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# 一般加速带电带磁的动态黑洞中标量场的熵*

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在一般加速带电带磁的动态黑洞中,化简 Klein-Gordon 场方程,利用乌龟坐标变换,得到在视界面附近的辐射温度. 用薄膜 brick-wall 模型,选择适当的截断因子和薄膜厚度,得到在视界面附近薄膜上的熵,结果表明黑洞熵与视界面积成正比.

关键词:黑洞，Hawking 温度，薄膜 brick-wall 模型，熵
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## 1. 引 言

自从 Bekenstein[1] 和 Hawking[2] 等提出黑洞的熵与其视界面积成正比以来,人们在黑洞热力学方面做了大量的工作,取得了许多有价值的成果. 1985 年,Hooft[3] 提出的 brick-wall 模型对黑洞熵的起源给出了一个统计解释. 2000 年,文献[4,5]把 brick-wall 模型发展成为薄膜模型. 人们用此模型计算各种类黑洞的熵,特别是在计算动态黑洞熵时,采用通常的乌龟坐标变换,要想得到黑洞熵与视界面积成正比,所选择的截断因子会变得很复杂[6—11]. 而文献[12]采用新的乌龟坐标变换,使得黑洞的辐射温度有所变化,得到了与静态或稳态一样简单的截断因子.

本文计算一般加速带电带磁的动态黑洞熵,采用新的乌龟坐标变换,在视界面附近将 Klein-Gordon 场方程化为标准的波动方程,得到了黑洞辐射温度. 它与旧的乌龟坐标变换下的结果不同,得到的是与静态或稳态情况下一样简捷的截断因子和薄膜厚度,使黑洞熵与面积成正比的表达式更为简单明了.

## 2. 一般加速带电带磁的动态黑洞的时空线元

一般加速带电带磁的动态黑洞的时空线元为

$$\begin{aligned}\mathrm{d}s^2 = {} & g_{00}\mathrm{d}v^2 + 2g_{01}\mathrm{d}v\mathrm{d}r + 2g_{02}\mathrm{d}v\mathrm{d}\theta \\ & + 2g_{03}\mathrm{d}v\mathrm{d}\varphi + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2,\end{aligned}\tag{1}$$

式中

$$\begin{aligned}g_{00} &= g_{00}(v,r,\theta,\varphi),\\ g_{01} &= g_{01}(v,r,\theta,\varphi),\\ g_{02} &= -r^2 f,\\ g_{03} &= -r^2 h\sin^2\theta,\\ g_{22} &= -r^2,\\ g_{33} &= -r^2\sin^2\theta.\end{aligned}\tag{2}$$

这里

$$\begin{aligned}f &= -a\sin\theta + b\sin\varphi + c\cos\varphi,\\ h &= \cot\theta(b\cos\varphi - c\sin\varphi),\end{aligned}$$

其中

$$\begin{aligned}a &= a(v),\\ b &= b(v),\\ c &= c(v)\end{aligned}$$

是加速度参量,$a$ 是加速度的大小,$b$ 和 $c$ 是描述加速度方向的改变率.

容易计算出度规行列式和非零逆变度规分量为

$$\begin{aligned}g &= -g_{01}^2 r^4\sin^2\theta,\\ g^{01} &= g^{10} = \frac{1}{g_{01}},\\ g^{11} &= -\frac{g_{00} + r^2f^2 + r^2h^2\sin^2\theta}{g_{01}^2},\\ g^{12} &= g^{21} = -\frac{f}{g_{01}},\\ g^{13} &= g^{31} = -\frac{h}{g_{01}},\\ g^{22} &= -\frac{1}{r^2},\end{aligned}$$

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$$g^{33} = -\frac{1}{r^2\sin^2\theta}. \tag{3}$$

## 3. 黑洞的温度

我们讨论一般加速带电带磁的动态黑洞的 Hawking 辐射．把黑洞的度规行列式和非零逆变度规分量代入 Klein-Gordon 方程中并进行化简，作乌龟坐标变换后得到在视界面附近的辐射温度．在弯曲时空中，Schwinger[13] 给出的带电荷 $e$ 和带磁荷 $q$ 的粒子 Klein-Gordon 方程为

$$\frac{1}{\sqrt{-g}}\left(\frac{\partial}{\partial x^\mu} - \mathrm{i}eA_\mu - \mathrm{i}qB_\mu\right) \times \left[\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu} - \mathrm{i}eA_\nu - \mathrm{i}qB_\nu\right)\Phi\right] - \mu_0^2\Phi = 0, \tag{4}$$

式中 $A_\mu$ 和 $B_\mu$ 为黑洞所带电荷和磁荷产生的电磁四势，$\mu_0$ 为粒子的静止质量．利用 Lorentz 条件

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}[\sqrt{-g}g^{\mu\nu}(eA_\nu + qB_\nu)] = 0, \tag{5}$$

并将(3)式代入(4)式，经过整理得到

$$\begin{aligned} &g^{11}\frac{\partial^2\Phi}{\partial r^2} + 2g^{1j}\frac{\partial^2\Phi}{\partial r\partial x^j} + g^{jj}\frac{\partial^2\Phi}{\partial x^{j2}} \\ &+ \left[\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{1\mu} + \frac{\partial g^{1\mu}}{\partial x^\mu} - \mathrm{i}2g^{1\mu}(eA_\mu + qB_\mu)\right]\frac{\partial\Phi}{\partial r} \\ &+ \left[\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{j\mu} + \frac{\partial g^{j\mu}}{\partial x^\mu} - \mathrm{i}2g^{j\mu}(eA_\mu + qB_\mu)\right]\frac{\partial\Phi}{\partial x^j} \\ &- [g^{\mu\nu}(eA_\mu + qB_\mu)(eA_\nu + qB_\nu) + \mu_0^2]\Phi = 0, \\ &\qquad (j = 0,2,3). \end{aligned} \tag{6}$$

给出乌龟坐标变换

$$\begin{aligned} r_* &= \frac{1}{2\kappa(v_0,\theta_0,\varphi_0)}\ln[r - r_{\mathrm{H}}(v,\theta,\varphi)], \\ v_* &= v - v_0, \\ \theta_* &= \theta - \theta_0, \\ \varphi_* &= \varphi - \varphi_0, \end{aligned} \tag{7}$$

式中 $r_{\mathrm{H}}$ 是黑洞的事件视界的位置，$\kappa$ 是一个可调节的待定参数，$v_0, \theta_0, \varphi_0$ 为任意固定参数，它们在乌龟坐标变换下保持不变．微分算子用乌龟坐标变换表出，有

$$\begin{aligned} \frac{\partial}{\partial r} &= \frac{1}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial}{\partial r_*}, \\ \frac{\partial}{\partial x^j} &= \frac{\partial}{\partial x^j_*} - \frac{r_{\mathrm{H},j}}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial}{\partial r_*}, \\ \frac{\partial^2}{\partial r^2} &= \frac{1}{[2\kappa(r - r_{\mathrm{H}})]^2}\frac{\partial^2}{\partial r_*^2} - \frac{1}{2\kappa(r - r_{\mathrm{H}})^2}\frac{\partial}{\partial r_*}, \\ \frac{\partial^2}{\partial r\partial x^j} &= -\frac{r_{\mathrm{H},j}}{[2\kappa(r - r_{\mathrm{H}})]^2}\frac{\partial^2}{\partial r_*^2} + \frac{r_{\mathrm{H},j}}{2\kappa(r - r_{\mathrm{H}})^2}\frac{\partial}{\partial r_*} + \frac{1}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial^2}{\partial r_*\partial x^j_*}, \\ \frac{\partial^2}{\partial x^{j2}} &= \frac{r^2_{\mathrm{H},j}}{[2\kappa(r - r_{\mathrm{H}})]^2}\frac{\partial^2}{\partial r_*^2} + \frac{\partial^2}{\partial x^{j2}_*} - \frac{2r_{\mathrm{H},j}}{2\kappa(r - r_{\mathrm{H}})}\frac{\partial^2}{\partial r_*\partial x^j_*} \\ &\quad - \frac{r_{\mathrm{H},jj}(r - r_{\mathrm{H}}) + r^2_{\mathrm{H},j}}{2\kappa(r - r_{\mathrm{H}})^2}\frac{\partial}{\partial r_*}, \end{aligned} \tag{8}$$

式中

$$r_{\mathrm{H},j} = \frac{\partial r_{\mathrm{H}}}{\partial x^j},$$

$$r_{\mathrm{H},jj} = \frac{\partial^2 r_{\mathrm{H}}}{\partial x^{j2}}.$$

将(8)式代入(6)式经整理后得

$$\begin{aligned} &A\frac{\partial^2\Phi}{\partial r_*^2} + 2(g^{1j} - r_{\mathrm{H},j}g^{jj})\frac{\partial^2\Phi}{\partial r_*\partial x^j_*} \\ &+ \Big\{-2\kappa A - r_{\mathrm{H},jj}g^{jj} + \frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{1\mu} + \frac{\partial g^{1\mu}}{\partial x^\mu} \\ &- \mathrm{i}2g^{1\mu}(eA_\mu + qB_\mu) - r_{\mathrm{H},j} \\ &\times \left[\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{j\mu} + \frac{\partial g^{j\mu}}{\partial x^\mu} - \mathrm{i}2g^{j\mu}(eA_\mu + qB_\mu)\right]\Big\} \\ &\times \frac{\partial\Phi}{\partial r_*} + 2\kappa(r - r_{\mathrm{H}})\Big\{g^{jj}\frac{\partial^2}{\partial x^{j2}_*} \\ &+ \left[\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{j\mu} + \frac{\partial g^{j\mu}}{\partial x^\mu} - \mathrm{i}2g^{j\mu}(eA_\mu + qB_\mu)\right]\frac{\partial\Phi}{\partial x^j_*} \\ &- [g^{\mu\nu}(eA_\mu + qB_\mu)(eA_\nu + qB_\nu)\Phi + \mu_0^2]\Phi\Big\} = 0, \end{aligned} \tag{9}$$

式中

$$A = \frac{g^{11} - 2r_{\mathrm{H},j}g^{1j} + r^2_{\mathrm{H},j}g^{jj}}{2\kappa(r - r_{\mathrm{H}})}. \tag{10}$$

在黑洞的事件视界附近，Klein-Gordon 方程应化为标准波动方程，就要求方程(9)中$\frac{\partial^2\Phi}{\partial r_*^2}$的系数 $A$ 在 $r \to r_{\mathrm{H}}$（表示 $r \to r_{\mathrm{H}}(v_0,\theta_0,\varphi_0)$，$v \to v_0$，$\theta \to \theta_0$，$\varphi \to \varphi_0$）时应等于一个常数，其分子必须满足

$$g^{11} - 2r_{\mathrm{H},j}g^{1j} + r^2_{\mathrm{H},j}g^{jj} = 0, \tag{11}$$

即

$$\frac{g_{00} + r^2f^2 + r^2h^2\sin^2\theta}{g_{01}^2} + \frac{2r_{\mathrm{H},v}}{g_{01}}$$

$$-\frac{2fr_{\mathrm{H},\varphi}}{g_{01}}-\frac{2hr_{\mathrm{H},\varphi}}{g_{01}}+\frac{r_{\mathrm{H},\theta}^2}{r^2}+\frac{r_{\mathrm{H},\varphi}^2}{r^2\sin^2\theta}\bigg|_{r=r_{\mathrm{H}}}=0. \tag{12}$$

这正好为确定黑洞事件视界面位置的方程. 在 $r\to r_{\mathrm{H}}$ 时调节参数 $\kappa$, 要使得$\frac{\partial^2\Phi}{\partial r_*^2}$与$\frac{\partial^2\Phi}{\partial r_*\partial v_*}$的系数比的极限为 1/2, 有

$$\kappa=\frac{1}{2g^{01}}\frac{\partial}{\partial r}\left(g^{11}-2r_{\mathrm{H},j}g^{1j}+r_{\mathrm{H},j}^2g^{jj}\right)\bigg|_{r=r_{\mathrm{H}}}, \tag{13}$$

即

$$\begin{aligned}\kappa=&\left(\frac{g_{00}+r^2f^2+r^2h^2\sin^2\theta}{g_{01}^2}\right.\\&\left.+\frac{r_{\mathrm{H},v}}{g_{01}}-\frac{fr_{\mathrm{H},\theta}}{g_{01}}-\frac{hr_{\mathrm{H},\varphi}}{g_{01}}\right)g_{01,1}\\&-\frac{g_{00,1}/2+rf^2+rh^2\sin^2\theta}{g_{01}}\\&+\frac{g_{01}r_{\mathrm{H},\theta}^2}{r^3}+\frac{g_{01}r_{\mathrm{H},\varphi}^2}{r^3\sin^2\theta}\bigg|_{r=r_{\mathrm{H}}}.\end{aligned} \tag{14}$$

这正好为黑洞的辐射温度函数. 在事件视界面附近方程(9)化为标准的波动方程

$$\begin{aligned}&\frac{\partial^2\Phi}{\partial r_*^2}+2\frac{\partial^2\Phi}{\partial r_*\partial v_*}+2X\frac{\partial^2\Phi}{\partial r_*\partial\theta_*}\\&+2Y\frac{\partial^2\Phi}{\partial r_*\partial\varphi_*}+(Z+\mathrm{i}2\omega_0)\frac{\partial\Phi}{\partial r_*}=0,\end{aligned} \tag{15}$$

式中

$$\begin{aligned}X&=-\left(f-\frac{g_{01}r_{\mathrm{H},\theta}}{r^2}\right)\bigg|_{r=r_{\mathrm{H}}},\\Y&=-\left(h-\frac{g_{01}r_{\mathrm{H},\varphi}}{r^2\sin^2\theta}\right)\bigg|_{r=r_{\mathrm{H}}},\\Z&=g_{01}\left[\frac{\partial g^{11}}{\partial r}-2r_{\mathrm{H},j}\frac{\partial g^{1j}}{\partial r}+r_{\mathrm{H},j}^2\frac{\partial g^{jj}}{\partial r}\right.\\&\quad-r_{\mathrm{H},jj}g^{jj}+\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{1\mu}-\frac{\partial g^{1\mu}}{\partial x^\mu}\\&\quad\left.-r_{\mathrm{H},j}\left(\frac{\partial\ln\sqrt{-g}}{\partial x^\mu}g^{j\mu}+\frac{\partial g^{j\mu}}{\partial x^\mu}\right)\right]\bigg|_{r=r_{\mathrm{H}}},\\\omega_0&=g_{01}r_{\mathrm{H},j}g^{j\mu}(eA_\mu+qB_\mu)\\&\quad-g_{01}g^{1\mu}(eA_\mu+qB_\mu)|_{r=r_{\mathrm{H}}}.\end{aligned} \tag{16}$$

在视界面附近,方程(15)的解可写为

$$\Phi=R(r_*)\Theta(\theta_*)\psi(\varphi_*)\exp(-\mathrm{i}\omega v_*). \tag{17}$$

根据 Damour-Ruffini[14] 和 Sannan[15] 方法,不难得到出射波的黑体谱 $N_\omega$ 和黑洞辐射温度 $T$ 的表达式,

$$N_\omega=\left[\exp\left(\frac{\omega-\omega_0}{T}\right)-1\right]^{-1}, \tag{18}$$

$$T=\frac{\kappa}{2\pi}. \tag{19}$$

从(19)式可以看出, $\kappa$ 的确是黑洞的辐射温度的函数,它不仅依赖于时间 $v$ 还依赖于极角 $\theta$ 和方位角 $\varphi$.

## 4. 黑洞的熵

采用薄膜 brick-wall 模型来计算黑洞的熵,薄膜就是在视界附近 $r_{\mathrm{H}}+\varepsilon\to r_{\mathrm{H}}+\varepsilon+\delta$(其中截断因子 $\varepsilon$ 和薄膜厚度 $\delta$ 都远远小于 $r_{\mathrm{H}}$)的区域. 一般加速带电带磁的动态黑洞属于非球对称动态黑洞,在视界附近的辐射温度要随着时间和位置发生变化. 因此,还要把薄膜分成许多小的子系统 $r_{\mathrm{H}}(v,\theta_i,\varphi_i)+\varepsilon\to r_{\mathrm{H}}(v,\theta_i,\varphi_i)+\varepsilon+\delta$, 在每个小子系统内的量子场是准热平衡的,且统计规律是有效的. 下面先计算薄膜上各个小子系统的熵,然后对其求和得到黑洞的总熵.

引入坐标变换[16]

$$R=r-r_{\mathrm{H}}(v,\theta,\varphi),$$

时空线元(1)式可化为

$$\begin{aligned}\mathrm{d}s^2=&\hat g_{00}\mathrm{d}v^2+2\hat g_{01}\mathrm{d}v\mathrm{d}R+2\hat g_{02}\mathrm{d}v\mathrm{d}\theta\\&+2\hat g_{03}\mathrm{d}v\mathrm{d}\varphi+\hat g_{22}\mathrm{d}\theta^2+\hat g_{33}\mathrm{d}\varphi^2,\end{aligned} \tag{20}$$

式中

$$\begin{aligned}\hat g_{00}&=g_{00}+2g_{01}r_{\mathrm{H},v},\\\hat g_{01}&=g_{01},\\\hat g_{02}&=g_{01}r_{\mathrm{H},\theta}-r^2f,\\\hat g_{03}&=g_{01}r_{\mathrm{H},\varphi}-r^2h\sin^2\theta,\\\hat g_{22}&=-r^2,\\\hat g_{33}&=-r^2\sin^2\theta.\end{aligned} \tag{21}$$

而非零逆变度规分量为

$$\begin{aligned}\hat g^{11}=&-\frac{g_{00}+r^2f^2+r^2h^2\sin^2\theta}{g_{01}^2}\\&-\frac{2r_{\mathrm{H},v}}{g_{01}}+\frac{2fr_{\mathrm{H},\theta}}{g_{01}}+\frac{2hr_{\mathrm{H},\varphi}}{g_{01}}\\&-\frac{r_{\mathrm{H},\theta}^2}{r_{\mathrm{H}}^2}-\frac{r_{\mathrm{H},\varphi}^2}{r_{\mathrm{H}}^2\sin^2\theta},\end{aligned}$$

$$\hat g^{01}=\frac{1}{\hat g_{01}},$$

$$\hat g^{12}=\frac{r_{\mathrm{H},\theta}}{r^2}-\frac{f}{\hat g_{01}},$$

$$\hat g^{13}=\frac{r_{\mathrm{H},\varphi}}{r^2\sin^2\theta}-\frac{h}{\hat g_{01}},$$

$$\hat{g}^{22}=-\frac{1}{r^2},$$

$$\hat{g}^{33}=-\frac{1}{r^2\sin^2\theta}. \tag{22}$$

注意到 $\hat{g}^{11}=0$ 正是视界面方程(见(12)式)，而

$$\left.\frac{\partial\hat{g}^{11}}{\partial r}\right|_{r=r_{\mathrm{H}}}=2\kappa g^{01}.$$

在坐标变换[16]下，Klein-Gorden 方程写为

$$\frac{1}{\sqrt{-\hat{g}}}\left(\frac{\partial}{\partial x^{\mu}}-\mathrm{i}e\hat{A}_{\mu}-\mathrm{i}q\hat{B}_{\mu}\right)$$

$$\times\left[\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\left(\frac{\partial}{\partial x^{\nu}}-\mathrm{i}e\hat{A}_{\nu}-\mathrm{i}q\hat{B}_{\nu}\right)\Phi\right]-\mu_0^2\Phi=0, \tag{23}$$

式中 $\hat{A}_{\mu}$ 和 $\hat{B}_{\mu}$ 是黑洞所产生的电磁四势经过坐标变换后的形式．设出射波为

$$\Phi=\exp(-\mathrm{i}Ev+\mathrm{i}G(R,\theta,\varphi)),$$

并利用 Lorentz 条件和 Wentzel-Kramwrs-Brillouin 近似可得到

$$\begin{aligned}&\hat{g}^{11}k_R^2-2[\hat{g}^{01}E-\hat{g}^{1j}k_j+\hat{g}^{1\mu}(e\hat{A}_{\mu}+q\hat{B}_{\mu})]k_R\\&+\hat{g}^{ij}k_j^2-2[(e\hat{A}_1+q\hat{B}_1)\hat{g}^{1j}+(e\hat{A}_j+q\hat{B}_j)\hat{g}^{ij}]k_j\\&+2\hat{g}^{01}(e\hat{A}_1+q\hat{B}_1)E\\&+\hat{g}^{\mu\nu}(e\hat{A}_{\mu}+q\hat{B}_{\mu})(e\hat{A}_{\nu}+q\hat{B}_{\nu})+\mu_0^2=0\\&\quad(j=2,3),\end{aligned} \tag{24}$$

式中

$$k_R=\frac{\partial G}{\partial R},$$

$$k_{\theta}=\frac{\partial G}{\partial\theta},$$

$$k_{\varphi}=\frac{\partial G}{\partial\varphi}.$$

从(24)式中可以得到 $k_R$ 与 $k_j$ 的关系

$$k_R^{\pm}=\frac{\hat{g}^{01}}{\hat{g}^{11}}\left[\tilde{E}+(e\hat{A}_1+q\hat{B}_1)\frac{\hat{g}^{11}}{\hat{g}^{01}}\right]\pm\frac{\hat{g}^{01}}{\hat{g}^{11}}\times\sqrt{\tilde{E}^2-\frac{\hat{g}^{11}}{(\hat{g}^{01})^2}[\hat{g}^{ij}(k_j-e\hat{A}_j-q\hat{B}_j)^2+\mu_0^2]}, \tag{25}$$

式中

$$\tilde{E}=E-\frac{\hat{g}^{1j}}{\hat{g}^{01}}(k_j-e\hat{A}_j-q\hat{B}_j)\qquad(j=0,2,3).$$

根据量子统计理论，第 $i$ 个子系统的自由能可以表示为

$$\Delta F_i=-\int_0^{\infty}\mathrm{d}\tilde{E}\,\frac{\Gamma(\tilde{E})}{\exp(\beta\tilde{E})-1}, \tag{26}$$

其中 $\Gamma(\tilde{E})$ 是能量不大于 $\tilde{E}$ 的微观态的数目．根据半经典量子化条件和薄膜 brick-wall 模型，有

$$\Gamma(\tilde{E})=\frac{1}{4\pi^3}\int\mathrm{d}k_{\theta}\int\mathrm{d}k_{\varphi}\int_{\theta_i}^{\theta+\Delta\theta_i}\mathrm{d}\theta\int_{\varphi_i}^{\varphi+\Delta\varphi_i}\mathrm{d}\varphi\times\left(\int_{\varepsilon}^{\varepsilon+\delta}k_R^+\mathrm{d}R+\int_{\varepsilon+\delta}^{\varepsilon}k_R^-\mathrm{d}R\right). \tag{27}$$

将(25)式代入(27)式，对 $k_j$ 进行积分．考虑到

$$\tilde{E}^2-\hat{g}^{11}[\hat{g}^{ij}(k_j-e\hat{A}_j-q\hat{B}_j)^2+\mu_0^2]/(\hat{g}^{01})^2\geqslant 0,$$

要限定 $k_j$ 的积分上下限，积分结果为

$$\begin{aligned}\Gamma(\tilde{E})&\approx-\frac{\tilde{E}^3}{6\pi^2}\int\mathrm{d}\theta\mathrm{d}\varphi\int_{\varepsilon}^{\varepsilon+\delta}\frac{(\hat{g}^{01})^3}{(\hat{g}^{11})^2}(\hat{g}^{22}\hat{g}^{33})^{-1/2}\mathrm{d}R\\&=-\frac{\tilde{E}^3}{6\pi^2}\int\mathrm{d}A_i\int_{\varepsilon}^{\varepsilon+\delta}\frac{(\hat{g}^{01})^3}{(\hat{g}^{11})^2}\mathrm{d}R,\end{aligned} \tag{28}$$

式中

$$\int\mathrm{d}A_i=\int_{\theta_i}^{\theta+\Delta\theta_i}\int_{\varphi_i}^{\varphi+\Delta\varphi_i}\sqrt{\hat{g}_{22}\hat{g}_{33}}\,\mathrm{d}\theta\mathrm{d}\varphi$$

是第 $i$ 个子系统在视界面上的小面积，记为 $\Delta A_i$．把(28)式代入(26)式，对 $\tilde{E}$ 积分并保留低阶项，得

$$\begin{aligned}\Delta F_i&=\frac{\Delta A_i}{6\pi^2}\int_{\varepsilon}^{\varepsilon+\delta}\frac{(\hat{g}^{01})^3}{(\hat{g}^{11})^2}\mathrm{d}R\int_0^{\infty}\frac{\tilde{E}^3}{\exp(\beta\tilde{E})-1}d\tilde{E}\\&=\frac{\pi^2\Delta A_i}{90\beta^4}\int_{\varepsilon}^{\varepsilon+\delta}\frac{(\hat{g}^{01})^3}{(\hat{g}^{11})^2}\mathrm{d}R.\end{aligned} \tag{29}$$

由于 $\hat{g}^{11}=0$ 为视界位置的方程(见(12)式)，可令 $\hat{g}^{11}=(r-r_{\mathrm{H}})f(v,r,\theta,\varphi)$，并代入(29)式，第 $i$ 个子系统的自由能为

$$\begin{aligned}\Delta F_i&=\frac{\pi^2\Delta A_i}{90\beta^4}\int_{\varepsilon}^{\varepsilon+\delta}\frac{(\hat{g}^{01})^3}{(r-r_{\mathrm{H}})^2f^2(v,r,\theta,\varphi)}\mathrm{d}R\\&=\frac{\pi^2(\hat{g}^{01})^3\Delta A_i}{90\beta^4f^2(r_{\mathrm{H}})}\frac{\delta}{\varepsilon(\varepsilon+\delta)}.\end{aligned} \tag{30}$$

由熵与自由能的关系，可得到第 $i$ 个子系统的熵为

$$\begin{aligned}\Delta S_i&=\beta^2\left.\frac{\partial\Delta F_i}{\partial\beta}\right|_{\beta=\beta_{\mathrm{H}}}\\&=-\left.\frac{4\pi^2(\hat{g}^{01})^3\Delta A_i}{90\beta^3f^2(r_{\mathrm{H}})}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\right|_{r=r_{\mathrm{H}}}.\end{aligned} \tag{31}$$

考虑到(13)式，有

$$\begin{aligned}f(r_{\mathrm{H}})&=\left|\frac{\partial\hat{g}^{11}}{\partial r}\right|_{r=r_{\mathrm{H}}}\\&=2\kappa g^{01}\big|_{r=r_{\mathrm{H}}},\end{aligned}$$

$$\beta_{\mathrm{H}}=\frac{2\pi}{\kappa},$$

因此有

$$\Delta S_i = -\frac{g^{01}}{90\beta}\frac{\delta}{\varepsilon(\varepsilon+\delta)}\frac{\Delta A_i}{4}\bigg|_{r=r_{\rm H}}. \tag{32}$$

选择适当的截断因子和薄膜厚度,使之满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = -90\beta g_{01}\big|_{r=r_{\rm H}},$$

则第 $i$ 个子系统的熵为

$$\Delta S_i = \frac{\Delta A_i}{4}. \tag{33}$$

进而,黑洞的总熵为

$$\begin{aligned} S &= \sum_i \Delta S_i \\ &= \frac{1}{4}\sum_i \Delta A_i = \frac{1}{4}A_{\rm H}. \end{aligned} \tag{34}$$

## 5. 几种典型动态黑洞的辐射温度函数和熵

对于下面的各种动态黑洞,只要给出时空线元,按照(14)式计算出黑洞辐射温度函数,选择合适的截断因子和薄膜厚度,就可直接给出黑洞标量场的熵,并能过渡到静态或稳态黑洞标量场的熵.

### 5.1. 动态 Vaidya-Bonner 黑洞

动态 Vaidya-Bonner 黑洞[8] 的时空线元为

$$\begin{aligned} ds^2 &= \left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)dv^2 - 2dvdr \\ &\quad - r^2(d\theta^2+\sin^2\theta d\varphi^2), \end{aligned} \tag{35}$$

式中 $m = m(v)$, $Q = Q(v)$. 将(35)式中的度规代入(14)式,得到该黑洞的辐射温度函数

$$\kappa = \frac{m}{r^2} - \frac{Q^2}{r^3}\bigg|_{r=r_{\rm H}}, \tag{36}$$

而

$$\begin{aligned} f(r_{\rm H}) &= \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\rm H}} \\ &= -2\kappa. \end{aligned}$$

选择的截断因子和薄膜厚度满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\rm H},$$

则(34)式就是动态 Vaidya-Bonner 黑洞熵的表达式.

### 5.2. 球对称带电动态黑洞

球对称带电动态黑洞[9] 的时空线元为

$$\begin{aligned} ds^2 &= e^{2\psi}\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)dv^2 \\ &\quad - 2e^{\psi}dvdr - r^2(d\theta^2+\sin^2\theta d\varphi), \end{aligned} \tag{37}$$

式中 $m = m(v,r)$, $Q = Q(v)$, $\psi = \psi(v,r)$. 将(37)式中的度规代入(14)式,得到该黑洞的辐射温度函数

$$\begin{aligned} \kappa &= \frac{e^{\psi}}{2}\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)\psi' \\ &\quad + e^{\psi}\left(\frac{m}{r^2}-\frac{m'}{r}-\frac{Q^2}{r^3}\right)\bigg|_{r=r_{\rm H}}, \end{aligned} \tag{38}$$

而

$$\begin{aligned} f(r_{\rm H}) &= \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\rm H}} \\ &= -2\kappa e^{-\psi}. \end{aligned}$$

选择的截断因子和薄膜厚度满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\rm H}e^{\psi},$$

则(34)式就是球对称带电动态黑洞熵的表达式.

### 5.3. 一般球对称动态黑洞

一般球对称动态黑洞[10] 的时空线元为

$$\begin{aligned} ds^2 &= g_{00}dv^2 + 2g_{01}dvdr \\ &\quad - r^2(d\theta^2+\sin^2\theta d\varphi^2), \end{aligned} \tag{39}$$

式中 $g_{00} = g_{00}(v,r)$, $g_{01} = g_{01}(v,r)$. 将(39)式中的度规代入(14)式,得到该黑洞的辐射温度函数

$$\kappa = \frac{g_{00}g_{01,1}}{2g_{01}^2} - \frac{g_{00,1}}{2g_{01}}\bigg|_{r=r_{\rm H}}, \tag{40}$$

而

$$\begin{aligned} f(r_{\rm H}) &= \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\rm H}} \\ &= 2\kappa g^{01}. \end{aligned}$$

选择的截断因子和薄膜厚度满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = -90\beta_{\rm H}g_{01},$$

则(34)式就是一般球对称动态黑洞熵的表达式.

### 5.4. 直线加速运动动态黑洞

直线加速运动动态黑洞[11] 的时空线元为

$$\begin{aligned} ds^2 &= \left(1-\frac{2m}{r}-2ar\cos\theta-r^2f^2\right)dv^2 - 2dvdr \\ &\quad - 2r^2fdvd\theta - r^2(d\theta^2+\sin^2\theta d\varphi^2), \end{aligned} \tag{41}$$

式中 $f = -a\sin\theta$,参量 $m = m(v)$ 是黑洞的质量,$a = a(v)$ 是加速度参量. 将(41)式中的度规代入(14)式,得到该黑洞的辐射温度函数

$$\kappa = \frac{m}{r^2} - \frac{r_{\rm H\theta}^2}{r^3} - a\cos\theta\bigg|_{r=r_{\rm H}}, \tag{42}$$

而

$$f(r_{\mathrm{H}}) = \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\mathrm{H}}} = -2\kappa.$$

选择的截断因子和薄膜厚度满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\mathrm{H}},$$

则(34)式就是直线加速运动动态黑洞熵的表达式.

### 5.5. 带有电荷与磁荷的直线加速动态黑洞

带有电荷与磁荷的直线加速动态黑洞[10]的时空线元为

$$\begin{aligned} \mathrm{d}s^2 = &\left[1-\frac{2m}{r}+\frac{e^2+q^2}{r^2}-2ar\cos\theta\right.\\ &\left.-4a\frac{e^2+q^2}{r}\cos\theta - r^2f^2-\frac{1}{3}\lambda r^2\right]\mathrm{d}v^2\\ &-2\mathrm{d}v\mathrm{d}r-2r^2f\mathrm{d}v\mathrm{d}\theta - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \end{aligned} \tag{43}$$

式中 $f=-a\sin\theta$，$m=m(v)$，$e=e(v)$和 $q=q(v)$分别为黑洞的质量、电荷和磁荷，参数 $a=a(v)$为加速度大小. 将(43)式中的度规代入(14)式，得到该黑洞的辐射温度函数

$$\kappa = \frac{m}{r^2}+2a\frac{e^2+q^2}{r^2}\cos\theta-\frac{e^2+q^2}{r^3} - \frac{r_{\mathrm{H}\theta}^2}{r^3}-a\cos\theta-\frac{1}{3}\lambda r\bigg|_{r=r_{\mathrm{H}}}, \tag{44}$$

而

$$f(r_{\mathrm{H}}) = \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\mathrm{H}}} = -2\kappa.$$

选择的截断因子和薄膜厚度满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\mathrm{H}},$$

则(34)式就是带有电荷与磁荷的直线加速动态黑洞熵的表达式.

### 5.6. 任意加速带电动态黑洞

任意加速带电动态黑洞[12]的时空线元为

$$\begin{aligned} \mathrm{d}s^2 = &\left[1-\frac{2m}{r}-2ar\cos\theta+\frac{Q^2}{r^2}-4a\frac{Q^2}{r}\cos\theta\right.\\ &\left.-r^2(f^2+h^2\sin^2\theta)\right]\mathrm{d}v^2\\ &-2\mathrm{d}v\mathrm{d}r-2r^2f\mathrm{d}v\mathrm{d}\theta-2r^2h\sin^2\theta\mathrm{d}v\mathrm{d}\varphi\\ &-r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \end{aligned} \tag{45}$$

其中 $f=-a\sin\theta+b\sin\varphi+c\cos\varphi$，$h=\cot\theta(b\cos\varphi-c\sin\varphi)$，参量 $m=m(v)$，$Q=Q(v)$分别为黑洞的质量和所带电荷，$a=a(v)$，$b=b(v)$，$c=c(v)$为加速度参量，$a$ 是加速度的大小，$b$ 和 $c$ 是描述加速度方向的改变率. 将(45)式中的度规代入(14)式，得到该黑洞的辐射温度函数

$$\kappa = \frac{1}{r^2}(m+2aQ^2\cos\theta) - \frac{1}{r^3}\left(Q^2+r_{\mathrm{H}\theta}^2+\frac{r_{\mathrm{H}\varphi}^2}{\sin^2\theta}\right)-a\cos\theta\bigg|_{r=r_{\mathrm{H}}}, \tag{46}$$

而

$$f(r_{\mathrm{H}}) = \frac{\partial \hat{g}^{11}}{\partial r}\bigg|_{r=r_{\mathrm{H}}} = -2\kappa.$$

选择的截断因子和薄膜厚度，满足

$$\frac{\delta}{\varepsilon(\varepsilon+\delta)} = 90\beta_{\mathrm{H}},$$

则(34)式就是任意加速带电动态黑洞熵的表达式.

## 6. 结 论

在一般加速带电带磁的动态黑洞中，利用新的乌龟坐标变换，得到视界面附近黑洞的 Hawking 辐射温度. 采用薄膜 brick-wall 模型，计算出在黑洞视界面附近薄膜上的熵，选择合适的截断因子和薄膜厚度，就得到了黑洞总熵与视界面积成正比的结论. 由于采用新的乌龟坐标变换，使黑洞辐射温度表达式变得更简捷，所选择的截断因子和薄膜厚度就与静态或稳态一样简单明了.


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# Entropy of the scalar field in general accelerating non-stationary black holes with electric charge and magnetic charge*


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## Abstract

Using tortoise coordinate transformation, Klein-Gordon field equation is simplified and thermal radiation temperature near the event horizon is obtained. Meanwhile, adopting thin film brick-wall model and regulating the cut-off parameter and the thin film's thickness properly, the entropy of thin film near the event horizon is acquired. The results show that the entropy of the black hole is proportional to the area of the event horizon.



**Keywords**: black hole, Hawking temperature, film brick-wall model, entropy

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