相对运动动力学系统 Nielsen 方程的 Lie 对称性与 Hojman 守恒量*

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研究相对运动动力学系统 Nielsen 方程的 Lie 对称性和 Lie 对称性直接导致的 Hojman 守恒量. 在群的无限小变换下,给出相对运动动力学系统 Nielsen 方程 Lie 对称性的定义和判据;得到相对运动动力学系统 Nielsen 方程 Lie 对称性的确定方程以及 Lie 对称性直接导致的 Hojman 守恒量的表达式. 举例说明结果的应用.

关键词: 相对运动动力学, Nielsen 方程, Lie 对称性, Hojman 守恒量

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1. 引 言

动力学系统的对称性和守恒量的研究在现代数理科学中占有重要地位,也是分析力学的一个发展方向.

对称性原理是物理学中更高层次的法则. 对称性理论有许多用途,其中之一就是寻找守恒量,力学系统对称性与守恒量的研究源自 Noether,她揭示了对称性与守恒量间的潜在关系[1],20 世纪 70 年代开始蓬勃发展[2—5]. 近年来我国学者深入而广泛地研究了动力学系统的 Noether 对称性,Lie 对称性和形式不变性[6—19]. 本文具体研究相对运动动力学系统 Nielsen 方程的 Lie 对称性与 Hojman 守恒量,最后给出一个算例来说明本文结果的应用.

2. 相对运动动力学系统的 Nielsen 方程

设载体的运动由基点 O 的速度 v 和它的角速度 $\boldsymbol{\omega}$ 来确定. 被载质点相对惯性坐标系 $\bar{O}xyz$ 的位置由矢径 $\boldsymbol{r}_i$ 确定,相对于载体固联的坐标系 $Ox'y'z'$ 的位置由矢径 $\boldsymbol{r}'_i$ 确定,设 $\boldsymbol{r}'_i$ 中不显含时间 t,即

$$\boldsymbol{r}'_i = \boldsymbol{r}'_i(q_s),(i = 1,\cdots,N;s = 1,\cdots,n). \quad (1)$$

用 $\boldsymbol{r}_0$ 表示载体基点的矢径,则有

$$\boldsymbol{r}_i = \boldsymbol{r}_0 + \boldsymbol{r}'_i,$$

其中 $\boldsymbol{r}_0$ 为时间的已知函数. 将(1)式对时间 t 求导数,得

$$\boldsymbol{r}'_i = \overset{*}{\boldsymbol{r}}'_i + \boldsymbol{\omega} \times \boldsymbol{r}'_i = \sum_{s=1}^{n} \frac{\partial r'_i}{\partial q_s}\dot{q}_s + \boldsymbol{\omega} \times \boldsymbol{r}'_i, \quad (2)$$

其中 $\overset{*}{\boldsymbol{r}}'_i$ 为 $\boldsymbol{r}'_i$ 的相对导数. 系统绝对运动的动能为

$$\begin{aligned} T &= \frac{1}{2}\sum_{i=1}^{N} m_i \dot{\boldsymbol{r}}_i \cdot \dot{\boldsymbol{r}}_i \\ &= \frac{1}{2}M\boldsymbol{v}_0^2 + M\boldsymbol{v}_0 \cdot (\boldsymbol{\omega} \times \boldsymbol{r}'_c) + \frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{\theta}^0 \cdot \boldsymbol{\omega} \\ &\quad + \frac{1}{2}\sum_{i=1}^{N} m_i \overset{*}{\boldsymbol{r}}'_i \cdot \overset{*}{\boldsymbol{r}}'_i + \boldsymbol{v}_0 \cdot \boldsymbol{Q}_r + \boldsymbol{\omega} \cdot \boldsymbol{K}_r^0, \end{aligned} \quad (3)$$

其中 $M = \sum_{i=1}^{N} m_i$ 为被载体的总质量,$\boldsymbol{r}'_c = \frac{1}{M}\sum_{i=1}^{N} m_i \boldsymbol{r}'_i$ 为被载体质心在 $Ox'y'z'$ 中的矢径,$\boldsymbol{\theta}^0$ 为系统在 O 点的惯性张量,$\boldsymbol{Q}_r$ 为系统相对动量主矢:$\boldsymbol{Q}_r = \sum_{i=1}^{N} m_i \overset{*}{\boldsymbol{r}}'_i$,$\boldsymbol{K}_r^0$ 为系统对基点 O 的相对动量矩主矢:$\boldsymbol{K}_r^0 = \sum_{i=1}^{N} m_i \boldsymbol{r}'_i \times \overset{*}{\boldsymbol{r}}'_i$. 将(3) 式写成形式

$$T = T_e + T_m + T_r, \quad (4)$$

则

$$T_e = \frac{1}{2}M v_0^2 + M v_0 \cdot (\omega \times \boldsymbol{r}'_c) + \frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{\theta}^0 \cdot \boldsymbol{\omega},$$

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$$T_{\mathrm{m}} = \boldsymbol{v}_0 \cdot \boldsymbol{Q}_r + \omega \cdot \boldsymbol{K}_{\mathrm{r}}^0,$$

$$T_{\mathrm{r}} = \frac{1}{2}\sum_{i=1}^{N} m_i \overset{*}{\boldsymbol{r}}'_i \cdot \overset{*}{\boldsymbol{r}}'_i.$$

它们分别为牵连运动动能,混合动能和相对运动动能.

将(4)式代入$\frac{\partial \dot{f}}{\partial \dot{q}_s} - 2\frac{\partial f}{\partial q_s} = Q_s$,得

$$N_s(T_{\mathrm{e}}) + N_s(T_{\mathrm{m}}) + N_s(T_{\mathrm{r}}) = Q_s,$$
$$(s = 1,\cdots,n). \tag{5}$$

计算得

$$N_s(T_{\mathrm{e}}) = -M(\boldsymbol{v}_0 \times \boldsymbol{\omega}) \cdot \frac{\partial \boldsymbol{r}'_c}{\partial q_s} - \frac{1}{2}\boldsymbol{\omega} \cdot \frac{\partial \boldsymbol{\theta}^0}{\partial q_s} \cdot \boldsymbol{\omega}, \tag{6}$$

$$N_s(T_{\mathrm{m}}) = M\overset{*}{\boldsymbol{v}}_0 \cdot \frac{\partial \boldsymbol{r}'_{\mathrm{c}}}{\partial q_s} + \dot{\omega} \cdot \frac{\partial \boldsymbol{K}_{\mathrm{r}}^0}{\partial \dot{q}_s} + \boldsymbol{\omega} \cdot N_s^*(\boldsymbol{K}_{\mathrm{r}}^0). \tag{7}$$

将(6)式和(7)式代入方程(5),得

$$N_s(T_{\mathrm{r}}) = Q_s - M(\overset{*}{\boldsymbol{v}}_0 + \boldsymbol{\omega} \times \boldsymbol{v}_0) \cdot \frac{\partial r'_{\mathrm{c}}}{\partial q_s} + \frac{1}{2}\boldsymbol{\omega} \cdot \frac{\partial \boldsymbol{\theta}^0}{\partial q_s} \cdot \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \frac{\partial \boldsymbol{K}_r^0}{\partial \dot{q}_s} - \boldsymbol{\omega} \cdot N_s^*(\boldsymbol{K}_{\mathrm{r}}^0). \tag{8}$$

其中矢量$-M\dot{\boldsymbol{v}}_0 = -M(\overset{*}{\boldsymbol{v}}_0 + \boldsymbol{\omega} \times \boldsymbol{v}_0)$称为平动运动的惯性力,而

$$Q_s^0 = -M\dot{\boldsymbol{v}}_0 \cdot \frac{\partial \boldsymbol{r}'_c}{\partial q_s} = -\frac{\partial}{\partial q_s}M(\overset{*}{\boldsymbol{v}}_0 + \boldsymbol{\omega} \times \boldsymbol{v}_0) \cdot \boldsymbol{r}'_c = -\frac{\partial V^0}{\partial q_s} \tag{9}$$

称为平动运动的广义惯性力,其中 V^0 为这些力的均匀场势能.

$$V^\omega = -\frac{1}{2}\boldsymbol{\omega} \cdot \boldsymbol{\theta}^0 \cdot \boldsymbol{\omega} \tag{10}$$

为离心力势能. 而

$$Q_s^\omega = -\frac{\partial V^\omega}{\partial q_s} = \frac{1}{2}\boldsymbol{\omega} \cdot \frac{\partial \boldsymbol{\theta}^0}{\partial q_s} \cdot \boldsymbol{\omega} \tag{11}$$

是广义离心力.

$$-\dot{\boldsymbol{\omega}} \cdot \frac{\partial \boldsymbol{K}_r^0}{\partial \dot{q}_s} = -\sum_{i=1}^{N} m_i(\dot{\boldsymbol{\omega}} \times \boldsymbol{r}'_i) \cdot \frac{\partial \boldsymbol{r}'_i}{\partial q_s} = Q_s^{\dot{\omega}} \tag{12}$$

为广义转动惯性力.

$$-\boldsymbol{\omega} \cdot N_s^*(\boldsymbol{K}_{\mathrm{r}}^0) = -\sum_{i=1}^{N} m_i(2\boldsymbol{\omega} \times \overset{*}{\boldsymbol{r}}'_i) \cdot \frac{\partial \boldsymbol{r}'_i}{\partial q_s} \tag{13}$$

表示广义 Coriolis 惯性力,同时也可作为广义陀螺力

$$-\boldsymbol{\omega} \cdot N_s^*(\boldsymbol{K}_{\mathrm{r}}^0) = \Gamma_s = \sum_{k=1}^{n} \gamma_{sk}\dot{q}_k. \tag{14}$$

其中陀螺系数为

$$\gamma_{sk} = 2\boldsymbol{\omega} \cdot \sum_{i=1}^{N} m_i \frac{\partial \boldsymbol{r}'_i}{\partial q_s} \times \frac{\partial \boldsymbol{r}'_i}{\partial q_k} = -\gamma_{ks}. \tag{15}$$

将(9),(11),(12)和(14)式代入方程(8),可得

$$\frac{\partial \dot{T}_{\mathrm{r}}}{\partial \dot{q}_s} - 2\frac{\partial T_{\mathrm{r}}}{\partial q_s} = Q_s - \frac{\partial}{\partial q_s}(V^0 + V^\omega) + Q_s^{\dot{\omega}} + \Gamma_s,$$
$$(s = 1,\cdots,n). \tag{16}$$

将广义力 Q_s 分为有势的 Q'_s 和非势的 Q''_s,有

$$Q_s = Q'_s + Q''_s, Q'_s = -\frac{\partial V}{\partial q_s}. \tag{17}$$

令 $L_{\mathrm{r}} = T_{\mathrm{r}} - V - V^0 - V^\omega$,及对任意函数 $f(q_s, \dot{q}_s, t)$,有

$$N_s = E_s, \ (s = 1,\cdots,n),$$

可得

$$\frac{\partial \dot{L}_{\mathrm{r}}}{\partial \dot{q}_s} - 2\frac{\partial L_{\mathrm{r}}}{\partial q_s} = Q''_s + Q_s^{\dot{\omega}} + \Gamma_s. \tag{18}$$

设系统非奇异,即设

$$\det\left(\frac{\partial^2 L_{\mathrm{r}}}{\partial \dot{q}_s \partial \dot{q}_k}\right) \neq 0, \tag{19}$$

则可由方程(18)解出所有广义加速度,记作

$$\ddot{q}_s = \alpha_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}). \tag{20}$$

3. 相对运动动力学系统 Nielsen 方程的 Lie 对称性和 Hojman 守恒量

相对运动动力学系统的微分方程(18)的 Lie 对称性确定方程为

$$X^{(2)}\{N_s(L_{\mathrm{r}})\} = X^{(1)}(Q''_s + Q_s^{\dot{\omega}} + \Gamma_s), \tag{21}$$

其中

$$X^{(2)} = X^{(1)} + [(\dot{\xi}_k - \dot{q}_k\dot{\xi}_0)^{\cdot} - \ddot{q}_k\dot{\xi}_0]\frac{\partial}{\partial \ddot{q}_k}.$$

方程(20)的 Lie 对称性确定方程为

$$\ddot{\xi}_s - \dot{q}_s\ddot{\xi}_0 - 2\ddot{\xi}_0\alpha_s = \frac{\partial \alpha_s}{\partial t}\xi_0 + \frac{\partial \alpha_s}{\partial q_k}\xi_k + \frac{\partial \alpha_s}{\partial \dot{q}_k}(\dot{\xi}_k - \dot{q}_k\dot{\xi}_0). \tag{22}$$

对相对运动动力学系统,由 Lie 对称性可直接导出 Hojman 守恒量.

取时间不变的特殊无限小变换

$$t^* = t, q_s^*(t^*) = q_s(t) + \varepsilon\xi_s(t,\boldsymbol{q},\dot{\boldsymbol{q}}) . \quad (23)$$

方程(20)在时间不变的特殊无限小变换下 Lie 对称性的确定方程为

$$\frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\xi_s = \frac{\partial\alpha_s}{\partial q_k}\xi_k + \frac{\partial\alpha_s}{\partial\dot{q}_k}\frac{\bar{d}}{dt}\xi_k, \quad (24)$$

其中 $\frac{\bar{d}}{dt} = \frac{\partial}{\partial t} + \dot{q}_k\frac{\partial}{\partial q_k} + \alpha_k\frac{\partial}{\partial\dot{q}_k}$.

命题 在特殊无限小变换下,如果生成元 ξ_s 满足(24)式,且存在某函数 $\mu = \mu(t,\boldsymbol{q},\dot{\boldsymbol{q}})$ 使得

$$\frac{\partial\alpha_s}{\partial\dot{q}_s} + \frac{\bar{d}}{dt}\ln\mu = 0, \quad (25)$$

则相对运动动力学系统的 Lie 对称性直接导致 Hojman 守恒量

$$I_H = \frac{1}{\mu}\frac{\partial}{\partial q_s}(\mu\xi_s) + \frac{1}{\mu}\frac{\partial}{\partial\dot{q}_s}\left(\mu\frac{\bar{d}}{dt}\xi_s\right) = \mathrm{const}. \quad (26)$$

证明 将(26)式按方程(20)求对时间的导数,得

$$\frac{\bar{d}}{dt}I_H = \frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial q_s}\xi_s\right) + \frac{\bar{d}}{dt}\frac{\partial\xi_s}{\partial q_s} + \frac{\bar{d}}{dt}\left[\frac{1}{\mu}\frac{\bar{d}}{dt}\frac{\partial\mu}{\partial\dot{q}_s}\xi_s + \frac{\partial}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_s\right]. \quad (27)$$

容易证明

$$\frac{\bar{d}}{dt}\frac{\partial}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_s = \frac{\partial}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) - \frac{\partial}{\partial q_s}\frac{\bar{d}}{dt}\xi_s - \frac{\partial\alpha_k}{\partial\dot{q}_s}\left(\frac{\bar{d}}{dt}\xi_s\right),$$

$$\frac{\bar{d}}{dt}\frac{\partial\xi_s}{\partial q_s} = \frac{\partial}{\partial q_s}\frac{\bar{d}}{dt}\xi_s - \frac{\partial\alpha_k}{\partial q_s}\frac{\partial\xi_s}{\partial\dot{q}_k}. \quad (28)$$

将(24)式对 $\dot{q}_s$ 求偏导数,并对 s 求和,得到

$$\frac{\partial}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) = \frac{\partial}{\partial\dot{q}}\left(\frac{\partial\alpha_s}{\partial q_k}\xi_k\right) + \frac{\partial}{\partial\dot{q}_s}\left(\frac{\partial\alpha_s}{\partial\dot{q}_k}\frac{\bar{d}}{dt}\xi_k\right). \quad (29)$$

将(28)式和(29)式代入(27)式,得

$$\frac{\bar{d}}{dt}I_H = \frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial q_s}\xi_s\right) + \frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_s\right) + \frac{\partial^2\alpha_s}{\partial q_k\partial\dot{q}_s}\xi_k + \frac{\partial^2\alpha_s}{\partial\dot{q}_k\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_k. \quad (30)$$

利用(25)式,得

$$\frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial q_s}\xi_s\right) = -\frac{\partial^2\alpha_s}{\partial q_k\partial\dot{q}_s}\xi_k + \frac{1}{\mu}\frac{\partial\mu}{\partial q_s}\frac{\bar{d}}{dt}\xi_s - \frac{1}{\mu}\frac{\partial\mu}{\partial\dot{q}_k}\frac{\partial\alpha_k}{\partial q_s}\xi_s, \quad (31)$$

$$\frac{\bar{d}}{dt}\left(\frac{1}{\mu}\frac{\partial\mu}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_s\right) = -\frac{\partial^2\alpha_s}{\partial\dot{q}_k\partial\dot{q}_s}\frac{\bar{d}}{dt}\xi_k - \frac{1}{\mu}\frac{\partial\alpha_k}{\partial\dot{q}_s}\frac{\partial\mu}{\partial\dot{q}_k}\frac{\bar{d}}{dt}\xi_s + \frac{1}{\mu}\frac{\partial\mu}{\partial\dot{q}_s}\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) - \frac{1}{\mu}\frac{\partial\mu}{\partial q_s}\frac{\bar{d}}{dt}\xi_s. \quad (32)$$

将(31)式和(32)式代入(30)式,得

$$\frac{\bar{d}}{dt}I_H = \frac{1}{\mu}\frac{\partial\mu}{\partial\dot{q}_s}\left\{\frac{\bar{d}}{dt}\left(\frac{\bar{d}}{dt}\xi_s\right) - \frac{\partial\alpha_s}{\partial\dot{q}_k}\frac{\bar{d}}{dt}\xi_k - \frac{\partial\alpha_s}{\partial q_k}\xi_k\right\} = 0. \quad (33)$$

证毕.

4. 算 例

相对运动动力学系统为

$$L_r = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) + \frac{1}{2}m\omega^2(q_1^2 + q_2^2),$$

$$Q''_1 = k\dot{q}_2,\ Q''_2 = -k\dot{q}_1, \quad (34)$$

$$\Gamma_1 = \Gamma_2 = Q^{\dot{\omega}} = Q'' = 0,$$

其中 m,k,ω 为常数,试研究系统的 Lie 对称性与 Lie 对称性导致的 Hojman 守恒量.

系统的运动微分方程为

$$m\ddot{q}_1 = m\omega^2 q_1 + k\dot{q}_2,$$

$$m\ddot{q}_2 = m\omega^2 q_2 - k\dot{q}_2,$$

$$m\ddot{q}_3 = 0. \quad (35)$$

Lie 对称性的确定方程(24)给出

$$\frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\xi_1 = \omega^2\xi_1 + \frac{k}{m}\frac{\bar{d}}{dt}\xi_2,$$

$$\frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\xi_2 = \omega^2 - \frac{k}{m}\frac{\bar{d}}{dt}\xi_1, \quad (36)$$

$$\frac{\bar{d}}{dt}\frac{\bar{d}}{dt}\xi_3 = 0.$$

取生成元为

$$\xi_1 = \xi_2 = 0, \xi_3 = 1, \quad (37)$$

$$\xi_1 = \xi_2 = 0, \xi_3 = \dot{q}_3^2 t - \dot{q}_3 q_3, \tag{38}$$

方程(25)给出

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\ln\mu = 0, \tag{39}$$

它有解

$$\mu = 1. \tag{40}$$

将(37)式和(40)式代入(26)式得到的 Hojman 守恒量为平凡解. 将(38)式和(40)式代入(26)式得到 Hojman 守恒量

$$I_{\mathrm{H}} = 2(q_3 - \dot{q}_3 t) = \text{const.}$$

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Lie symmetry and Hojman conserved quantity of Nielsen equation in a dynamical system of the relative motion*

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Abstract

Lie symmetry and Hojman conserved quantity for Nielsen equations in a dynamical system of the relative motion are investigated. The definition and the criterion of Lie symmetry of Nielsen equations in a dynamical system of the relative motion under the infinitesimal transformations of groups are given. The expressions of the determining equation of Lie symmetry of Nielsen equations and Hojman conserved quantity deduced directly from Lie symmetry in a dynamical system of the relative motion are obtained. An example is given to illustrate the application of the results.

Keywords: dynamics of relative motion, Nielsen equations, Lie symmetry, Hojman conserved quantity
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