xx} + H_x^2 \Delta\widehat{\chi}^{2}_{xx} + H_x^1 \Delta\widehat{\chi}^{3}_{xx} \\
&= H_x^3 \Delta\widehat{\chi}^{1}_{xx} + \widehat{\psi}^{3}_{xx} \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})\Delta t\right). \quad \text{(A7)}
\end{aligned}
$$

综上可归纳得到

$$
\begin{aligned}
\widehat{\psi}^{n}_{xx} &= \sum_{m=1}^{n-1} H_x^{n-m} \Delta\widehat{\chi}^{m}_{xx} \\
&= H_x^{n-1} \Delta\widehat{\chi}^{1}_{xx} + \exp\left(-\frac{\omega_0}{1+\alpha^2}(\alpha - \mathrm{j})\Delta t\right)\widehat{\psi}^{n-1}_{xx}. \quad \text{(A8)}
\end{aligned}
$$

$\widehat{\psi}^{n}_{yxy}$，$\widehat{\psi}^{n}_{yy}$，$\widehat{\psi}^{n}_{xyx}$的推导过程与$\widehat{\psi}^{n}_{xx}$相同，在此从略.

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A recursive convolution-finite-difference time-domain implementation of electromagnetic scattering by magnetized ferrite medium*

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(Received 15 April 2007; revised manuscript received 25 June 2007)

Abstract

The permeability of magnetized ferrite medium in the frequency domain is transformed to the time domain, and the complex magnetic susceptibility dyadic matrix and the complex magnetic strength vector in time domain are introduced. A recursive convolution-finite-difference time-domain (RC-FDTD) method of electromagnetic scattering by magnetized ferrite medium is analysed in detail based on the convolution principle. To exemplify the availability of the algorithm, the backscattering radar scattering section of a magnetized ferrite sphere is computed, and the numerical results are the same as the reference values, which shows that the RC-FDTD method is correct and efficient.

Keywords: recursive convolution, magnetized ferrite, electromagnetic scattering, finite-difference time-domain method

PACC: 4110H, 5170

* Project supported by the National Natural Science Foundation of China (Grand No. 40674060).

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