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规范变换对 Birkhoff 系统对称性的影响

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研究 Birkhoff 系统规范变换对其 Noether 对称性、Lie 对称性和 Mei 对称性的影响.在一定条件下,Noether 对称性和守恒量不改变.Lie 对称性和 Hojaman 守恒量仍保持不变.Mei 对称性和新型守恒量可能变化,得到了 Mei 对称性和新型守恒量保持不变的条件.举例说明结果的应用.

关键词:Birkhoff 系统,规范变换,对称性,守恒量
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1.引 言

Birkhoff 系统动力学是近代分析力学的一个重要分支[1—3],研究力学系统的对称性并由此导出守恒量是数学、力学和物理学中重要课题,对 Birkhoff 系统对称性的研究取得多方面进展和丰富成果[4—17].在变量保持不变情形下,Birkhoff 函数和函数组可以发生变换,其中之一是 Birkhoff 规范变换,在此变换下 Birkhoff 方程保持不变.本文研究规范变换对 Birkhoff 系统的 Noether 对称性、Lie 对称性和 Mei 对称性及相应的守恒量的影响,并举例说明所得到的结果.

2.Birkhoff 方程和规范变换

Birkhoff 系统运动微分方程为

$$\Omega_{\mu\nu}\dot{a}^{\nu}-\frac{\partial B}{\partial a^{\mu}}-\frac{\partial R_{\mu}}{\partial t}=0,\quad (\mu,\nu=1,\cdots,2n),\tag{1}$$

其中 a^{μ} 为变量,$B(t,\boldsymbol{a})$ 称为 Birkhoff 函数,$R_{\mu}(t,\boldsymbol{a})$ 称为 Birkhoff 函数组,而

$$\Omega_{\mu\nu}=\frac{\partial R_{\nu}}{\partial a^{\mu}}-\frac{\partial R_{\mu}}{\partial a^{\nu}}\tag{2}$$

称为 Birkhoff 张量.设系统是规则的,即

$$\det(\Omega_{\mu\nu})\neq 0.\tag{3}$$

则由方程(1)可解出全部 $\dot{a}^{\mu}$,得到

$$\dot{a}^{\mu}=\Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^{\nu}}+\frac{\partial R_{\nu}}{\partial t}\right),\tag{4}$$

其中 $\Omega^{\mu\nu}$ 称为 Birkhoff 逆变张量,且有

$$\Omega^{\mu\nu}\Omega_{\nu\tau}=\delta^{\mu}_{\tau},\tag{5}$$

当时间 t 和变量 a^{μ} 保持不变,动力学函数 B 和 R_{μ} 可以作如下 Birkhoff 规范变换:

$$R_{\mu}\rightarrow R'_{\mu}=R_{\mu}(t,\boldsymbol{a})+\frac{\partial G(t,\boldsymbol{a})}{\partial a^{\mu}},$$

$$B\rightarrow B'=B(t,\boldsymbol{a})-\frac{\partial G(t,\boldsymbol{a})}{\partial t}.\tag{6}$$

其中 $G(t,\boldsymbol{a})$ 称为规范变换函数,在此变换下,

$$\Omega'_{\mu\nu}=\frac{\partial R'_{\nu}}{\partial a^{\mu}}-\frac{\partial R'_{\mu}}{\partial a^{\nu}}=\frac{\partial R_{\nu}}{\partial a^{\mu}}-\frac{\partial R_{\mu}}{\partial a^{\nu}}=\Omega_{\mu\nu},\tag{7}$$

$$\frac{\partial B'}{\partial a^{\mu}}+\frac{\partial R'_{\mu}}{\partial t}=\frac{\partial B}{\partial a^{\mu}}+\frac{\partial R_{\mu}}{\partial t},\tag{8}$$

[illegible]Birkhoff 方程(1)和(4)是不变的.

3.规范变换对 Noether 对称性和守恒量的影响

取时间 t 和变量 a^{μ} 的无限小变换

$$t^{*}=t+\varepsilon\xi_{0}(t,\boldsymbol{a}),\ a^{\mu^{*}}=a^{\mu}+\varepsilon\xi_{\mu}(t,\boldsymbol{a}),\tag{9}$$

其中 ε 为无限小参数,ξ_{o},ξ_{μ} 为无限小生成元.如果 Birkhoff 系统(1)满足 Noether 等式

$$X^{(1)}(R_{\nu}\dot{a}^{\nu}-B)+(R_{\nu}\dot{a}^{\nu}-B)\dot{\xi}_{0}+\dot{G}_{N}=0,\tag{10}$$

其中 $G_{N}=G_{N}(t,\boldsymbol{a})$ 称为规范函数,

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$$X^{(1)} = X^{(0)} + (\dot{\xi}_\mu - \dot{a}^\mu \dot{\xi}_0)\frac{\partial}{\partial \dot{a}^\mu}, \tag{11}$$

$$X^{(0)} = \xi_0 \frac{\partial}{\partial t} + \xi_\mu \frac{\partial}{\partial a^\mu}, \tag{12}$$

则系统这种不变性为 Noether 对称性,且可以导出 Noether 守恒量

$$I_N = R_\mu \xi_\mu - B\xi_0 + G_N = \text{const}. \tag{13}$$

以 Birkhoff 系统作规范变换(6)后的 B' 和 R'_μ 代入 Noether 等式(10),得到

$$\begin{aligned} &X^{(1)}(R'_\nu \dot{a}^\nu - B') + (R'_\nu \dot{a}^\nu - B')\dot{\xi}_0 + \dot{G}_N \\ &= X^{(1)}(R_\nu \dot{a}^\nu - B) + (R_\nu \dot{a}^\nu - B)\dot{\xi}_0 \\ &\quad + \dot{G}_N + \frac{d}{dt}X^{(0)}(G) = \frac{d}{dt}X^{(0)}(G), \end{aligned} \tag{14}$$

调整规范函数,取

$$G'_N = G_N - X^{(0)}(G), \tag{15}$$

则使得 Noether 等式仍成立,

$$X^{(1)}(R'_\nu \dot{a}^\nu - B') + (R'_\nu \dot{a}^\nu - B')\dot{\xi}_0 + \dot{G}'_N = 0. \tag{16}$$

系统在变换(9)下的不变性仍为 Noether 对称性,导出的 Noether 守恒量为

$$\begin{aligned} I'_N &= R'_\mu \xi_\mu - B'\xi_0 + G'_N \\ &= R_\mu \xi_\mu - B\xi_0 + X^{(0)}(G) + G'_N \\ &= R_\mu \xi_\mu - B\xi_0 + G_N = I_N, \end{aligned} \tag{17}$$

即 Birkhoff 规范变换下 Noether 守恒量不变.

4. 规范变换对 Lie 对称性和 Hojman 守恒量的影响

如果 Birkhoff 系统(1)在变换(9)下满足 Lie 对称性确定方程

$$X^{(1)}\left\{\Omega_{\mu\nu}\dot{a}^\nu - \frac{\partial B}{\partial a^\mu} - \frac{\partial R_\mu}{\partial t}\right\} = 0, \tag{18}$$

则对应的不变性为系统的 Lie 对称性. 由 Lie 对称性可以直接导出 Hojman 守恒量,设变换(9)中时间不变,即 $\xi_0 = 0$,而生成元 ξ_μ 满足

$$\frac{\bar{d}}{dt}\xi_\mu = \frac{\partial}{\partial a^\rho}\left\{\Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^\nu} + \frac{\partial R_v}{\partial t}\right)\right\}\xi_\rho, \tag{19}$$

其中

$$\frac{\bar{d}}{dt} = \frac{\partial}{\partial t} + \Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^\nu} + \frac{\partial R_v}{\partial t}\right)\frac{\partial}{\partial a^\mu}, \tag{20}$$

并且存在函数 $\mu = \mu(t, a)$ 满足条件

$$\frac{\partial}{\partial a^\mu}\left\{\Omega^{\mu\nu}\left(\frac{\partial B}{\partial a^\nu} + \frac{\partial R_v}{\partial t}\right)\right\} + \frac{\bar{d}}{dt}\ln\mu = 0, \tag{21}$$

则系统存在 Hojman 守恒量

$$I_H = \frac{1}{\mu}\frac{\partial}{\partial a^\nu}(\mu\xi_\nu) = \text{const}. \tag{22}$$

当 Birkhoff 系统(1)作规范变换(6)后,对新的动力学函数 B' 和 R'_μ,已经证明了(7)和(8)式的结果,由(7)和(5)式可以得到

$$\Omega'^{\mu\nu} = \Omega^{\mu\nu}. \tag{23}$$

这就是说,Lie 对称性确定方程和 Hojman 守恒量存在的条件,在规范变换后仍成立,故 Birkhoff 规范变换对系统的 Lie 对称性和 Hojman 守恒量无影响.

5. 规范变换对 Mei 对称性和新型守恒量的影响

如果 Birkhoff 系统(1)在变换(9)下满足形式不变性判据方程

$$\begin{aligned} &\left\{\frac{\partial}{\partial a^\mu}X^{(0)}(R_\nu) - \frac{\partial}{\partial a^\nu}X^{(0)}(R_\mu)\right\}\dot{a}^\nu \\ &- \frac{\partial}{\partial a^\mu}X^{(0)}(B) - \frac{\partial}{\partial t}X^{(0)}(R_\mu) = 0, \end{aligned} \tag{24}$$

则相应的不变性为系统的 Mei 对称性. 由 Mei 对称性可以导出新型不变量,设生成元 ξ_0,ξ_μ 和规范函数 $G_F = G_F(t, a)$ 满足结构方程

$$\begin{aligned} &[X^{(0)}(R_\mu)\dot{a}^\mu - X^{(0)}(B)]\dot{\xi}_0 + X^{(1)}[X^{(0)}(R_\mu)\dot{a}^\mu \\ &- X^{(0)}(B)] + \dot{G}_F = 0, \end{aligned} \tag{25}$$

则系统存在新型守恒量

$$\begin{aligned} I_F = &X^{(0)}(R_\mu)\xi_\mu - X^{(0)}(B)\xi_0 + G_F \\ &= \text{const}. \end{aligned} \tag{26}$$

将 Birkhoff 系统(1)作规范变换(6)后的 R'_μ 和 B' 代入形式不变性判据方程(24),得到

$$\begin{aligned} &\left\{\frac{\partial}{\partial a^\mu}X^{(0)}(R'_\nu) - \frac{\partial}{\partial a^\nu}X^{(0)}(R'_\mu)\right\}\dot{a}^\nu \\ &- \frac{\partial}{\partial a^\mu}X^{(0)}(B') - \frac{\partial}{\partial t}X^{(0)}(R'_\mu) \\ &= \left\{\frac{\partial}{\partial a^\mu}X^{(0)}\left(\frac{\partial G}{\partial a^\nu}\right) - \frac{\partial}{\partial a^\nu}X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\right\}\dot{a}^\nu \\ &+ \frac{\partial}{\partial a^\mu}X^{(0)}\left(\frac{\partial G}{\partial t}\right) - \frac{\partial}{\partial t}X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right) \\ &= \left(\frac{\partial \xi_o}{\partial a^\mu}\frac{\partial}{\partial t} + \frac{\partial \xi_\rho}{\partial a^\mu}\frac{\partial}{\partial a^\rho}\right)\dot{G} \\ &- \left(\dot{\xi}_0\frac{\partial}{\partial t} + \dot{\xi}_\rho\frac{\partial}{\partial a^\rho}\right)\frac{\partial G}{\partial a^\mu}. \end{aligned} \tag{27}$$

这就表明,在一般情况下规范变换后的 Birkhoff 系统将破坏 Mei 对称性. 然而,如果规范变换函数 G 满

足下列条件：

$$\left(\frac{\partial \xi_0}{\partial a^\mu}\frac{\partial}{\partial t}+\frac{\partial \xi_\rho}{\partial a^\mu}\frac{\partial}{\partial a^\rho}\right)\dot G - \left(\dot\xi_0\frac{\partial}{\partial t}+\dot\xi_\rho\frac{\partial}{\partial a^\rho}\right)\frac{\partial G}{\partial a^\mu}=0, \tag{28}$$

变换后的 Birkhoff 系统保持 Mei 对称性不变.

将 B' 和 R'_μ 代入结构方程式(25)，得到

$$\begin{aligned}&[X^{(0)}(R'_\mu)\dot a^\mu - X^{(0)}(B')]\dot\xi_0 + X^{(1)}[X^{(0)}(R'_\mu)\dot a^\mu - X^{(0)}(B')] + \dot G_F\\ &=\left[X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot a^\mu + X^{(0)}\left(\frac{\partial G}{\partial t}\right)\right]\dot\xi_0 + X^{(1)}\left[X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot a^\mu + X^{(0)}\left(\frac{\partial G}{\partial t}\right)\right]\\ &=X^{(0)}\left(\frac{\partial G}{\partial t}\right)\dot\xi_0 + X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot\xi_\mu + X^{(0)}\left\{X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\right\}\dot a^\mu + X^{(0)}\left\{X^{(0)}\left(\frac{\partial G}{\partial t}\right)\right\}\\ &=X^{(0)}\left(\frac{\partial G}{\partial t}\right)\dot\xi_0 + X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot\xi_\mu + X^{(0)}\{X^{(0)}(\dot G)\}.\end{aligned}\tag{29}$$

换句话说，规范变换函数 G 不仅要满足(28)式，系统才能保持 Mei 对称性，而且 G 还要满足

$$X^{(0)}\{X^{(0)}(\dot G)\} + X^{(0)}\left(\frac{\partial G}{\partial t}\right)\dot\xi_0 + X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot\xi_\mu = 0, \tag{30}$$

才能保持新型不变量不变. 如果(30)式不能满足，但是存在新的规范函数 G'_F，使得

$$\dot G'_F = \dot G_F - \left[X^{(0)}\left(\frac{\partial G}{\partial t}\right)\dot\xi_0 + X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot\xi_\mu + X^{(0)}\{X^{(0)}(\dot G)\}\right], \tag{31}$$

则可以导出新的新型守恒量

$$I'_F = X^{(0)}(R'_\mu)\xi_\mu - X^{(0)}(B')\xi_0 + G'_F = \text{const}. \tag{32}$$

6. 算 例

4 阶 Birkhoff 系统为

$$R_1 = a^3, R_2 = a^4, R_3 = R_4 = 0,\quad B = a^2 + \frac{1}{2}\{(a^3)^2 + (a^4)^2\}, \tag{33}$$

经过规范变换(6)后为

$$R'_1 = R'_2 = 0, R'_3 = -a^1, R'_4 = -a^2, B' = B, \tag{34}$$

规范变换函数为

$$G = -(a^1a^3 + a^2a^4). \tag{35}$$

研究上述规范变换对系统 Noether 对称性和守恒量，以及 Mei 对称性和新型守恒量的影响.

对系统(33)，Noether 等式(10)给出

$$\dot a^1\xi_3 + a^3\dot\xi_1 + \dot a^2\xi_4 + a^4\dot\xi_2 - \xi_2 - a^3\xi_3 - a^4\xi_4 - \left\{a^2 + \frac{1}{2}[(a^3)^2 + (a^4)^2]\right\}\dot\xi_0 + \dot G_N = 0. \tag{36}$$

上式的解不是唯一的，下面给出 5 组解：

$$\xi_0 = -1, \xi_1 = \xi_2 = \xi_3 = \xi_4 = 0, G_N = 0; \tag{37}$$

$$\xi_1 = 1, \xi_0 = \xi_2 = \xi_3 = \xi_4 = 0, G_N = 0; \tag{38}$$

$$\xi_2 = 1, \xi_0 = \xi_1 = \xi_3 = \xi_4 = 0, G_N = t; \tag{39}$$

$$\xi_1 = -t, \xi_3 = -1, \xi_0 = \xi_2 = \xi_4 = 0, G_N = a^1; \tag{40}$$

$$\xi_2 = -t, \xi_4 = -1, \xi_0 = \xi_1 = \xi_3 = 0, G_N = a^2 - \frac{1}{2}t^2. \tag{41}$$

将(37)—(41)式代入(13)式，分别得到 5 个守恒量：

$$I_N = B = a^2 + \frac{1}{2}[(a^3)^2 + (a^4)^2] = \text{const}; \tag{42}$$

$$I_N = a^3 = \text{const}; \tag{43}$$

$$I_N = a^4 + t = \text{const}; \tag{44}$$

$$I_N = a^1 - a^3t = \text{const}; \tag{45}$$

$$I_N = a^2 - a^4t - \frac{1}{2}t^2 = \text{const}. \tag{46}$$

对规范变换后的系统(34)，Noether 等式(10)给出

$$-a^1\dot\xi_3 - a^2\dot\xi_4 - \dot a^3\xi_1 - \dot a^4\xi_2 - \xi_2 - a^3\xi_3 - a^4\xi_4 - \left\{a^2 + \frac{1}{2}[(a^3)^2 + (a^4)^2]\right\}\dot\xi_0 + \dot G'_N = 0. \tag{47}$$

对应(37)—(41)式，(47)式也有 5 组解：

$$\xi_0 = -1, \xi_1 = \xi_2 = \xi_3 = \xi_4 = 0, G'_N = 0; \tag{48}$$

$$\xi_1 = 1, \xi_0 = \xi_2 = \xi_3 = \xi_4 = 0, G'_N = a^3; \tag{49}$$

$$\xi_2 = 0, \xi_0 = \xi_1 = \xi_3 = \xi_4 = 0, G'_N = t + a^4; \tag{50}$$

$$\xi_1 = -t, \xi_3 = -1, \xi_0 = \xi_2 = \xi_4 = 0, G'_N = -ta^3; \tag{51}$$

$$\xi_2 = -t, \xi_4 = -1, \xi_0 = \xi_1 = \xi_3 = 0, G'_N = -ta^4 - \frac{1}{2}t^2. \tag{52}$$

容易验证，(48)—(52)式中各组的 G'_N 与(37)—(41)

式中各组的 G_N 之间,均满足(15)式关系.以(41)式与(52)式为例,

$$X^{(0)}(G)=\left(-t\frac{\partial}{\partial a^2}-\frac{\partial}{\partial a^4}\right)[-(a'a^3+a^2a^4)]$$
$$=a^2+ta^4, \qquad (53)$$

$$G'_N=G_N-X^{(0)}(G)=-ta^4-\frac{1}{2}t^2. \qquad (54)$$

将(48)—(52)式代入(13)式,仍得到与(42)—(46)式相同的 5 个守恒量,即验证了(17)式.

讨论系统(33)的 Mei 对称性,容易得到

$$X^{(0)}(R_1)=\xi_3, X^{(0)}(R_2)=\xi_4,$$
$$X^{(0)}(R_3)=X^{(0)}(R_4)=0,$$
$$X^{(0)}(B)=\xi_2+a^3\xi_3+a^4\xi_4. \qquad (55)$$

形式不变性判据方程(24)给出一组解:

$$\xi_0=\xi_1=\xi_3=0, \xi_2=a^4, \xi_4=-1, \qquad (56)$$

代入结构方程(25),得到

$$G_F=-t. \qquad (57)$$

代入(26)式,得新型守恒量

$$I_F=-a^4-t=\text{const}. \qquad (58)$$

对规范变换得到的系统(34),有

$$X^{(0)}(R'_1)=X^{(0)}(R'_2)=0,$$
$$X^{(0)}(R'_3)=-\xi_1, X^{(0)}(R'_4)=-\xi_2,$$
$$X^{(0)}(B')=X^{(0)}(B)=\xi_2+a^3\xi_3+a^4\xi_4, \qquad (59)$$

对无限小变换(56),计算表明对 $\mu=1,2,3,4$,(28)式均成立,换句话说,形式不变性判据方程(24)满足,系统(34)保持 Mei 对称性.

然而,计算表明(30)式不能成立,

$$X^{(0)}\{X^{(0)}(\dot{G})\}+X^{(0)}\left(\frac{\partial G}{\partial t}\right)\dot{\xi}_0$$
$$+X^{(0)}\left(\frac{\partial G}{\partial a^\mu}\right)\dot{\xi}_\mu=-2\neq 0, \qquad (60)$$

即将 R'_μ 和 B' 代入结构方程(25),但不改变规范函数 G_F 时,该方程不能满足.根据(31)式,引入新的规范函数

$$G'_F=t, \qquad (61)$$

则修正后的结构方程(25)成立.

$$[X^{(0)}(R'_\mu)\dot{a}^\mu-X^{(0)}(B')]\dot{\xi}_0+X^{(1)}$$
$$\times[X^{(0)}(R'_\mu)\dot{a}^\mu-X^{(0)}(B')]+\dot{G}'_F=0, \qquad (62)$$

由此得到新的新型守恒量

$$I'_F=X^{(0)}(R'_\mu)\xi_\mu-X^{(0)}(B')\xi_0+G'_F$$
$$=a^4+t=\text{const}. \qquad (63)$$

7. 结 论

变换理论是分析力学中重要的课题.本文涉及 Birkhoff 系统的两种变换 变量的无限小连续变换和动力学函数(Birkhoff 函数 B 和函数组 R_μ)的规范等效变换,讨论了后者对与前者相关联的三种对称性和守恒量的影响.结论如下:

1. Birkhoff 系统规范变换下,系统 Noether 对称性和守恒量保持不变,有些情况下,需要对规范函数 G_H 作适当调整.

2. Birkhoff 系统规范变换下,系统的 Lie 对称性及由此导出的 Hojman 守恒量保持不变.

3. Birkhoff 系统规范变换下,系统的 Mei 对称性和导出的新型守 恒量可能改变,但是如果 Birkhoff 规范变换函数满足相应的条件,那么可能出现 Mei 对称性和新型守恒量均不改变的情况,也可能出现 Mei 对称性不变而新型守恒量却发生改变的情况.

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Effects of gauge transformations on symmetries of Birkhoffian system

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Abstract

The effects of gauge transformation on Noether symmetry, Lie symmetry and Mei symmetry of a Birkhoff system are studied. Under certain conditions, the system can keep its Noether symmetry and conserved quantity. The Lie symmetry and Hojman conserved quantity of the system will still remain unchanged. The Mei symmetry and a new-type conserved quantity of the system may very, and the conditions under which the Mei symmetry and the new-type conserved quantity will remain are obtained. An example is given to illustrate the application of the results.

Keywords: Birkhoffian system, gauge transformation, symmetry, conserved quantity
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