第 59 卷 第 5 期 2010 年 5 月
1000-3290/2010/59(05)/3309-06
物 理 学 报
ACTA PHYSICA SINICA
Vol. 59, No. 5, May, 2010

层结流体中具有 β 效应与地形效应的强迫 Rossby 孤立波*

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(2009 年 7 月 21 日收到; 2009 年 8 月 21 日收到修改稿)

层结流体中,从绝热位涡的扰动方程出发采用摄动方法和时空伸长变换推导了具有 β 效应和地形效应的强迫 Rossby 孤立波方程,得到孤立 Rossby 波振幅的演变满足带有地形强迫的非齐次 mKdV 方程的结论. 通过分析孤立 Rossby 波振幅的演变,即使基本气流没有切变,仍可能激发出 Rossby 孤立波. 指出了科氏力效应、地形效应以及 Vaisala-Brunt 频率都是诱导 Rossby 孤立波产生的重要因素,说明了在地形强迫效应和非线性作用相平衡的假定下,Rossby 孤立波振幅的演变满足非齐次的 mKdV 方程. 讨论了地形和层结流体中 Rossby 波的相互作用.

关键词: 非齐次 mKdV 方程, β 效应, 地形, Vaisala-Brunt 频率
PACC: 4735, 0340K

1. 引 言

地球物理流体中,Rossby 波是指生命史很长结构上有组织的前后一致的大尺度永久性波动,并且这些波动具有稳定的、大振幅孤立波特征. 对于正压流体,Long[1] 在 1964 年作了开创性的研究,得到在 β 平面近似下 Rossby 波振幅演变满足 Korteweg-de Vries(KdV)方程, Benney[2] 在 1966 年推广了 Long 的结论,同时还得到 Rossby 孤立波波速与波振幅有关的结论,刻画了非线性的重要性. Larsen[3] 和 Clarker[4] 也研究了 Rossby 孤立波振幅的演变,他们得到了一系列与文献[1]类似的结果. Redekopp[5] 和 Wadati[6] 从正压流体和分层流体的模式推导了 Rossby 孤立波振幅演变的方程分别满足 KdV 方程和改进的 KdV(mKdV)方程的结论,极大地推广了文献[1]的结果. Redekopp[7] 研究了切变气流中 Rossby 孤立波的产生,指出在纬向流中 Rossby 孤立波存在的必要条件. Maslowe 等人[8] 讨论了在分层流体中纬向流的切变对 Rossby 波的影响. Charney 和 Straus[9] 基于准地转位涡度方程构造了一个 β 平面通道中考虑地形、非绝热加热和摩擦的正压大气模式,这项工作开创了大气多平衡态非线性动力学的研究[10]. Boyd[11,12] 用多重尺度方法,从基本方程导出在正压流体中小振幅 Rossby 孤立波振幅演变满足非线性 KdV 方程和 mKdV 方程. 刘式适和谭本馗[13] 研究了 Rossby 参数随纬度的变化,罗德海[14,15] 用推广的 β 平面近似模式研究了 Rossby 孤立波和 β 随纬度变化的关系,得到 β 随纬度变化可能是偶极子阻塞的原因. 赵强[16] 讨论了地形对热带大气超长尺度 Rossby 波的影响,指出了地形随纬度的变化能够导致热带大气超长尺度 Rossby 波波动的不稳定. 吕克利和蒋后硕[17] 利用扰动展开和时空伸长变换导出了包括地形强迫的非齐次 KdV 方程,讨论了近共振地形强迫 Rossby 孤立波的产生,显示出地形对扰动具有明显的增幅作用. 达朝究和丑纪范[18] 研究了地形随时间缓变时 Rossby 波振幅的演变问题. 宋健和杨联贵等人[19—21] 在正压流体与层结流体中分别给出 β 效应与地形效应对 Rossby 孤立波振幅的影响,张亮[22] 给出了正压 Rossby 波扰动能量问题,汪萍和戴新刚[23] 讨论了外强迫作用下正压大气非线性特征数值模拟,说明了大气大尺度非线性运动的某些特征. 孤立波解在非线性问题中占有重要地位,给出了许多求孤立波解

* 内蒙古教育厅基金(批准号:NJZY08005, NJ09066),内蒙古自然科学基金(批准号:2009ZD01)和内蒙古工业大学科学研究项目(批准号:X200933)资助的课题.

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的方法[24—28],如 Hirota's 双线性方法[29]、椭圆函数展开法[30,31]等被广泛应用. 在本文中,研究了 β 平面近似与非线性地形变化及其 Vaisala-Brunt 频率对 Rossby 孤立波振幅的演变.

2. 方程的推导

2.1. 控制方程与边界条件

由绝热位涡方程可得到准地转无量纲扰动位涡方程[17,32,33]

$$\frac{\partial q'}{\partial t}+\bar{u}\frac{\partial q'}{\partial x}+v'\frac{\partial \bar{q}}{\partial y}+\varepsilon\left(u'\frac{\partial q'}{\partial x}+v'\frac{\partial q'}{\partial y}\right)=0, \quad (1)$$

方程(1)中, $\varepsilon\ll 1$, 是度量非线性程度的强弱, 带横线的量为基本量,带撇号的量为扰动量,它们有下述关系:

$$\bar{u}=-\frac{\partial\bar{\psi}}{\partial y},\ u'=\frac{\partial\psi'}{\partial y},\ v'=\frac{\partial\psi'}{\partial x},$$

$$\bar{q}=\frac{\partial^2\bar{\psi}}{\partial y^2}+f+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi'}{\partial z}\right),$$

$$\rho=\rho(z),\ N^2=N^2(z),$$

这里, ρ 是密度, $N(z)$ 是 Vaisala-Brunt 频率,它是度量层结稳定的物理量, f 是科氏力.

β 平面近似取为 $f=f_0+\beta(y)y$[19—21], f_0 是局地科氏参数. 方程(1)可以改写为

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)+\left[\nabla^2\psi+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi}{\partial z}\right)\right]$$
$$+\left[(\beta(y)y)'-\frac{\partial^2\bar{u}}{\partial y^2}-\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\bar{u}}{\partial z}\right)\right]\frac{\partial\psi}{\partial x}$$
$$+\varepsilon J[\psi,\nabla^2\psi]+\varepsilon J\left[\psi,\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi}{\partial z}\right)\right]=0, \quad (2)$$

式中扰动量的撇号已略去, $(\beta(y)y)'$ 是纬度变量 y 的一阶导数,

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},$$

$$J[A,B]=\frac{\partial A}{\partial x}\frac{\partial B}{\partial y}-\frac{\partial A}{\partial y}\frac{\partial B}{\partial x}$$

为 Jacobi 算子.

侧边界条件为刚壁条件的无量纲形式,取

$$\frac{\partial\psi}{\partial x}=0,\qquad y=0,1, \quad (3)$$

垂直方向的无量纲边界条件,有

$$\rho\psi\to 0,\qquad z\to\infty. \quad (4)$$

在下边界上,考虑地形存在,无量纲下边界条件为

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z}-\frac{\partial\bar{u}}{\partial z}\frac{\partial\psi}{\partial x}+\frac{N^2}{f}\bar{u}\frac{\partial h}{\partial x}$$
$$+\varepsilon\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\frac{\partial\psi}{\partial z}$$
$$+\frac{N^2}{f}\varepsilon J[\psi,h]=0,\ z=0, \quad (5)$$

式中 $h=h(x,y)$ 为无量纲形式的地廓线.

2.2. 非齐次 mKdV 方程

为使方程(2)式中的非线性效应与频散项相平衡,为此对 x,t 引进缓变坐标[19,20,21]

$$X=\varepsilon x,\qquad T=\varepsilon^3 t, \quad (6)$$

将(6)式代入方程(2)—(5),化简后得

$$\left(\varepsilon^3\frac{\partial}{\partial T}+\varepsilon\bar{u}\frac{\partial}{\partial X}\right)$$
$$+\left[\varepsilon^2\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi}{\partial z}\right)\right]$$
$$+\varepsilon\left[(\beta(y)y)'-\frac{\partial^2\bar{u}}{\partial y^2}-\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\bar{u}}{\partial z}\right)\right]\frac{\partial\psi}{\partial X}$$
$$+\varepsilon^4 J\left[\psi,\frac{\partial^2\psi}{\partial X^2}\right]+\varepsilon^2 J\left[\psi,\frac{\partial^2\psi}{\partial y^2}\right.$$
$$\left.+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi}{\partial z}\right)\right]=0, \quad (7)$$

$$\frac{\partial\psi}{\partial X}=0,\quad y=0,1, \quad (8)$$

$$\rho\psi\to 0,\quad z\to\infty, \quad (9)$$

$$\left(\varepsilon^3\frac{\partial}{\partial T}+\varepsilon\bar{u}\frac{\partial}{\partial X}\right)\frac{\partial\psi}{\partial z}-\varepsilon\frac{\partial\bar{u}}{\partial z}\frac{\partial\psi}{\partial X}$$
$$+\varepsilon\frac{N^2}{f}\bar{u}\frac{\partial h}{\partial X}+\varepsilon^2 J\left[\psi,\frac{\partial\psi}{\partial z}\right]$$
$$+\frac{N^2}{f}\varepsilon J[\psi,h]=0,z=0. \quad (10)$$

假设基本气流有

$$\bar{u}=U(y,z)+\varepsilon^2\alpha, \quad (11)$$

其中 $U(y,z)$ 是纬向流, α 为常值,其量级为 1,它是为考虑线性情况下系统是非共振的,称为失谐参数[33]. 为了使地形强迫效应与非线性作用相平衡[33],可设

$$h=\varepsilon^2\Omega(X,y), \quad (12)$$

式中 $\Omega=O(1)$.

将(11),(12)式代入方程(7)—(10)得到

$$\varepsilon L_0(\psi)+\varepsilon^3\alpha L_1(\psi)+\varepsilon^3 L_2(\psi)$$
$$+\varepsilon^2 J\left[\psi,\frac{\partial^2\psi}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi}{\partial z}\right)\right]$$

$$+\varepsilon^4\alpha\frac{\partial^3\psi}{\partial X^3}+\varepsilon^4 J\left[\psi,\frac{\partial^2\psi}{\partial X^2}\right]+\varepsilon^5\frac{\partial}{\partial T}\frac{\partial^2\psi}{\partial X^2}=0,\tag{13}$$

$$\frac{\partial\psi}{\partial X}=0,\quad y=0,1,\tag{14}$$

$$\rho\psi\to 0,\quad z\to\infty,\tag{15}$$

$$\varepsilon^3\frac{\partial}{\partial T}\frac{\partial\psi}{\partial z}+\varepsilon U\frac{\partial}{\partial X}\frac{\partial\psi}{\partial z}+\varepsilon^3\alpha\frac{\partial}{\partial X}\frac{\partial\psi}{\partial z}-\varepsilon\frac{\partial U}{\partial z}\frac{\partial\psi}{\partial X}+\varepsilon^3\frac{N^2}{f}U\frac{\partial\Omega}{\partial X}+\varepsilon J\left[\psi,\frac{\partial\psi}{\partial z}\right]+\varepsilon^4\frac{N^2}{f}J[\psi,\Omega]=0,\quad z=0,\tag{16}$$

上述方程中已没有快变量 x 和 t，只含慢变量 X,T. 其中三个算子 L_0,L_1,L_2 分别为

$$L_0=\left[U\left(\frac{\partial^2}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial}{\partial z}\right)+P(y,z)\right]\frac{\partial}{\partial X},\tag{17}$$

$$L_1=\left[\frac{\partial^2}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial}{\partial z}\right)\right]\frac{\partial}{\partial X},\tag{18}$$

$$L_2=\frac{\partial}{\partial T}\left[\frac{\partial^2}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial}{\partial z}\right)\right]+U\frac{\partial^3}{\partial X^3},\tag{19}$$

(17)式中

$$P(y,z)=(\beta(y)y)'-\frac{\partial^2 U}{\partial y^2}-\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial U}{\partial z}\right).$$

假设扰动流函数有如下的小参数展开形式[34]：

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\cdots,\tag{20}$$

将(20)式代入方程(13)—(16)，得到各阶摄动问题的方程与边界条件.

对于 $O(\varepsilon^1)$ 阶，有

$$L_0(\psi_0)=0,\tag{21}$$

$$\frac{\partial\psi_0}{\partial X}=0,\quad y=0,1,\tag{22}$$

$$\rho\psi_0\to 0,\quad z\to\infty,\tag{23}$$

$$U\frac{\partial}{\partial z}\frac{\partial\psi_0}{\partial X}-\frac{\partial U}{\partial z}\frac{\partial\psi_0}{\partial X}=0,\quad z=0.\tag{24}$$

假设 ψ_0 有如下形式的分离变量解：

$$\psi_0=A(X,T)\Phi_0(y,z),\tag{25}$$

将(25)式代入方程(21)—(24)得

$$\frac{\partial^2\Phi_0}{\partial y^2}+\frac{P(y,z)}{U}\Phi_0+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\Phi_0}{\partial z}\right)=0,\tag{26}$$

$$\Phi_0(y,z)=0,\quad y=0,1,\tag{27}$$

$$\rho\Phi_0\to 0,\quad z\to\infty,\tag{28}$$

$$\frac{\partial\Phi_0}{\partial z}-\frac{1}{U}\frac{\partial U}{\partial z}\Phi_0=0,\quad z=0.\tag{29}$$

在方程(26)—(29)，对于确定的 $P(y,z)$，$\Phi_0(y,z)$ 就能被确定. 由于 $P(y,z)$ 是关于变量 y,z 的非线性函数，方程(26)—(29)很难获得解析解. 另外，在本阶问题中，只能确定波的空间结构，而不能确定波振幅随时间的演变. 为了确定波振幅 $A(X,T)$ 的演变，继续求解高阶问题.

对于 $O(\varepsilon^2)$ 阶，有

$$L_0(\psi_1)=-J\left[\psi_0,\frac{\partial^2\psi_0}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi_0}{\partial z}\right)\right]\equiv F_1,\tag{30}$$

$$\frac{\partial\psi_1}{\partial X}=0,\quad y=0,1,\tag{31}$$

$$\rho\psi_1\to 0,\quad z\to\infty,\tag{32}$$

$$U\frac{\partial}{\partial z}\frac{\partial\psi_1}{\partial X}-\frac{\partial U}{\partial z}\frac{\partial\psi_1}{\partial X}=-J\left[\psi_0,\frac{\partial\psi_0}{\partial z}\right],\quad z=0,\tag{33}$$

(30)式中 $F_1=A\frac{\partial A}{\partial X}\left(\frac{P(y,z)}{U}\right)_y\Phi_0^2$，$\left(\frac{P(y,z)}{U}\right)_y$ 是 $\frac{P(y,z)}{U}$ 关于变量 y 的一阶偏导数.

我们通过分析可以得到

$$\psi_1=\frac{1}{2}A^2(X,T)\Phi_1(y,z),\tag{34}$$

将(34)式代入方程(30)—(33)得

$$\frac{\partial^2\Phi_1}{\partial y^2}+\frac{P(y,z)}{U}\Phi_1+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\Phi_1}{\partial z}\right)=\left(\frac{P(y,z)}{U}\right)_y\frac{\Phi_0^2}{U},\tag{35}$$

$$\Phi_1(y,z)=0,\quad y=0,1,\tag{36}$$

$$\rho\Phi_1\to 0,\quad z\to\infty,\tag{37}$$

$$\frac{\partial\Phi_1}{\partial z}-\frac{1}{U}\frac{\partial U}{\partial z}\Phi_1=-\frac{1}{U}\left(\Phi_0\frac{\partial^2\Phi_0}{\partial y\partial z}-\frac{\partial\Phi_0}{\partial y}\frac{\partial\Phi_0}{\partial z}\right),\quad z=0.\tag{38}$$

为了得到 Rossby 振幅的数学模型，我们继续求解更高阶的问题.

对于 $O(\varepsilon^3)$ 阶，有

$$L_0(\psi_2)=-\alpha L_1(\psi_0)-L_2(\psi_0)-J\left[\psi_0,\frac{\partial^2\psi_1}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi_1}{\partial z}\right)\right]-J\left[\psi_1,\frac{\partial^2\psi_0}{\partial y^2}+\frac{f^2}{\rho}\frac{\partial}{\partial z}\left(\frac{\rho}{N^2}\frac{\partial\psi_0}{\partial z}\right)\right]\equiv F_2,\tag{39}$$

$$\frac{\partial\psi_2}{\partial X}=0,\quad y=0,1,\tag{40}$$

$$\rho\psi_2 \to 0, \quad z \to \infty, \tag{41}$$

$$\frac{\partial}{\partial z}\frac{\partial \psi_2}{\partial X} - \frac{1}{U}\frac{\partial U}{\partial z}\frac{\partial \psi_2}{\partial X}$$

$$= -\left(\frac{\partial A}{\partial T} + \alpha\frac{\partial A}{\partial X}\right)\frac{\partial \Phi_0}{\partial z} - \frac{N^2}{f}\frac{\partial \Omega}{\partial X}$$

$$- A^2\frac{\partial A}{\partial X}\left(\frac{\Phi_0}{2U}\frac{\partial^2 \Phi_1}{\partial y\partial z} - \frac{1}{U}\frac{\partial \Phi_0}{\partial y}\frac{\partial \Phi_1}{\partial z}\right.$$

$$\left.+ \frac{\Phi_1}{U}\frac{\partial^2 \Phi_0}{\partial y\partial z} - \frac{1}{2U}\frac{\partial \Phi_1}{\partial y}\frac{\partial \Phi_0}{\partial z}\right). \quad z = 0, \tag{42}$$

(39)式中

$$F_2 = \left(\frac{\partial A}{\partial T} + \alpha\frac{\partial A}{\partial X}\right)\frac{P(y,z)\Phi_0}{U}$$

$$+ A^2\frac{\partial A}{\partial X}\left\{\frac{3\Phi_0\Phi_1}{2U}\left(\frac{P(y,z)}{U}\right)_y\right.$$

$$\left.- \frac{\Phi_0^3}{2}\left(\frac{1}{U}\left(\frac{P(y,z)}{U}\right)_y\right)_y\right\} - U\Phi_0\frac{\partial^3 A}{\partial X^3},$$

这里 $\left(\frac{1}{U}\left(\frac{P(y,z)}{U}\right)_y\right)_y$ 是 $\frac{1}{U}\left(\frac{P(y,z)}{U}\right)_y$ 对变量 y 的导数. 由(39)式可知,在本阶出现了弱非线性效应与弱频散效应. 用 $\frac{\rho\Phi_0}{U}$ 乘方程(39)的两端,并在变量 $y(0 \leqslant y \leqslant 1)$ 和 $z(0 \leqslant z \leqslant \infty)$ 积分得

$$\int_0^1\int_0^\infty \frac{\rho\Phi_0}{U}L_0(\psi_2)\,\mathrm{d}y\mathrm{d}z = \int_0^1\int_0^\infty \frac{\rho\Phi_0}{U}F_2\,\mathrm{d}y\mathrm{d}z. \tag{43}$$

这表明,若摄动问题(20)式有效, F_2 必须满足方程(43),否则将出现无穷大振幅的奇异效应,即共振现象. 将 F_2 与(36),(40)和(42)式代入方程(43),有

$$\left\{\int_0^1\int_0^\infty \frac{\rho P(y,z)}{U}\Phi_0^2\,\mathrm{d}y\mathrm{d}z\right.$$

$$\left.- \int_0^1 f^2\left[\frac{\rho}{N^2}\Phi_0\frac{\partial \Phi_0}{\partial z}\right]\Big|_{z=0}\mathrm{d}y\right\}\left(\frac{\partial A}{\partial T} + \alpha\frac{\partial A}{\partial X}\right)$$

$$+ \left\{\int_0^1\int_0^\infty \left[\frac{\rho\Phi_0^2\Phi_1}{U}\left(\frac{P(y,z)}{U}\right)_y\right.\right.$$

$$\left.- \frac{\rho\Phi_0^4}{2U}\left(\frac{1}{U}\left(\frac{P(y,z)}{U}\right)_y\right)_y\right]\mathrm{d}y\mathrm{d}z$$

$$- \int_0^1 f^2\left[\frac{\rho}{N^2}\Phi_0\left(\frac{\Phi_0}{2U}\frac{\partial^2 \Phi_1}{\partial y\partial z} - \frac{1}{U}\frac{\partial \Phi_0}{\partial y}\frac{\partial \Phi_1}{\partial z}\right.\right.$$

$$\left.\left.\left.+ \frac{\Phi_1}{U}\frac{\partial^2 \Phi_0}{\partial y\partial z} - \frac{1}{2U}\frac{\partial \Phi_1}{\partial y}\frac{\partial \Phi_0}{\partial z}\right)\right]\Big|_{z=0}\mathrm{d}y\right\}A^2\frac{\partial A}{\partial X}$$

$$- \int_0^1\int_0^\infty \rho\Phi_0^2\,\mathrm{d}y\mathrm{d}z\,\frac{\partial^3 A}{\partial X^3} = \frac{\partial G}{\partial X}, \tag{44}$$

方程(44)中 $G = \int_0^1 f^2\left[\frac{\rho}{f}\Phi_0\Omega\right]\Big|_{z=0}\mathrm{d}y.$

若记

$$\sigma = \left\{\int_0^1\int_0^\infty \frac{\rho P(y,z)}{U}\Phi_0^2\,\mathrm{d}y\mathrm{d}z - \int_0^1 f^2\left[\frac{\rho}{N^2}\Phi_0\frac{\partial \Phi_0}{\partial z}\right]\Big|_{z=0}\mathrm{d}y\right\}^{-1},$$

$$\gamma = \sigma\left\{\int_0^1\int_0^\infty \left[\frac{\rho\Phi_0^2\Phi_1}{U}\left(\frac{P(y,z)}{U}\right)_y - \frac{\rho\Phi_0^4}{2U}\left(\frac{1}{U}\left(\frac{P(y,z)}{U}\right)_y\right)_y\right]\mathrm{d}y\mathrm{d}z\right.$$

$$- \int_0^1 f^2\left[\frac{\rho}{N^2}\Phi_0\left(\frac{\Phi_0}{2U}\frac{\partial^2 \Phi_1}{\partial y\partial z} - \frac{1}{U}\frac{\partial \Phi_0}{\partial y}\frac{\partial \Phi_1}{\partial z}\right.\right.$$

$$\left.\left.\left.+ \frac{\Phi_1}{U}\frac{\partial^2 \Phi_0}{\partial y\partial z} - \frac{1}{2U}\frac{\partial \Phi_1}{\partial y}\frac{\partial \Phi_0}{\partial z}\right)\right]\Big|_{z=0}\mathrm{d}y\right\},$$

$$\lambda = -\sigma\int_0^1\int_0^\infty \rho\Phi_0^2\,\mathrm{d}y\mathrm{d}z,$$

$$F = \sigma G,$$

式中的 $\Phi_0(y,z)$, $\Phi_1(y,z)$ 分别由方程(26)—(29)和方程(35)—(38)确定. 这样,方程(44)可简写为

$$\frac{\partial A}{\partial T} + \alpha\frac{\partial A}{\partial X} + \gamma A^2\frac{\partial A}{\partial X} + \lambda\frac{\partial^3 A}{\partial X^3} = \frac{\partial F}{\partial X}. \tag{45}$$

方程(45)说明层结流体中 Rossby 孤立波振幅的演变满足非齐次 mKdV 方程,系数 γ, λ 依赖于函数 $\beta(y)$, $U(y,z)$ 和 $N(z)$. 系数 F 与地形 $\Omega(X,y)$ 有关,如果 $\Omega(X,y)$ 为常数,即不存在地形效应,此时方程(45)是齐次 mKdV 方程. $\beta(y)$, 地形 $\Omega(X,y)$ 和 Vaisala-Brunt 频率能够诱导 Rossby 孤立波.

3. 结 论

在分层流体中,应用摄动法导出了 Rossby 孤立波振幅的演变满足非齐次 mKdV 方程,只要地形存在,即使基本气流没有切变,仍可能诱导出 Rossby 孤立波. 非线性 β 效应,地形效应以及 Vaisala-Brunt 频率 显然都是 Rossby 孤立波产生的重要因子.

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Force solitary Rossby waves with beta effect and topography effect in stratified flows*

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(Received 21 July 2009; revised manuscript received 21 August 2009)

Abstract

For the stratified fluids, based on the quasi-geostrophic potential vorticity equation, an inhomogeneous modified Korteweg-de Vried (mKdV) equation including topographic forcing is derived by employing the perturbation method and stretching transforms of time and space. With inspection of the evolution of the amplitude of Rossby waves, it is found that Coridis effect, topography effect and Vaisala-Brunt frequency are the important factors, that induce the solitary Rossby wave, and it is induced even though the basic stream function has not a shear. Assuming that there is a balance between nonlinear and topographic effects, an inhomogeneous mKdV equation is derived, the results show that the topography and Rossby waves interact in the stratified flows. The inhomogeneous mKdV equation describing the evolution of the amplitude of solitary Rossby waves as a function of the change of Rossby parameter $\beta(y)$ with latitude y, topographic forcing and the Vaisala-Brunt frequency is obtained.

Keywords: inhomogeneous mKdV equation, β effect, topographic, Vaisala-Brunt frequency

PACC: 4735, 0340K

* Project supported by the Education Department of Inner Mongolia (Grnat Nos. NJZY08005, NJ09066), the Natural Science Foundation of Inner Mongolia (Grant No. 2009ZD01) and the Sciences of Inner Mongolia University of Technology (Grnat No. X200933).

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