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磁偶极和电四极在磁各向异性介质中的辐射功率*

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在经典电动力学的框架下,研究了磁各向异性介质中的电磁辐射问题,得到了磁偶极和电四极在磁各向异性介质中的辐射功率表达式. 进一步地,通过把各向同性介质中的 μ_{rii} 代入所得辐射功率表达式,得到了与文献相符合的结果,验证了所得结果的正确性. 研究结果表明磁偶极和电四极在磁各向异性介质中的辐射功率大小与磁各向异性介质的 μ_{rii} 大小有关,对判断磁偶极和电四极在磁各向异性介质中的辐射效果有较大的帮助.

关键词: 磁各向异性介质, 磁偶极, 电四极, 辐射功率
PACC: 0350, 4100, 4110H, 4190

1. 引 言

近些年来,随着人们对各向异性介质的电磁散射研究的深入以及各向异性介质材料应用的推广,各向异性介质电磁特性的理论与实验研究成为热点[1—12]. 尽管对电磁波在各向异性介质中的传播问题已有较成熟和深刻的认识[13—15],但是,在宏观电动力学范畴内对磁各向异性介质中的电磁辐射问题的研究还不多.

研究各向异性介质电磁特性的方法很多,主要有有限差分法[1—5]、有限元法[16]、积分方程法[17]等. 我们则是引入各向异性直角坐标系,把各向异性介质的介电系数和磁导率张量写成并矢,推导出电各向同性磁各向异性介质中满足 Maxwell 方程组的推迟磁矢势 $\boldsymbol{A}$ 的表达式[18],并由此得到了电偶极、磁偶极和电四极在电各向同性磁各向异性介质中辐射的电场强度 $\boldsymbol{E}$、磁感应强度 $\boldsymbol{B}$ 和能流密度 $\boldsymbol{S}$ 的表达式[19—22],本文在此基础上进一步对磁偶极和电四极在磁各向异性介质中的辐射功率表达式进行推导和验证.

2. 磁偶极的辐射功率

2.1. 磁偶极辐射的能流密度

磁偶极在磁各向异性介质中辐射的能流密度公式用各向异性直角坐标系可表示为[21]

$$\begin{aligned}
\boldsymbol{S} = {} & \frac{|\ddot{\boldsymbol{m}}_y|^2}{32\pi^2\varepsilon\sqrt{\varepsilon_r}c^5y^7}\Bigg[\frac{\mu_{r22}}{\mu_{r11}\sqrt{\mu_{r33}}}y_3^2(y_1^2+y_2^2) \\
& +\frac{\mu_{r11}^2\mu_{r22}}{\mu_{r33}^3\sqrt{\mu_{r11}}}(y_1^2+y_2^2)^2 \\
& -\left(\frac{\mu_{r11}^2\mu_{r33}}{\mu_{r22}^3\sqrt{\mu_{r11}}}-\frac{\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right)y_2^2y_3^2\Bigg]y_1\boldsymbol{e}_1 \\
& +\Bigg[\frac{\mu_{r11}}{\mu_{r22}\sqrt{\mu_{r33}}}(y_1^2+y_2^2)y_3^2 \\
& +\frac{\mu_{r11}\mu_{r22}^2}{\mu_{r33}^3\sqrt{\mu_{r22}}}(y_1^2+y_2^2)^2 \\
& -\left(\frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r11}}}-\frac{\mu_{r22}^2\mu_{r33}}{\mu_{r11}^3\sqrt{\mu_{r22}}}\right)y_1^2y_3^2\Bigg]y_2\boldsymbol{e}_2 \\
& +\Bigg[\frac{\mu_{r11}\mu_{r33}^2}{\mu_{r22}^3\sqrt{\mu_{r33}}}y_2^2y_3^2+\frac{\mu_{r22}\mu_{r33}^2}{\mu_{r11}^3\sqrt{\mu_{r33}}}y_1^2y_3^2 \\
& +\Bigg(\frac{\mu_{r11}}{\mu_{r33}\sqrt{\mu_{r22}}}(y_1^2+y_2^2)y_2^2 \\
& +\frac{\mu_{r22}}{\mu_{r33}\sqrt{\mu_{r11}}}(y_1^2+y_2^2)y_1^2\Bigg]y_3\boldsymbol{e}_3,
\end{aligned} \tag{1}$$

式中 $\ddot{\boldsymbol{m}}_y$ 代表在 $y_1y_2y_3$ 各向异性直角坐标系中沿极轴振荡的磁偶极矩对时间的二次导数,且有

$$\begin{aligned}
|\ddot{\boldsymbol{m}}_y|^2 = {} & \Big|\sqrt{\varepsilon_r^3\mu_{r11}\mu_{r22}\mu_{r33}}\sqrt{\varepsilon_r}(\sqrt{\mu_{r11}}\boldsymbol{e}_1\boldsymbol{e}_1 \\
& +\sqrt{\mu_{r22}}\boldsymbol{e}_2\boldsymbol{e}_2+\sqrt{\mu_{r33}}\boldsymbol{e}_3\boldsymbol{e}_3)\cdot\ddot{\boldsymbol{m}}_x\Big|^2,
\end{aligned}$$

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式中 $\boldsymbol{m}_x$ 代表在 $x_1x_2x_3$ 各向同性直角坐标系中沿极轴振荡的磁偶极矩. 各向异性直角坐标系与通常的直角坐标系的关系为

$$y_i = \sqrt{\varepsilon_r\mu_{rii}}x_i.$$

把各向异性直角坐标系中球坐标与直角坐标的关系

$$y_1 = y\sin\theta_y\cos\phi_y,$$
$$y_2 = y\sin\theta_y\sin\phi_y,$$
$$y_3 = y\cos\theta_y,$$

代入(1)式得磁偶极在磁各向异性介质中辐射的能流密度公式用各向异性球坐标系表示为

$$\begin{aligned}S =& \frac{|\ddot{\boldsymbol{m}}_y|^2}{32\pi^2\varepsilon\sqrt{\varepsilon_r}c^5y^2}\Bigg[\frac{\mu_{r22}}{\mu_{r11}\sqrt{\mu_{r33}}}\sin^3\theta_y\\
&\times\cos^2\theta_y\cos\phi_y+\frac{\mu_{r11}^2\mu_{r22}}{\mu_{r33}^3\sqrt{\mu_{r11}}}\sin^5\theta_y\cos\phi_y\\
&-\left(\frac{\mu_{r11}^2\mu_{r33}}{\mu_{r22}^3\sqrt{\mu_{r11}}}-\frac{\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right)\\
&\times\sin^3\theta_y\cos^2\theta_y\sin^2\phi_y\cos\phi_y\Bigg]\boldsymbol{e}_1\\
&+\Bigg[\frac{\mu_{r11}}{\mu_{r22}\sqrt{\mu_{r33}}}\sin^3\theta_y\cos^2\theta_y\sin\phi_y\\
&+\frac{\mu_{r11}\mu_{r22}^2}{\mu_{r33}^3\sqrt{\mu_{r22}}}\sin^5\theta_y\sin\phi_y\\
&-\left(\frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r11}}}-\frac{\mu_{r22}^2\mu_{r33}}{\mu_{r11}^3\sqrt{\mu_{r22}}}\right)\\
&\times\sin^3\theta_y\cos^2\theta_y\cos\phi_y^2\sin\phi_y\Bigg]\boldsymbol{e}_2\\
&+\Bigg[\frac{\mu_{r11}\mu_{r33}^2}{\mu_{r22}^3\sqrt{\mu_{r33}}}\sin^2\theta_y\cos^3\theta_y\sin^2\phi_y\\
&+\frac{\mu_{r22}\mu_{r33}^2}{\mu_{r11}^3\sqrt{\mu_{r33}}}\sin^2\theta_y\cos^3\theta_y\cos^2\phi_y\\
&+\frac{\mu_{r11}}{\mu_{r33}\sqrt{\mu_{r22}}}\sin^4\theta_y\cos\theta_y\sin^2\phi_y\\
&+\frac{\mu_{r22}}{\mu_{r33}\sqrt{\mu_{r11}}}\sin^4\theta_y\cos\theta_y\cos^2\phi_y\Bigg]\boldsymbol{e}_3.\end{aligned}\tag{2}$$

2.2. 磁偶极的辐射功率

有了磁偶极辐射的能流密度公式便可求出其辐射功率表达式. 为计算方便,在各向异性球坐标系中进行计算,由(2)式可得磁偶极在磁各向异性介质中的辐射功率为

$$\begin{aligned}P =& \oint|\boldsymbol{S}|R^2\mathrm{d}\Omega\\
=& \frac{|\ddot{\boldsymbol{m}}_y|^2R^2}{32\pi^2\varepsilon\sqrt{\varepsilon_r}c^5y^2}\int_0^{\pi}\int_0^{2\pi}\Bigg|\Bigg[\frac{\mu_{r22}}{\mu_{r11}\sqrt{\mu_{r33}}}\\
&\times\sin^3\theta_y\cos^2\theta_y\cos\phi_y\\
&+\frac{\mu_{r11}^2\mu_{r22}}{\mu_{r33}^3\sqrt{\mu_{r11}}}\sin^5\theta_y\cos\phi_y\\
&-\left(\frac{\mu_{r11}^2\mu_{r33}}{\mu_{r22}^3\sqrt{\mu_{r11}}}-\frac{\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right)\\
&\times\sin^3\theta_y\cos^2\theta_y\sin^2\phi_y\cos\phi_y\Bigg]\boldsymbol{e}_1\\
&+\Bigg[\frac{\mu_{r11}}{\mu_{r22}\sqrt{\mu_{r33}}}\sin^3\theta_y\cos^2\theta_y\sin\phi_y\\
&+\frac{\mu_{r11}\mu_{r22}^2}{\mu_{r33}^3\sqrt{\mu_{r22}}}\sin^5\theta_y\sin\phi_y\\
&-\left(\frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r11}}}-\frac{\mu_{r22}^2\mu_{r33}}{\mu_{r11}^3\sqrt{\mu_{r22}}}\right)\\
&\times\sin^3\theta_y\cos^2\theta_y\cos^2\phi_y\sin\phi_y\Bigg]\boldsymbol{e}_2\\
&+\Bigg[\frac{\mu_{r11}\mu_{r33}^2}{\mu_{r22}^3\sqrt{\mu_{r33}}}\sin^2\theta_y\cos^3\theta_y\sin^2\phi_y\\
&+\frac{\mu_{r22}\mu_{r33}^2}{\mu_{r11}^3\sqrt{\mu_{r33}}}\sin^2\theta_y\cos^3\theta_y\cos^2\phi_y\\
&+\frac{\mu_{r11}}{\mu_{r33}\sqrt{\mu_{r22}}}\sin^4\theta_y\cos\theta_y\sin^2\phi_y\\
&+\frac{\mu_{r22}}{\mu_{r33}\sqrt{\mu_{r11}}}\sin^4\theta_y\cos\theta_y\cos^2\phi_y\Bigg]\boldsymbol{e}_3\Bigg|\\
&\times\sin\theta_y d\theta_y d\phi_y,\end{aligned}\tag{3}$$

式中 $R^2 = x_1^2+x_2^2+x_3^2 = \frac{1}{\varepsilon_r}\left(\frac{y_1^2}{\mu_{r11}}+\frac{y_2^2}{\mu_{r22}}+\frac{y_3^2}{\mu_{r33}}\right)$, 又由于磁偶极辐射的能流是沿球半径方向辐射出去的,所以作坐标变换时只取 $\boldsymbol{e}_1 = \sin\theta_y\cos\phi_y\boldsymbol{e}_r$, $\boldsymbol{e}_2 = \sin\theta_y\sin\phi_y\boldsymbol{e}_r$, $\boldsymbol{e}_3 = \cos\theta_y\boldsymbol{e}_r$ 代入(3)式中,而且有 $|\boldsymbol{e}_r| = 1$,于是求得磁偶极在磁各向异性介质中的辐射功率表达式为

$$\begin{aligned}P =& \frac{2|\ddot{\boldsymbol{m}}_y|^2R^2}{32\pi^2\varepsilon\sqrt{\varepsilon_r}c^5y^2}\int_0^{\pi}\int_0^{\pi}\Bigg\{\Bigg[\frac{\mu_{r22}}{\mu_{r11}\sqrt{\mu_{r33}}}\\
&\times\sin^5\theta_y\cos^2\theta_y\cos^2\phi_y\\
&+\frac{\mu_{r11}^2\mu_{r22}}{\mu_{r33}^3\sqrt{\mu_{r11}}}\sin^7\theta_y\cos^2\phi_y\end{aligned}$$

$$
\begin{aligned}
&- \left(\frac{\mu_{r11}^{2}\mu_{r33}}{\mu_{r22}^{3}\sqrt{\mu_{r11}}} - \frac{\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right) \\
&\times \sin^{5}\theta_{y}\cos^{2}\theta_{y}\sin^{2}\phi_{y}\cos^{2}\phi_{y}\Big] \\
&+ \Big[\frac{\mu_{r11}}{\mu_{r22}\sqrt{\mu_{r33}}}\sin^{5}\theta_{y}\cos^{2}\theta_{y}\sin^{2}\phi_{y} \\
&+ \frac{\mu_{r11}\mu_{r22}^{2}}{\mu_{r33}^{3}\sqrt{\mu_{r22}}}\sin^{7}\theta_{y}\sin^{2}\phi_{y} \\
&- \left(\frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r11}}} - \frac{\mu_{r22}^{2}\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right) \\
&\times \sin^{5}\theta_{y}\cos^{2}\theta_{y}\cos^{2}\phi_{y}\sin^{2}\phi_{y}\Big] \\
&+ \Big[\frac{\mu_{r11}\mu_{r33}^{2}}{\mu_{r22}^{3}\sqrt{\mu_{r33}}}\sin^{3}\theta_{y}\cos^{4}\theta_{y}\sin^{2}\phi_{y} \\
&+ \frac{\mu_{r22}\mu_{r333}^{2}}{\mu_{r11}^{3}\sqrt{\mu_{r33}}}\sin^{3}\theta_{y}\cos^{4}\theta_{y}\cos^{2}\phi_{y} \\
&+ \frac{\mu_{r11}}{\mu_{r33}\sqrt{\mu_{r22}}}\sin^{5}\theta_{y}\cos^{2}\theta_{y}\sin^{2}\phi_{y} \\
&+ \frac{\mu_{r22}}{\mu_{r33}\sqrt{\mu_{r11}}}\sin^{5}\theta_{y}\cos^{2}\theta_{y}\cos^{2}\phi_{y}\Big]\Big\}\mathrm{d}\theta_{y}\mathrm{d}\phi_{y} \\
={}& \frac{|\ddot{\boldsymbol{m}}_{y}|^{2}R^{2}}{16\pi^{2}\varepsilon\sqrt{\varepsilon_{r}}c^{5}y^{2}}\Big\{\Big[\frac{\mu_{r22}}{\mu_{r11}\sqrt{\mu_{r33}}}\frac{8}{105}\pi \\
&+ \frac{\mu_{r11}^{2}\mu_{r22}}{\mu_{r33}^{3}\sqrt{\mu_{r11}}}\frac{48}{105}\pi \\
&- \left(\frac{\mu_{r11}^{2}\mu_{r33}}{\mu_{r22}^{3}\sqrt{\mu_{r11}}} - \frac{\mu_{r33}}{\mu_{r11}\sqrt{\mu_{r22}}}\right)\frac{2}{105}\pi\Big)\Big] \\
&+ \Big[\frac{\mu_{r11}}{\mu_{r22}\sqrt{\mu_{r23}}}\frac{8}{105}\pi + \frac{\mu_{r11}\mu_{r22}^{2}}{\mu_{r33}^{3}\sqrt{\mu_{r22}}}\frac{48}{105}\pi \\
&- \left(\frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r11}}} - \frac{\mu_{r22}^{2}\mu_{r33}}{\mu_{r11}^{3}\sqrt{\mu_{r22}}}\right)\frac{2}{105}\pi\Big] \\
&+ \Big[\frac{\mu_{r11}\mu_{r33}^{2}}{\mu_{r22}^{3}\sqrt{\mu_{r33}}}\frac{6}{105}\pi + \frac{\mu_{r22}\mu_{r33}^{2}}{\mu_{r11}^{3}\sqrt{\mu_{r33}}}\frac{6}{105}\pi \\
&+ \frac{\mu_{r11}}{\mu_{r33}\sqrt{\mu_{r22}}}\frac{8}{105}\pi + \frac{\mu_{r22}}{\mu_{r33}\sqrt{\mu_{r11}}}\frac{8}{105}\pi\Big]\Big\}, \qquad (4)
\end{aligned}
$$

式中 $y^{2} = \varepsilon_{r}(\mu_{r11}x_{1}^{2} + \mu_{r22}x_{2}^{2} + \mu_{r33}x_{3}^{2})$.

2.3. 讨论与验证所得结果

由辐射功率表达式(4)可知,对于同一个磁偶极子在不同磁各向异性介质中(μ_{rii}值不同),即使 R 和 y 值相同其辐射功率也是不同的. 换言之,磁偶极在磁各向异性介质中的辐射功率大小受介质的各向异性(μ_{rii}的大小)所影响.

为了检验磁偶极在磁各向异性介质中的辐射功率表达式(4)正确与否,假设当 $\mu_{r11} = \mu_{r22} = \mu_{r33} = \mu_{r}$ 时,(4)式变为

$$
P = \frac{|\ddot{\boldsymbol{m}}_{y}|^{2}R^{2}}{32\pi\varepsilon\sqrt{\varepsilon_{r}}c^{5}y^{2}} \times \frac{8\pi}{3\sqrt{\mu_{r}}} = \frac{\mu\omega^{4}m_{x}^{2}}{12\pi v^{3}}. \qquad (5)
$$

该式与文献[23]的结果 $\left(P = \dfrac{\mu_{0}\omega^{4}m_{x}^{2}}{12\pi c^{3}}\right)$ 相符合,从而证明了磁偶极在磁各向异性介质中的辐射功率表达式(4)的正确性. 式中用到了

$$
\begin{aligned}
\nu &= c/\sqrt{\varepsilon_{r}\mu_{r}}, \\
y_{i} &= \sqrt{\varepsilon_{r}\mu_{rii}}x_{i}, \\
R^{2} &= x_{1}^{2} + x_{2}^{2} + x_{3}^{2} \\
&= \frac{1}{\varepsilon_{r}}\left(\frac{y_{1}^{2}}{\mu_{r11}} + \frac{y_{2}^{2}}{\mu_{r22}} + \frac{y_{3}^{2}}{\mu_{r33}}\right), \\
|\ddot{\boldsymbol{m}}_{x}|^{2} &= (-i\omega)^{4}m_{x}^{2}, \\
|\ddot{\boldsymbol{m}}_{y}|^{2} &= |\sqrt{\varepsilon_{r}^{3}\mu_{r11}\mu_{r22}\mu_{r33}}\sqrt{\varepsilon_{r}}(\sqrt{\mu_{r11}}\boldsymbol{e}_{1}\boldsymbol{e}_{1} \\
&\quad + \sqrt{\mu_{r22}}\boldsymbol{e}_{2}\boldsymbol{e}_{2} + \sqrt{\mu_{r33}}\boldsymbol{e}_{3}\boldsymbol{e}_{3}) \cdot \ddot{\boldsymbol{m}}_{x}|^{2}, \\
\ddot{\boldsymbol{m}}_{x} &= |\ddot{\boldsymbol{m}}_{x}|\boldsymbol{e}_{3}.
\end{aligned}
$$

3. 电四极的辐射功率

3.1. 电四极辐射的能流密度

电四极在磁各向异性介质中辐射的能流密度公式为[22]

$$
\begin{aligned}
\boldsymbol{S} ={}& \frac{\sqrt{\varepsilon_{r}}|\dddot{\boldsymbol{D}}_{y}|^{2}}{4\times 288\mu_{0}\pi^{2}\varepsilon^{2}c^{7}y^{5}} \\
&\times\Big\{\Big[\frac{\sqrt{\mu_{r11}}\mu_{r33}}{\mu_{r22}^{2}}y_{1}^{2} + \frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r22}}}y_{2}^{2}\Big]y_{1}\boldsymbol{e}_{1} \\
&+ \Big[\frac{\mu_{r33}}{\sqrt{\mu_{r11}^{3}}}y_{1}^{2} + \frac{\sqrt{\mu_{r22}}\mu_{r33}}{\mu_{r11}^{2}}y_{2}^{2}\Big]y_{2}\boldsymbol{e}_{2} \\
&+ \Big[\frac{\mu_{r11}\sqrt{\mu_{r33}^{3}}}{\mu_{r22}^{3}}y_{1}^{2} + \frac{\mu_{r22}\sqrt{\mu_{r33}^{3}}}{\mu_{r11}^{3}}y_{2}^{2}\Big]y_{3}\boldsymbol{e}_{3}\Big\}, \qquad (6)
\end{aligned}
$$

式中 $\dddot{\boldsymbol{D}}_{y}$ 代表在各向异性坐标系 $y_{1}y_{2}y_{3}$ 中沿极轴震荡的电四极矩 $\boldsymbol{D}_{y}$ 对时间的三次导数, 且有

$$
\begin{aligned}
|\dddot{\boldsymbol{D}}_{y}| &= |\boldsymbol{n}_{y}\cdot\dddot{\boldsymbol{D}}_{y}| \\
&= \sqrt{\varepsilon_{r}^{3}\mu_{r11}\mu_{r22}\mu_{r33}}\,|\boldsymbol{n}_{y}\cdot\boldsymbol{\mu}\boldsymbol{\mu} : \dddot{D}'_{x}|, \\
\dddot{\boldsymbol{D}}'_{x} &= (-i\omega)^{3}3\int\frac{\boldsymbol{x}'}{\boldsymbol{\mu}\cdot\boldsymbol{x}'}\rho(x')\,\boldsymbol{x}'\boldsymbol{x}'\mathrm{d}v'_{x},
\end{aligned}
$$

$$\boldsymbol{\mu} = \sqrt{\varepsilon_r}\,(\sqrt{\mu_{r11}}\boldsymbol{e}_1\boldsymbol{e}_1 + \sqrt{\mu_{r22}}\boldsymbol{e}_2\boldsymbol{e}_2 + \sqrt{\mu_{r33}}\boldsymbol{e}_3\boldsymbol{e}_3),$$

$\boldsymbol{n}_y$ 代表在 $y_1y_2y_3$ 各向异性直角坐标系中沿 $\boldsymbol{y}$ 方向的单位矢量，$\boldsymbol{x}'$ 代表在 $x_1x_2x_3$ 各向同性直角坐标系中原点到源点的位矢，$\dddot{\boldsymbol{D}}'_x$ 代表用各向同性直角坐标系描述的各向异性介质中的电四极矩. 把各向异性直角坐标系中球坐标与直角坐标的关系式代入(6)式得电四极在磁各向异性介质中辐射的能流密度公式用各向异性球坐标系表示为

$$\begin{aligned}
\boldsymbol{S} = {} & \frac{\sqrt{\varepsilon_r}\,|\dddot{\boldsymbol{D}}_y|^2}{4\times 288\mu_0\pi^2\varepsilon^2c^7y^2} \\
& \times \Bigg\{\Bigg[\frac{\sqrt{\mu_{r11}}\mu_{r33}}{\mu_{r22}^2}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r22}}}\sin^2\theta_y\sin^2\phi_y\Bigg]\sin\theta_y\cos\phi_y\boldsymbol{e}_1 \\
& + \Bigg[\frac{\mu_{r33}}{\sqrt{\mu_{r11}^3}}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\sqrt{\mu_{r22}}\mu_{r33}}{\mu_{r11}^2}\sin^2\theta_y\sin^2\phi_y\Bigg]\sin\theta_y\sin\phi_y\boldsymbol{e}_2 \\
& + \Bigg[\frac{\mu_{r11}\sqrt{\mu_{r33}^3}}{\mu_{r22}^3}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\mu_{r22}\sqrt{\mu_{r33}^3}}{\mu_{r11}^3}\sin^2\theta_y\sin^2\phi_y\Bigg]\cos\theta_y\boldsymbol{e}_3\Bigg\}.
\end{aligned} \tag{7}$$

3.2. 电四极的辐射功率

由于电四极辐射的能流密度 $\boldsymbol{S}$ 分布的各向异性，且沿各个方向大小不同，因此，求电四极的辐射功率时可在各向异性球坐标系中进行. 表达式(7)中 $\boldsymbol{D}_y$ 代表在各向异性坐标系 $y_1y_2y_3$ 中沿极轴振荡的电四极矩，若用符号 $\boldsymbol{D}_{y_3}$ 表示则有 $|\dddot{\boldsymbol{D}}_{y_3}| = |\boldsymbol{n}_y\cdot\dddot{\boldsymbol{D}}_y| = |\dddot{\boldsymbol{D}}_y\cos\theta_y|$，$\theta_y$ 是各向异坐标系中 $\boldsymbol{n}_y$ 与 $\boldsymbol{D}_y$ 的夹角，于是有

$$\begin{aligned}
P = {} & \oint|\boldsymbol{S}|R^2\mathrm{d}\Omega \\
= {} & \frac{\sqrt{\varepsilon_r}\,|\dddot{\boldsymbol{D}}_y|^2R^2\cos^2\theta_y}{4\times 288\mu_0\pi^2\varepsilon^2c^8y^2} \\
& \times\Bigg|\int_0^\pi\int_0^{2\pi}\Bigg[\frac{\sqrt{\mu_{r11}}\mu_{r33}}{\mu_{r22}^2}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r22}}}\sin^2\theta_y\sin^2\phi_y\Bigg] \\
& \times\sin^2\theta\cos\phi_y\,\mathrm{d}\theta_y\phi_y\boldsymbol{e}_1 \\
& + \int_0^\pi\int_0^\pi\Bigg[\frac{\mu_{r33}}{\mu_{r11}^3}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\sqrt{\mu_{r22}}\mu_{r33}}{\mu_{r11}^2}\sin^2\theta_y\sin^2\phi_y\Bigg] \\
& \times\sin^2\theta_y\sin\phi_y\,d\theta_y\,d\phi_y\boldsymbol{e}_2 \\
& + \int_0^\pi\int_0^\pi\Bigg[\frac{\mu_{r11}\sqrt{\mu_{r33}^3}}{\mu_{r22}^2}\sin^2\theta_y\cos^2\phi_y \\
& + \frac{\mu_{r22}\mu_{r33}^3}{\mu_{r11}^3}\sin^2\theta_y\sin^2\phi\Bigg] \\
& \times\sin\theta_y\cos\theta_y\,d\theta_y\,d\phi_y\boldsymbol{e}_3\Bigg| \\
= {} & \frac{\sqrt{\varepsilon_r}\,|\dddot{\boldsymbol{D}}_y|^2R^2}{4\times 288\mu_0\pi^2\varepsilon^2c^7y^2} \\
& \times\int_0^\pi\int_0^{2\pi}\Bigg|\Bigg[\frac{\sqrt{\mu_{r11}}\mu_{r33}}{\mu_{r22}^2}\sin^4\theta_y\cos^2\theta_y\cos^2\phi_y \\
& + \frac{\mu_{r33}}{\mu_{r22}\sqrt{\mu_{r22}}}\sin^4\theta_y\cos^2\theta_y\cos\phi_y\sin^2\phi_y\Bigg]\boldsymbol{e}_1 \\
& + \Bigg[\frac{\mu_{r33}}{\sqrt{\mu_{r11}^3}}\sin^4\theta_y\cos^2\theta_y\cos^2\phi_y\sin\phi_y \\
& + \frac{\sqrt{\mu_{r22}}\mu_{r33}}{\mu_{r11}^2}\sin^3\theta_y\cos^2\theta_y\sin^3\phi_y\Bigg]\boldsymbol{e}_2 \\
& + \Bigg[\frac{\mu_{r11}\sqrt{\mu_{r33}^3}}{\mu_{r22}^2}\sin^3\theta_y\cos\theta_y^3\cos^2\phi_y \\
& + \frac{\mu_{r22}\sqrt{\mu_{r33}^3}}{\mu_{r11}^3}\sin^3\theta_y\cos^3\theta_y\sin^2\phi_y\Bigg]\boldsymbol{e}_3\Bigg|\,\mathrm{d}\theta_y\mathrm{d}\phi_y \\
= {} & \frac{2\sqrt{\varepsilon_r}\,|\dddot{\boldsymbol{D}}_y|^2R^2\mu_{r33}}{4\times 288\mu_0\pi^2\varepsilon^2c^7y^2} \\
& \times\Bigg\{\Bigg[\frac{\sqrt{\mu_{r11}}}{\mu_{r22}^2}\times\frac{6\pi}{105} + \frac{1}{\mu_{r22}\sqrt{\mu_{r22}}}\frac{2\pi}{105}\Bigg] \\
& + \Bigg[\frac{1}{\sqrt{\mu_{r11}^3}}\frac{2\pi}{105} + \frac{\sqrt{\mu_{r22}}}{\mu_{r11}^2}\frac{6\pi}{105}\Bigg] \\
& + \Bigg[\frac{\mu_{r11}\sqrt{\mu_{r33}}}{\mu_{r22}^2}\frac{6\pi}{105} + \frac{\mu_{r22}\sqrt{\mu_{r33}}}{\mu_{r11}^3}\frac{6\pi}{105}\Bigg]\Bigg\}.
\end{aligned} \tag{8}$$

上式即电四极在磁各向异性介质中的辐射功率表达式.

3.3. 讨论与验证所得结果

由电四极在磁各向异性介质中的辐射功率表

达式(8)可知电四极的辐射功率大小具有一定的方向性，在不同的方向上其辐射功率大小不同，与介质的磁各向异性有关，与 μ_{r33} 大小成正比；而且和电四极矩在各向同性介质中的辐射一样，其辐射功率大小与电四极矩三次导数的模的平方成正比.

当 $\mu_{r11}=\mu_{r22}=\mu_{r33}=\mu_r$ 时，(8)式变换为

$$P=\frac{\sqrt{\varepsilon_r}\,|\dddot{\boldsymbol{D}}_y|^2R^2}{4\times 288\mu_0\pi^2\varepsilon^2c^7y^2}\left[\frac{8\pi}{15\sqrt{\mu_r}}\right]$$

$$=\frac{v\varepsilon_r\mu_r\,|\dddot{\boldsymbol{D}}_y|^2R^2}{144\mu\pi\varepsilon^2c^8y^2\times 15}. \tag{9}$$

把以下各式：

$$|\dddot{\boldsymbol{D}}_y|=\sqrt{\varepsilon_r^3\mu_{r11}\mu_{r22}\mu_{r33}}\,|\boldsymbol{n}_y\cdot\boldsymbol{\mu\mu}:\dddot{\boldsymbol{D}}'_x|$$

$$=\sqrt{\varepsilon_r^3\mu_{r11}\mu_{r22}\mu_{r33}}\,|\boldsymbol{n}_x\cdot\boldsymbol{\mu\mu}:\dddot{\boldsymbol{D}}'_x|$$

$$=\varepsilon_r^2\mu_r^2\,|\dddot{\boldsymbol{D}}_x|,$$

$$R^2=x_1^2+x_2^2+x_3^2,\ y_i=\sqrt{\varepsilon_r\mu_{rii}}x_i,$$

$$\boldsymbol{\mu\mu}=\varepsilon_r\mu_r\boldsymbol{II},\ |\dddot{\boldsymbol{D}}'_x|=\frac{|\dddot{\boldsymbol{D}}_x|}{\sqrt{\varepsilon_r\mu_r}}$$

代入(9)式得

$$P=\frac{v\varepsilon_r^3\mu_r^3\,|\dddot{\boldsymbol{D}}_x|^2}{15\times 144\mu_0\pi\varepsilon_0^2\varepsilon_rc^8}=\frac{v\varepsilon_r^3\mu_r^3\,|\dddot{\boldsymbol{D}}_x|^2}{15\times 144\pi\varepsilon_0\varepsilon_rc^6}$$

$$=\frac{v\,|\dddot{\boldsymbol{D}}_x|^2}{15\times 144\pi\varepsilon v^6}=\frac{|\dddot{\boldsymbol{D}}_x|^2}{15\times 144\pi\varepsilon v^5}, \tag{10}$$

式中用到了

$$v=\frac{c}{\sqrt{\varepsilon_r\mu_r}},\quad c=\frac{1}{\sqrt{\varepsilon_0\mu_0}},\quad \dddot{\boldsymbol{D}}_x=|\dddot{\boldsymbol{D}}_x|\boldsymbol{e}_3.$$

假设体系的电四极张量为 $\dddot{\boldsymbol{D}}=6Ql^2\boldsymbol{e}_3\boldsymbol{e}_3$，即沿极轴以频率 ω 振荡时，则把 $|\dddot{\boldsymbol{D}}_x|^2=36Q^2l^4\omega^6$ 代入(10)式得

$$P=\frac{36Q^2l^4\omega^6}{15\times 144\pi\varepsilon v^5}=\frac{Q^2l^4\omega^6}{60\pi\varepsilon v^5}. \tag{11}$$

这正是文献[23]的结果. 可见所得结果与文献符合很好，说明了所求电四极矩在磁各向异性介质中的辐射功率表达式(8)是正确的.

4. 结　　论

磁偶极和电四极在磁各向异性介质中的辐射功率问题，是研究、开发利用各向异性介质所必须解决的一个重要问题. 本文在已有工作的基础上导出了计算磁偶极和电四极在磁各向异性介质中的辐射功率表达式. 本文结果为研发新材料及测定磁偶极和电四极在磁各向异性介质中的辐射功率提供了理论参考.

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Radiation powers of magnetic dipole and electric quadrupole in magnetic anisotropic medium*

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Abstract

Under the frame of classical electrodynamics, the electromagnetic radiation in magnetic anisotropic medium is investigated in this paper. The radiation power expressions for magnetic dipole and electronic quadrupole in magnetic anisotropic medium are in derived. Furthermore, the results, which are achieved by inserting the μ_r in isotropic medium into the obtained expressions, are agreed with the references. And the correctness of the obtained expressions is verified. The research results demonstrate that the radiation powers of magnetic dipole and electronic quadrupole are related to the μ_r in magnetic anisotropic medium. This conclusion is very helpful for determining the radiation effects of magnetic dipole and electronic quadrupole in magnetic anisotropic medium.

Keywords: magnetic anisotropic medium, magnetic dipole, electric quadrupole, radiation power

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