第 58 卷 第 4 期 2009 年 4 月
1000-3290/2009/58(04)/2147-08

物 理 学 报
ACTA PHYSICA SINICA

Vol.58,No.4,April,2009

耦合相对转动非线性动力系统的稳定性与近似解*

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(2008 年 8 月 2 日收到;2008 年 9 月 3 日收到修改稿)

研究了一类含三次非线性耦合项的相对转动非线性动力系统的动力学行为. 建立了具有非线性弹性力、广义摩阻力耦合项的系统动力学方程. 运用多尺度法求解谐波激励下耦合非自治系统的近似解,通过讨论系统的主共振和内共振特性,分析了耦合项对系统响应的影响. 应用奇异性理论研究了主振稳态响应分岔方程的稳定性,得到了系统的转迁集和分岔曲线的拓扑结构.

关键词:相对转动, 非线性耦合动力系统, 奇异性理论, 稳定性

PACC:0340D, 0313, 0316

1. 引 言

转动运动是自然界中最普遍的运动之一,在研究转动运动的过程中,Carmeli 于 1985 年提出了转动相对论力学理论[1,2],Luo 于 1996 年建立了转动相对论分析力学理论[3,4],并构建了转动相对论系统分析力学的基本理论框架[5—7]. 随后转动相对论系统的研究受到学术界的广泛重视. 近年来,关于转动相对论系统动力学的研究,在转动相对论系统分析力学基本理论的进一步发展[8,9],在考虑具有质量分离或并入的转动相对论系统分析动力学的研究[10—12],以及转动相对论 Birkhoff 系统动力学的基本理论、对称性理论、积分的场理论、代数结构、几何理论、积分不变量及平衡稳定性等方面的研究取得了进展[13—25]. 基于相对性原理,建立了弹性转轴任意两横截面间的相对转动线性和非线性动力学方程并进行了定量分析[26—30],研究了可归结为一类周期变系数线性系统的非线性相对转动系统的稳定性[31],研究了非线性刚度相对转动系统的动力学特性[32—37]. 但是,以往的工作多限于双惯量扭转系统的动力学建模、求解及稳定性方面的研究,对于具有非线性耦合项的多惯量扭转系统的动力学研究甚少.

实际物理系统中,由于耦合作用普遍存在,研究具有非线性耦合项的相对转动动力系统的动态响应及稳定性具有更大的实际意义. 本文在文献[38,39]的基础上,基于具有耗散项的广义 Lagrange 方程,研究一类具有三次非线性耦合项的相对转动系统的动力学行为,建立了耦合系统的一般动力学方程. 运用多尺度法求解非自治系统的近似解,并对主振响应进行了奇异稳定性分析. 这对耦合动力传动系统的动力学行为分析与控制具有理论意义和应用价值.

2. 耦合相对转动非线性动力系统的动力学方程

在文献[38,39]的基础上考虑一类具有耦合项的非线性弹性力、广义摩阻力和外扰激励作用下的三惯量耦合系统,系统的动能为

$$
\begin{aligned}
E &= \sum_{i=1}^{3} \frac{1}{2} I_i \dot{\theta}_i^2 \\
&= \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_2 \dot{\theta}_2^2 + \frac{1}{2} I_3 \dot{\theta}_3^2 .
\end{aligned} \tag{1}
$$

考虑一次、三次扭转刚度作用下的系统势能为

$$
U = \frac{1}{2} K_{12} (\theta_1 - \theta_2)^2 + \frac{1}{4} K'_{12} (\theta_1 - \theta_2)^4
$$

* 国家"十五"重大科技攻关计划(批准号:ZZ02-13B-02-03-1)、河北省自然科学基金(批准号:2008000882)和河北省教育厅科学研究计划(批准号:2007496,ZH2007102)资助的课题.

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$$+\frac{1}{2}K_{23}(\theta_2-\theta_3)^2+\frac{1}{4}K'_{23}(\theta_2-\theta_3)^4. \quad (2)$$

广义力为

$$Q_j=\sum_{i=1}^{3}F_i^i\frac{\partial\theta_i}{\partial q_j}\quad (j=1,2,3), \quad (3)$$

其中 I_i $(i=1,2,3)$为系统集中质量的转动惯量，K_{12}，K_{23}为系统线性扭转刚度，K'_{12}，K'_{23}为系统非线性扭转刚度，θ_i $(i=1,2,3)$和 $\dot{\theta}_i$ $(i=1,2,3)$分别为系统集中质量的转角和转速. $F_i^i=F_i+F_i^c$，F_i 为广义外力，F_i^c 为系统广义阻尼力. q_j $(j=1,2,3)$为广义坐标.

考虑一端具有广义摩阻力情形，令

$$F_1^c=-\mu_{12}(\dot{\theta}_1-\dot{\theta}_2), \quad (4)$$

$$F_2^c=-\mu_{12}(\dot{\theta}_2-\dot{\theta}_1)-\mu_{23}(\dot{\theta}_3-\dot{\theta}_2), \quad (5)$$

$$F_3^c=-\mu_{23}(\dot{\theta}_2-\dot{\theta}_3)+f(\dot{\theta}_2-\dot{\theta}_3), \quad (6)$$

则

$$F_1^1=F_1-\mu_{12}(\dot{\theta}_1-\dot{\theta}_2), \quad (7)$$

$$F_2^2=F_2-\mu_{12}(\dot{\theta}_2-\dot{\theta}_1)-\mu_{23}(\dot{\theta}_3-\dot{\theta}_2), \quad (8)$$

$$F_3^3=F_3-\mu_{23}(\dot{\theta}_2-\dot{\theta}_3)+f(\dot{\theta}_2-\dot{\theta}_3), \quad (9)$$

其中 μ_{12}，μ_{23}为线性阻尼系数，$f(\dot{\theta}_2-\dot{\theta}_3)$为非线性摩阻力函数，代入具有耗散项的 Lagrange 方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial E}{\partial\dot{q}_j}-\frac{\partial E}{\partial q_j}+\frac{\partial U}{\partial q_j}=Q_j\quad (j=1,2,3). \quad (10)$$

得

$$I_1\ddot{\theta}_1+K_{12}(\theta_1-\theta_2)+K'_{12}(\theta_1-\theta_2)^3+\mu_{12}(\dot{\theta}_1-\dot{\theta}_2)=F_1, \quad (11)$$

$$I_2\ddot{\theta}_2-K_{12}(\theta_1-\theta_2)-K'_{12}(\theta_1-\theta_2)^3-\mu_{12}(\dot{\theta}_1-\dot{\theta}_2)+K_{23}(\theta_2-\theta_3)+K'_{23}(\theta_2-\theta_3)^3+\mu_{23}(\dot{\theta}_2-\dot{\theta}_3)=F_2, \quad (12)$$

$$I_3\ddot{\theta}_3-K_{23}(\theta_2-\theta_3)-K'_{23}(\theta_2-\theta_3)^3-\mu_{23}(\dot{\theta}_2-\dot{\theta}_3)=F_3+f(\dot{\theta}_2-\dot{\theta}_3), \quad (13)$$

式中 $\ddot{\theta}_i$ $(i=1,2,3)$为系统集中质量的角加速度. 考虑相对转角的变化，式(11)乘以 $1/I_1$ 减式(12)乘以 $1/I_2$ 和式(12)乘以 $1/I_2$ 减式(13)乘以 $1/I_3$ 得到

$$(\ddot{\theta}_1-\ddot{\theta}_2)+\frac{(I_1+I_2)}{I_1I_2}K_{12}(\theta_1-\theta_2)+\frac{(I_1+I_2)}{I_1I_2}K'_{12}(\theta_1-\theta_2)^3+\frac{(I_1+I_2)}{I_1I_2}\mu_{12}(\dot{\theta}_1-\dot{\theta}_2)-\frac{K_{23}}{I_2}(\theta_2-\theta_3)-\frac{K'_{23}}{I_2}(\theta_2-\theta_3)^3-\frac{\mu_{23}}{I_2}(\dot{\theta}_2-\dot{\theta}_3)=\frac{1}{I_1I_2}(I_2F_1-I_1F_2), \quad (14)$$

$$(\ddot{\theta}_2-\ddot{\theta}_3)+\frac{(I_2+I_3)}{I_2I_3}K_{23}(\theta_2-\theta_3)+\frac{(I_2+I_3)}{I_2I_3}K'_{23}(\theta_2-\theta_3)^3+\frac{(I_2+I_3)}{I_2I_3}\mu_{23}(\dot{\theta}_2-\dot{\theta}_3)-\frac{K_{12}}{I_2}(\theta_1-\theta_2)-\frac{K'_{12}}{I_2}(\theta_1-\theta_2)^3-\frac{\mu_{12}}{I_2}(\dot{\theta}_1-\dot{\theta}_2)=\frac{1}{I_2I_3}(I_3F_2-I_2F_3)-\frac{1}{I_3}f(\dot{\theta}_2-\dot{\theta}_3), \quad (15)$$

令

$$\varphi_1=\theta_1-\theta_2,$$
$$\dot{\varphi}_1=\dot{\theta}_1-\dot{\theta}_2,$$
$$\ddot{\varphi}_1=\ddot{\theta}_1-\ddot{\theta}_2,$$
$$\varphi_2=\theta_2-\theta_3,$$
$$\dot{\varphi}_2=\dot{\theta}_2-\dot{\theta}_3,$$
$$\ddot{\varphi}_2=\ddot{\theta}_2-\ddot{\theta}_3,$$
$$a_0=\frac{I_1+I_2}{I_1I_2},$$
$$b_0=\frac{I_2+I_3}{I_2I_3},$$
$$a_1=a_0K_{12},$$
$$a_2=b_0K_{23},$$
$$b_1=a_0K'_{12},$$
$$b_2=b_0K'_{23},$$
$$\mu_1=a_0\mu_{12},$$
$$\mu_2=b_0\mu_{23},$$
$$c_1=\frac{K_{23}}{I_2},$$
$$c_2=\frac{K_{12}}{I_2},$$
$$d_1=\frac{K'_{23}}{I_2},$$
$$d_2=\frac{K'_{12}}{I_2},$$
$$v_1=\frac{\mu_{23}}{I_2},$$

$$
\begin{aligned}
v_2 &= \frac{\mu_{12}}{I_2}, \\
T_1 &= \frac{1}{I_1 I_2}(I_2 F_1 - I_1 F_2), \\
T_2 &= \frac{1}{I_2 I_3}(I_3 F_2 - I_2 F_3), \\
f(\dot\theta_2 - \dot\theta_3) &= -e(\dot\theta_2 - \dot\theta_3) + f(\dot\theta_2 - \dot\theta_3)^3, \\
\beta &= \frac{f}{I_3}, \\
\alpha &= \frac{e}{I_3} - \mu_2,
\end{aligned} \tag{16}
$$

(14)和(15)式分别化为

$$
\ddot\varphi_1 + a_1\varphi_1 + b_1\varphi_1^3 + \mu_1\dot\varphi_1 - c_1\varphi_2 - d_1\varphi_2^3 - v_1\dot\varphi_2 = T_1, \tag{17}
$$

$$
\ddot\varphi_2 + a_2\varphi_2 + b_2\varphi_2^3 + \alpha\dot\varphi_2 - c_2\varphi_1 - d_2\varphi_1^3 - v_2\dot\varphi_1 + \beta\dot\varphi_2^3 = T_2. \tag{18}
$$

方程(17)和(18)为含三次耦合项的非线性弹性力、广义摩阻力和外扰激励作用下的耦合相对转动非线性动力系统的动力学方程，其中 $c_1, c_2, d_1, d_2, v_1, v_2$ 为耦合系数，表征了系统的耦合程度. 这是工程中描述可简化为该类耦合转动动力系统的动力学方程，是进一步分析该系统动力学行为的基础.

3. 耦合相对转动非线性动力系统的近似解

令 $T_1 = f_1\cos(\omega_1 t)$，$T_2 = f_2\cos(\omega_2 t)$，为求耦合系统的近似解，在 $b_i, \mu_i, c_i, d_i, v_i, \alpha, \beta$ 和 $f_i\ (i=1,2)$ 前冠以小参数 ε，应用多尺度法[40−42]求解耦合系统的近似解. 设方程(17)和方程(18)的解为

$$
\varphi_1 = \varphi_{10}(T_0, T_1) + \varepsilon\varphi_{11}(T_0, T_1) + \cdots, \tag{19}
$$

$$
\varphi_2 = \varphi_{20}(T_0, T_1) + \varepsilon\varphi_{21}(T_0, T_1) + \cdots, \tag{20}
$$

式中 $T_0 = t$，$T_1 = \varepsilon t$，分别为快、慢时间尺度. 将(19)和(20)式代入(17)和(18)式，比较方程两边 ε 同次幂的系数，可得下列方程组

$$
D_0^2\varphi_{10} + a_1\varphi_{11} = 0, \tag{21}
$$

$$
D_0^2\varphi_{20} + a_2\varphi_{21} = 0, \tag{22}
$$

$$
\begin{aligned}
D_0^2\varphi_{11} + a_1\varphi_{11} = {} & c_1\varphi_{20} + d_1\varphi_{20}^2 + v_1 D_0\varphi_{20} \\
& - 2D_0 D_1\varphi_{10} - \mu_1 D_0\varphi_{10} \\
& - b_1\varphi_{10}^3 + f_1\cos(\omega_1 T_0),
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
D_0^2\varphi_{21} + a_2\varphi_{21} = {} & c_2\varphi_{10} + d_2\varphi_{10}^2 + v_2 D_0\varphi_{10} \\
& - 2D_0 D_1\varphi_{20} - \alpha D_0\varphi_{20} - b_2\varphi_{20}^3 \\
& - \beta(D_0\varphi_{20})^3 + f_2\cos(\omega_2 T_0),
\end{aligned} \tag{24}
$$

式中 D_i 为偏微分算子，$D_i = \dfrac{\partial}{\partial T_i}$，$T_i = \varepsilon^n t\ (i = 0, 1, 2, \cdots, n)$.

设方程(21)和(22)的解为

$$
\varphi_{10} = A(T_1)\mathrm{e}^{\mathrm{i}\Omega_1 T_0} + \mathrm{c.c.}, \tag{25}
$$

$$
\varphi_{20} = B(T_1)\mathrm{e}^{\mathrm{i}\Omega_2 T_0} + \mathrm{c.c.}, \tag{26}
$$

式中 c.c.表示前面项的共轭复数，下同.

将(25)和(26)式代入方程(23)和方程(24)整理得

$$
\begin{aligned}
& D_0^2\varphi_{11} + a_1\varphi_{11} \\
& = (-2\mathrm{i}\Omega_1 A' - \mathrm{i}\mu_1\Omega_1 A - 3b_1 A^2\bar A)\mathrm{e}^{\mathrm{i}\Omega_1 T_0} \\
& \quad + (c_1 B + \mathrm{i}v_1\Omega_2 B + 3d_1 B^2\bar B)\mathrm{e}^{\mathrm{i}\Omega_2 T_0} \\
& \quad - b_1 A^3\mathrm{e}^{3\mathrm{i}\Omega_1 T_0} + d_1 B^3\mathrm{e}^{3\mathrm{i}\Omega_2 T_0} \\
& \quad + \frac{1}{2}f_1\mathrm{e}^{\mathrm{i}\omega_1 T_0} + \mathrm{c.c.},
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
& D_0^2\varphi_{21} + a_1\varphi_{21} \\
& = (-2\mathrm{i}\Omega_2 B' + \mathrm{i}\alpha\Omega_2 B - 3b_2 B^2\bar B - 3\mathrm{i}\beta\Omega_2^3 B^2\bar B)\mathrm{e}^{\mathrm{i}\Omega_2 T_0} \\
& \quad + (c_2 A + \mathrm{i}v_2\Omega_1 A + 3d_2 A^2\bar A)\mathrm{e}^{\mathrm{i}\Omega_1 T_0} \\
& \quad - b_2 B^3\mathrm{e}^{3\mathrm{i}\Omega_2 T_0} + d_2 A^3\mathrm{e}^{3\mathrm{i}\Omega_1 T_0} \\
& \quad + \frac{1}{2}f_2\mathrm{e}^{\mathrm{i}\omega_2 T_0} + \mathrm{c.c.}.
\end{aligned} \tag{28}
$$

情形1　主共振

考虑主共振情形. 令 Ω_1 远离 Ω_2，$\omega_1 = 0$，$f_1 = 0$，ω_2 与 Ω_2 的差别为 ε 的同阶小量，引入频率调制参数 σ，令 $\omega_2 = \omega = \Omega_2 + \varepsilon\sigma$，消除(27)和(28)式中的久期项得

$$
-2\mathrm{i}\Omega_1 A' - \mathrm{i}\mu_1\Omega_1 A - 3b_1 A^2\bar A = 0, \tag{29}
$$

$$
\begin{aligned}
& -2\mathrm{i}\Omega_2 B' + \mathrm{i}\alpha\Omega_2 B - 3b_2 B^2\bar B \\
& - 3\mathrm{i}\beta\Omega_2^3 B^2\bar B + \frac{1}{2}f_2\mathrm{e}^{\mathrm{i}\sigma T_1} = 0.
\end{aligned} \tag{30}
$$

引入 A，B 的极坐标形式：$A = \dfrac{1}{2}a(T_1)\mathrm{e}^{\mathrm{i}\varphi_1(T_1)}$，$B = \dfrac{1}{2}b(T_1)\mathrm{e}^{\mathrm{i}\varphi_2(T_1)}$，代入复方程(29)和(30)，并分解实、虚部得到

$$
\begin{aligned}
& \left(a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3\right)\cos\varphi_1 \\
& + \left(\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a\right)\sin\varphi_1 = 0,
\end{aligned} \tag{31}
$$

$$
\left(a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3\right)\sin\varphi_1
$$

$$-\left(\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a\right)\cos\varphi_1 = 0, \tag{32}$$

$$\left(b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3\right)\cos\varphi_2 + \left(\Omega_2 b' - \frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\sin\varphi_2 + \frac{1}{2}f_2\cos(\sigma T_1) = 0, \tag{33}$$

$$\left(b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3\right)\sin\varphi_2 - \left(\Omega_2 b' - \frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\cos\varphi_2 + \frac{1}{2}f_2\sin(\sigma T_1) = 0. \tag{34}$$

(31)和(32)式消去 φ_1,(33)和(34)式消去 φ_2,并令 $\theta = \sigma T_1 - \varphi_2$ 得

$$a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3 = 0, \tag{35}$$

$$\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a = 0, \tag{36}$$

$$b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3 + \frac{1}{2}f_2\cos\theta = 0, \tag{37}$$

$$\Omega_2 b' - \frac{1}{2}\alpha\Omega_1 b + \frac{3}{8}\beta\Omega_2^3 b^3 + \frac{1}{2}f_2\sin\theta = 0, \tag{38}$$

则系统首次近似解为

$$\varphi_1 = a\cos(\Omega_1 t + \varphi_1) + O(\varepsilon), \tag{39}$$

$$\varphi_2 = b\cos(\Omega_2 t + \varphi_2) + O(\varepsilon), \tag{40}$$

其中 $a, b, \varphi_1, \varphi_2$ 由方程(35)—(38)确定.

对于稳态响应有,$a' = b' = 0, \varphi_1' = 0, \theta' = 0$,从而 $a = 0, \varphi_2' = \sigma$. 消去方程(37)和(38)中的 θ 可以得到频响方程.

$$\frac{9}{16}(b_2^2 + \beta\Omega_2^6)b^6 + 3\Omega_2\left(\frac{1}{2}\alpha\beta\Omega_2^3 - b_2\sigma\right)b^4 + \Omega_2^2(4\sigma^2 + \alpha^2)b^2 - f_2^2 = 0, \tag{41}$$

方程(41)说明在主共振条件下,系统的一阶近似稳定解不受耦合项的影响. 图1给出了不同 f_2 时的系统的幅频响应曲线. 当 f_2 取较小值时,系统为单值曲线,随着 f_2 的增大,耦合系统的共振点向右偏移,并有跳跃现象.

情形2 内共振

假设此时频率满足关系:$\Omega_2 = \Omega_1 - \varepsilon\sigma_1$;$\omega_2 = \omega = \Omega_2 + \varepsilon\sigma$ 代入方程(27)和(28),并消去久期项得到:

$$-2\mathrm{i}\Omega_1 A' - \mathrm{i}\mu_1\Omega_1 A - 3b_1 A^2\bar{A} + (c_1 B + \mathrm{i}v_1\Omega_2 B + 3d_1 B^2\bar{B})\mathrm{e}^{-\mathrm{i}\sigma_1 T_1} = 0, \tag{42}$$

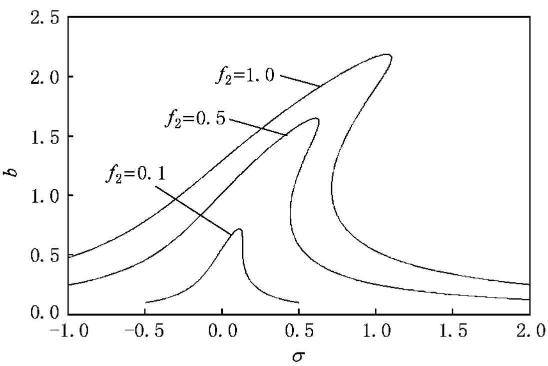

图1 耦合系统主共振幅频特性曲线 $\alpha = 0.1$, $\beta = 0.1$, $b_2 = 0.6$, $\mu = 0.15$, $\omega_2 = 1.0$

$$-2\mathrm{i}\Omega_2 B' + \mathrm{i}\alpha\Omega_2 B - 3b_2 B^2\bar{B} - 3\mathrm{i}\beta\Omega_2^3 B^2\bar{B} + (c_2 A + \mathrm{i}v_2\Omega_1 A + 3d_2 A^2\bar{A})\mathrm{e}^{\mathrm{i}\sigma_1 T_1} + \frac{1}{2}f_2\mathrm{e}^{\mathrm{i}\sigma T_1} = 0. \tag{43}$$

引入 A, B 的极坐标形式:$A = \frac{1}{2}a(T_1)\mathrm{e}^{\mathrm{i}\varphi_1(T_1)}$,$B = \frac{1}{2}b(T_1)\mathrm{e}^{\mathrm{i}\varphi_2(T_1)}$,代入复方程(42)和(43),并分解实、虚部得到

$$\left(a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3\right)\cos\varphi_1 + \left(\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a\right)\sin\varphi_1 + \left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\cos(\varphi_2 - \sigma_1 T_1) - \frac{1}{2}v_1\Omega_2 b\sin(\varphi_2 - \sigma_1 T_1) = 0, \tag{44}$$

$$\left(a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3\right)\sin\varphi_1 - \left(\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a\right)\cos\varphi_1 + \left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\sin(\varphi_2 - \sigma_1 T_1) + \frac{1}{2}v_1\Omega_2 b\cos(\varphi_2 - \sigma_1 T_1) = 0, \tag{45}$$

$$\left(b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3\right)\cos\varphi_2 + \left(\Omega_2 b' - \frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\sin\varphi_2 + \left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\cos(\varphi_1 + \sigma_1 T_1) - \frac{1}{2}v_2\Omega_1 a\sin(\varphi_1 + \sigma_1 T_1) + \frac{1}{2}f_2\cos(\sigma T_1) = 0, \tag{46}$$

$$\begin{aligned}&\left(b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3\right)\sin\varphi_2\\ &- \left(\Omega_2 b' - \frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\cos\varphi_2\\ &+ \left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\sin(\varphi_1 + \sigma_1 T_1)\\ &+ \frac{1}{2}v_2\Omega_1 a\cos(\varphi_1 + \sigma_1 T_1) + \frac{1}{2}f_2\sin(\sigma T_1) = 0.\end{aligned} \tag{47}$$

化简(44)和(45)式,(46)和(47)式,并令 $\theta_1 = \varphi_1 + \sigma_1 T_1 - \varphi_2$,$\theta = \sigma T_1 - \varphi_2$ 得

$$\begin{aligned}&a\Omega_1\varphi_1' - \frac{3}{8}b_1 a^3 + \left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\cos\theta_1\\ &+ \frac{1}{2}v_1\Omega_2 b\sin\theta = 0,\end{aligned} \tag{48}$$

$$\begin{aligned}&\Omega_1 a' + \frac{1}{2}\mu_1\Omega_1 a + \left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\sin\theta_1\\ &- \frac{1}{2}v_1\Omega_2 b\cos\theta = 0,\end{aligned} \tag{49}$$

$$\begin{aligned}&b\Omega_2\varphi_2' - \frac{3}{8}b_2 b^3 + \left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\cos\theta_1\\ &- \frac{1}{2}v_2\Omega_1 a\sin\theta_1 + \frac{1}{2}f_2\cos\theta = 0,\end{aligned} \tag{50}$$

$$\begin{aligned}&\Omega_2 b' - \frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\\ &- \left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\sin\theta_1 - \frac{1}{2}v_2\Omega_1 a\cos\theta_1\\ &- \frac{1}{2}f_2\sin\theta = 0.\end{aligned} \tag{51}$$

则系统首次近似解为

$$\varphi_1 = a\cos(\Omega_1 t + \varphi_1) + O(\varepsilon), \tag{52}$$

$$\varphi_2 = b\cos(\Omega_2 t + \varphi_2) + O(\varepsilon), \tag{53}$$

其中 a,b,φ_1,φ_2 由方程(48)—(51)确定.

对于稳态响应,有 $a' = b' = 0$,$\theta_1' = 0$,$\theta' = 0$,从而 $\varphi_1' = \sigma - \sigma_1$,$\varphi_2' = \sigma$. 可得耦合系统的频响方程为

$$\begin{aligned}&\left(\left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)^2 + \left(\frac{1}{2}v_1\Omega_2 b\right)^2\right)\\ &\left(\left(b\Omega_2\sigma - \frac{3}{8}b_2 b^3\right)^2 + \left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)^2\right.\\ &\left.+ \left(-\frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)^2 + \left(\frac{1}{2}v_2\Omega_1 a\right)^2\right)\\ &+ 2\left(\left(b\Omega_2\sigma - \frac{3}{8}b_2 b^3\right)\left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\right.\\ &\left.- \left(-\frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\left(\frac{1}{2}v_2\Omega_1 a\right)\right)\\ &\times\left(\left(\frac{1}{2}\mu_2\Omega_1 a\right)\left(\frac{1}{2}v_1\Omega_2 b\right)\right.\\ &\left.- \left((\sigma - \sigma_1)a\Omega_1 - \frac{3}{8}b_1 a^3\right)\left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\right)\\ &+ 2\left(\left(b\Omega_2\sigma - \frac{3}{8}b_2 b^3\right)\left(\frac{1}{2}v_2\Omega_1 a\right)\right.\\ &\left.+ \left(-\frac{1}{2}\alpha\Omega_2 b + \frac{3}{8}\beta\Omega_2^3 b^3\right)\left(\frac{1}{2}c_2 a + \frac{3}{8}d_2 a^3\right)\right)\\ &\left(\left((\sigma - \sigma_1)a\Omega_1 - \frac{3}{8}b_1 a^3\right)\left(\frac{1}{2}v_1\Omega_2 b\right)\right.\\ &\left.+ \left(\frac{1}{2}\mu_2\Omega_1 a\right)\left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)\right)\\ &- \frac{1}{4}\left(\left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)^2 + \left(\frac{1}{2}v_1\Omega_2 b\right)^2\right)f_2^2 = 0,\end{aligned} \tag{54}$$

$$\begin{aligned}&\left((\sigma - \sigma_1)a\Omega_1 - \frac{3}{8}b_1 a^3\right)^2 + J^2\\ &- \left(\frac{1}{2}c_1 b + \frac{3}{8}d_1 b^3\right)^2 - \left(\frac{1}{2}v_1\Omega_2 b\right)^2 = 0.\end{aligned} \tag{55}$$

方程(54)和(55)表明,1∶1 内共振条件下,系统的一阶近似稳定响应是耦合的. 图 2 和图 3 给出了耦合系统的响应曲线. 在一定参数范围内耦合项对系统响应影响较大,图中右侧的曲线弯曲较大是受到耦合项影响的结果.

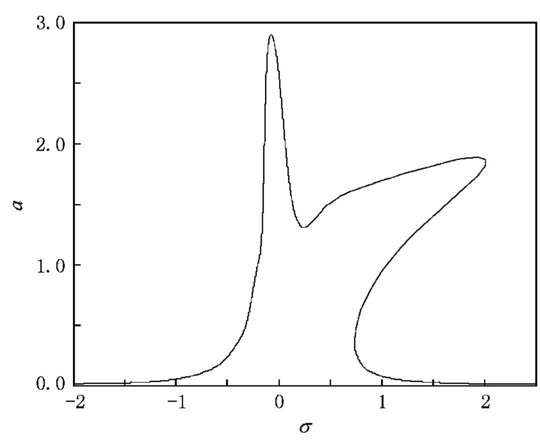

图 2　耦合系统内共振 σ-a 幅频特性曲线　$\alpha = 0.1$,$\beta = 0.1$,$b_2 = 0.6$,$\mu = 0.15$,$\omega_2 = 1.0$,$f_2 = 1.0$

4. 奇异性稳定性分析

(41)式变形得

$$\begin{aligned}&b^6 + \frac{8(\alpha\beta\Omega_2^3 - 2b_2\Omega_2\sigma)}{3(b_2^2 + \beta\Omega_2^6)}b^4\\ &+ \frac{16(4\Omega_2^2\sigma^2 + \Omega_2^2\alpha^2)}{9(b_2^2 + \beta\Omega_2^6)}b^2\\ &- \frac{16}{9(b_2^2 + \beta\Omega_2^6)}f_2^{\ 2} = 0,\end{aligned} \tag{56}$$

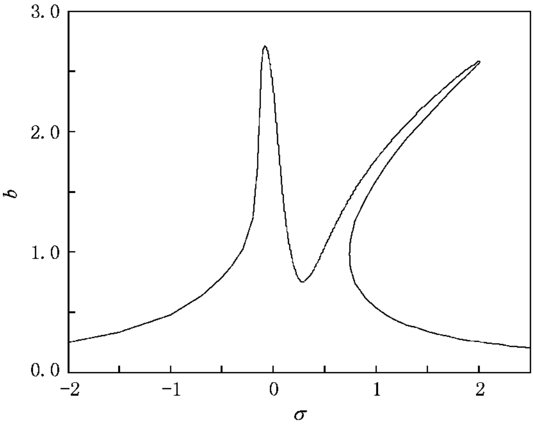

图 3 耦合系统内共振 σ-b 幅频特性曲线 $\alpha=0.1$，$\beta=0.1$，$b_2=0.6$，$\mu=0.15$，$\omega_2=1.0$，$f_2=1.0$

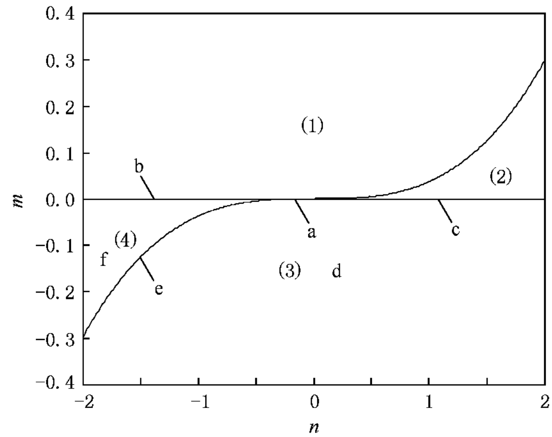

图 4 参数的转迁集

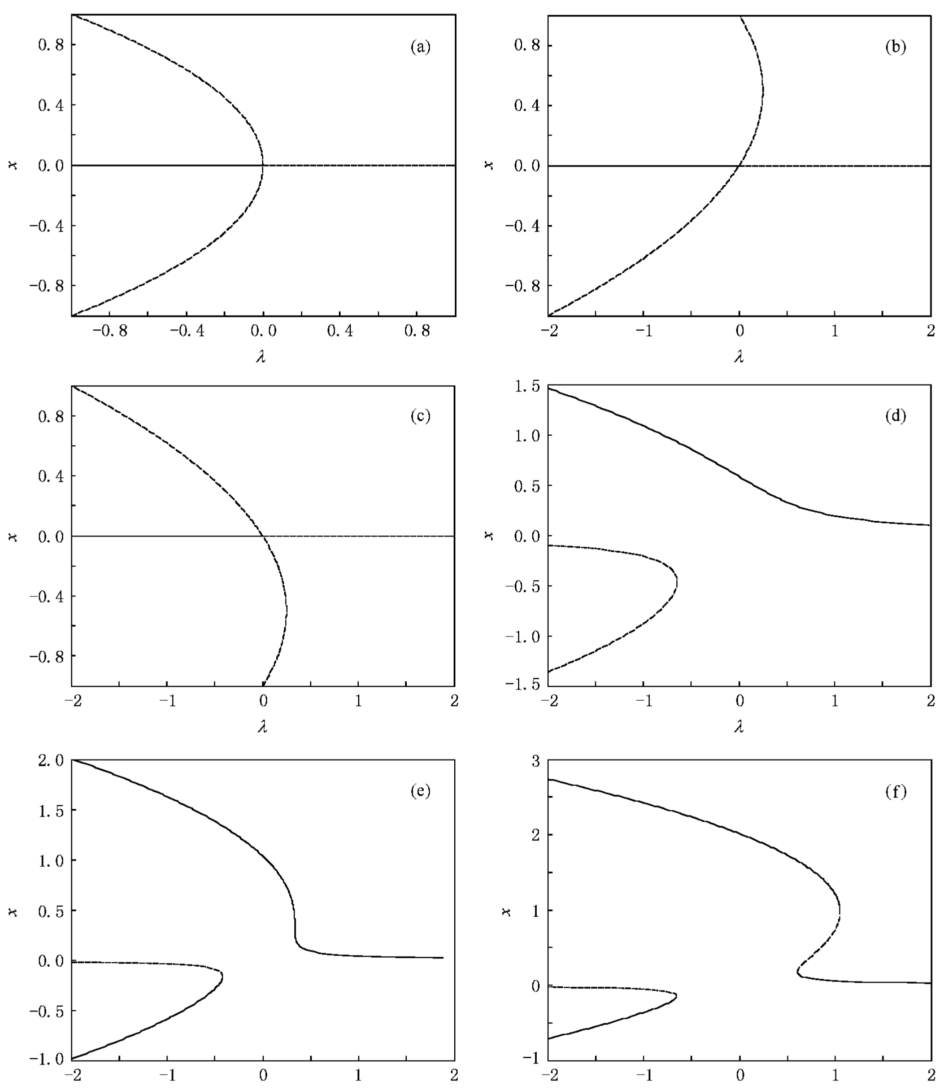

图 5 分岔曲线的拓扑结构

令

$$
\begin{aligned}
x &= b^2,\\
l &= b_2^2 + \beta^2 \Omega_2^6,\\
n &= \frac{8(\alpha\beta\Omega_2^3 - 2b_2\Omega_2\sigma)}{3(b_2^2 + \beta\Omega_2^6)},\\
\lambda &= \frac{16(4\Omega_2^2\sigma^2 + \Omega_2^2\alpha^2)}{9(b_2^2 + \beta\Omega_2^6)},\\
m &= -\frac{16f_2^2}{9l},
\end{aligned}
$$

则(56)式化为

$$
x^3 + \lambda x + m + nx^2 = 0, \qquad (57)
$$

利用奇异性理论研究方程(57)的拓扑结构．方程(57)等号左边为 GS 范式 $f = x^3 + \lambda x$ 的普适开折，且余维数为 2，根据转迁集的定义，可得转迁集如下：

(1)分岔点集：$B = \{m = 0\}$；

(2)滞后点集：$H = \{27m - n^3 = 0\}$；

(3)双极限点集：$D = \phi$；

(4)转迁集：$\Sigma = B \cup H \cup D$.

转迁集 Σ 将开折平面 m-n 平面划分为 4 个区域，如图 4 所示．根据奇异性理论，在不同区域中，解的拓扑结构不同，但在同一区域中，即便参数发生微小的变化，其分岔图也保持不变，这样的分岔图称为持久的分岔图，而转迁集上的分岔图在参数受到小扰动时，会改变其保持分岔结构，故称为非持久性的分岔图．由于 $m \leqslant 0$，故只讨论图中区域(3，4)部分．图 5 为每个区域的分岔曲线的拓扑结构，图中虚线部分为不稳定，在选择参数时应避开这一区域．

5．结　　论

应用具有耗散项的广义 Lagrange 方程建立了含三次耦合项的相对转动非线性动力系统的一般动力学方程．运用多尺度法求解了谐波激励下耦合非自治系统在主共振和内共振情形下的近似解，并分析了耦合项对系统响应的影响，在主共振情形下，存在类似于单自由度系统的跳跃现象，此时耦合项对系统无影响，在 1∶1 内共振情形下耦合项对系统响应影响较大．应用奇异性理论研究了主振稳态响应分岔方程的稳定性，给出了系统的转迁集上不同区域中分岔曲线的拓扑结构．这对工程中广泛存在的耦合动力传动系统的动力学行为分析与控制具有理论意义和应用价值．

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Stability and approximate solution of a relative-rotation nonlinear dynamical system with coupled terms*

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(Received 2 August 2008; revised manuscript received 3 September 2008)

Abstract

The dynamic behaviors of a relative-rotation nonlinear dynamic system with cubic coupled terms are studied. First, the dynamic equation of coupled system with nonlinear elastic force and generalized friction and harmonic excitation is deduced. The approximate solution of coupled unautonomous equation under harmonic excitation is obtained by the method of multiple scales. The effect of coupled terms on system resonance is analyzed in respect of principal resonance and internal resonance. The singularity stability of bifurcation function of principal resonance is studied by singularity theory, and the transfer concourse and topological structure of bifurcation function are obtained.

Keywords: relative rotation, nonlinear coupling dynamic system, singularity theory, stability

PACC: 0340D, 0313, 0316

* Project supported by the National Significant Science and Technology Program for the 10th "Five-Year" Plan of China (Grant No. ZZ02-13B-02-03-1), the Natural Science Foundation of Hebei Province, China (Grant No. 2008000882) and the Scientific Research Program of the Education Bureau of Hebei Province, China (Grant Nos. 2007496, ZH2007102).

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