

# 厄尔尼诺/拉尼娜-南方涛动机制 时滞海-气振子的渐近解\*

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研究了一类厄尔尼诺/拉尼娜-南方涛动机制. 利用渐近分析的摄动方法, 简单而有效地构造了一个厄尔尼诺/拉尼娜和南方涛动时滞模型解的渐近展开式. 讨论了相应问题解的渐近性态.

**关键词:** 非线性, 渐近性态, 厄尔尼诺/拉尼娜-南方涛动模型

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## 1. 引言

厄尔尼诺/拉尼娜和南方涛动分别是发生在热带大气和海洋中的异常事件. 它的发生严重地影响全球各地区气候和生态等方面的变化, 因此对其规律的研究和预防为当前学术界所关注<sup>[1-8]</sup>. 厄尔尼诺/拉尼娜和南方涛动的振荡性态是海-气流动的正负两种反馈的结果. 这两种反馈决定了海表温度的变化, 并与南方涛动流动强度有关, 导致了弱信风沿着赤道行进. 弱信风驱动着海洋流动变化, 加强了海表温度的异常. 海洋-大气的正反馈与耦合的海洋-大气的不稳定性导致了赤道太平洋温度变化, 因此需研究东、西太平洋振子模型的异常关系.

近来, 许多学者已经研究了非线性问题<sup>[9-12]</sup>. 近似方法不断地被优化, 包括平均方法、边界层方法、匹配渐近展开法和多重尺度法. 文献[13-20]也利用渐近理论研究了一类奇摄动问题. 本文利用渐近分析的摄动方法, 简单而有效地研究一类扰动时滞海-气振子模型.

## 2. 一类厄尔尼诺/拉尼娜和南方涛动时滞模型

首先研究一类扰动时滞厄尔尼诺/拉尼娜和南方涛动模型振子. 这是西太平洋时滞振子为一个非活动区域和在西边界提供负反馈反射波的振荡, 时滞振子模型由一组具有正负反馈的常微分时滞方程及其初始时滞条件来表示.

研究东、西太平洋之间的气候异常变化通常是进一步利用一些单纯的假设把它简化为振荡模型. 考虑耦合系统为如下模型<sup>[4]</sup>:

$$\frac{dT}{dt} = aT - bq(t - \delta) - \varepsilon_1 f(T, q), \quad (1)$$

$$c \frac{dq}{dt} = dT - R_q q - \varepsilon_2 g(T, q), \quad (2)$$

$$\begin{aligned} T(0) &= r, \\ q(t) &= s, \\ -\delta \leq t \leq 0, \end{aligned} \quad (3)$$

其中  $T$  为东太平洋海表温度,  $q$  为信风强度,  $a, b, c$  和  $d$  为正常数,  $r$  和  $s$  为初始常数,  $\delta$  为时滞时间,  $R_q$  为阻尼系数,  $\varepsilon_1 f$  和  $\varepsilon_2 g$  为扰动项. 设

$$\varepsilon_1 = C\delta,$$

$$\varepsilon_2 = D\delta,$$

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其中  $\delta$  为小参数,  $C$  和  $D$  为常数. (1) 式等号右边第一项表示耦合系统的正反馈, 第二项表示由西太平洋海岸反射波的负反馈, 而最后一项为限制异常增长的扰动项. (2) 式表示与温度异常有关的信风强度异常变化.

### 3. 模型的渐近解

我们首先按  $\delta$  展开时滞函数  $q(t - \delta)$ ,

$$q(t - \delta) = q(t) - \frac{dq(t)}{dt}\delta + \frac{1}{2} \frac{d^2q(t)}{dt^2}\delta^2 + \dots + (-1)^i \frac{1}{i!} \frac{d^i q(t)}{dt^i} \delta^i + \dots \quad (4)$$

设(1)–(3)式的解为

$$T(t, \delta) = \sum_{i=0}^{\infty} T_i(t) \delta^i, \quad (5)$$

$$q(t, \delta) = \sum_{i=0}^{\infty} q_i(t) \delta^i. \quad (6)$$

将(4)–(6)式代入(1)–(3)式, 并令  $\delta = 0$ , 我们得到(1)–(3)式的退化系统

$$\frac{dT_0}{dt} = aT_0 - bq_0, \quad (7)$$

$$c \frac{dq_0}{dt} = dT_0 - R_q q_0, \quad (8)$$

$$T(0) = r, \quad (9)$$

$$q(0) = s.$$

系统(7), (8)的特征根为

$$\lambda_{1,2} = \frac{1}{2c} [(ac - R_q) \pm \sqrt{(ac + R_q)^2 + 4bcd}]. \quad (10)$$

不难看出, 系统(7), (8)的特征根  $\lambda_{1,2}$  为异号实根. 于是系统(7), (8)的零解在相平面上为不稳定的鞍点, 它导致了原系统(1)–(4)解的不稳定性.

系统(7)–(9)的解为

$$T_0(t) = \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \exp(\lambda_1 t) + \left\{ T(0) - \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \right\} \exp(\lambda_2 t), \quad (11)$$

$$q_0(t) = \frac{(a - \lambda_1)c}{b \sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \exp(\lambda_1 t) + \frac{a - \lambda_2}{b} \left\{ T(0) - \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \right\} \exp(\lambda_2 t), \quad (12)$$

其中  $\lambda_i (i = 1, 2)$  由(10)式表示.

将(4)–(6)式代入(1)–(3)式, 对于  $\delta^i$  项的系数, 有

$$\frac{dT_1}{dt} = aT_1 - bq_1 + b \frac{dq_0}{dt} - Cf(T_0, q_0), \quad (13)$$

$$c \frac{dq_1}{dt} = dT_1 - R_q q_1 - Dg(T_0, q_0), \quad (14)$$

$$T_1(0) = q_1(0) = 0. \quad (15)$$

利用(7), (8)式, 得到(13)–(15)式的解为

$$T_1(t) = \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \left\{ bDg(T_0(t_1), q_0(t_1)) + c(a - \lambda_2) \left[ \frac{dT_0(t_1)}{dt_1} - Cf(T_0(t_1), q_0(t_1)) \right] \right\} \times \exp(\lambda_1(t - t_1)) dt_1, \quad (16)$$

$$\tau_1(t) = \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \left\{ -bDg(T_0(t_1), q_0(t_1)) - c(a - \lambda_1) \left[ \frac{dT_0(t_1)}{dt_1} - Cf(T_0(t_1), q_0(t_1)) \right] \right\} \times \exp(\lambda_2(t - t_1)) dt_1, \quad (17)$$

其中  $\lambda_i (i = 1, 2), T_0, q_0$  由(10)–(12)式表示.

将(4)–(6)式代入(1)–(3)式, 对于  $\delta^i (i = 2, 3, \dots)$  项的系数, 有

$$\frac{dT_i}{dt} = aT_i - bq_i + F_{1i}(t) + F_{2(i-1)}(t), \quad (18)$$

$$c \frac{dq_i}{dt} = dT_i - R_q q_i - G_{(i-1)}(t), \quad (19)$$

$$T_i(0) = q_i(0) = 0, \quad (20)$$

其中  $F_{1i}, F_{2i}, G_i$  为逐次已知的函数,

$$F_{1i}(t) = \frac{b}{i!} \left[ \frac{\partial^i}{\partial \delta^i} \left( \sum_{k=0}^{\infty} q_i(t-\delta) \delta^k \right) \right]_{\delta=0} \quad (i = 1, 2, \dots), \quad (21)$$

$$F_{2i}(t) = \frac{1}{i!} \left[ \frac{\partial^i}{\partial \delta^i} Cf \left( \sum_{k=0}^{\infty} T_i(t) \delta^k, \sum_{k=0}^{\infty} q_i(t) \delta^k \right) \right]_{\delta=0} \quad (i = 2, 3, \dots), \quad (22)$$

$$G_i(t) = \frac{1}{i!} \left[ \frac{\partial^i}{\partial \delta^i} Dg \left( \sum_{k=0}^{\infty} T_i(t) \delta^k, \sum_{k=0}^{\infty} q_i(t) \delta^k \right) \right]_{\delta=0} \quad (i = 2, 3, \dots). \quad (23)$$

可以依次得到线性非齐次耦合系统初值问题(18)–(20)式的解

$$T_i(t) = \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \{ bDg(T_0(t_1), q_0(t_1)) + c(a - \lambda_2) [F_{1i}(t_1) + F_{2(i-1)}(t_1)] \} \times \exp(\lambda_1(t - t_1)) dt_1 \quad (i = 2, 3, \dots), \quad (24)$$

$$\tau_i(t) = \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \{ -bg(T_0(t_1), q_0(t_1)) - c(a - \lambda_1) G_{i-1}(t_1) \} \times \exp(\lambda_2(t - t_1)) dt_1 \quad (i = 2, 3, \dots). \quad (25)$$

于是,我们得到耦合系统模型(1)–(3)式解的渐近展开式

$$\begin{aligned} T(t) &= \sum_{i=0}^m T_i(t) \delta^i + O(\delta^{m+1}) \\ &= \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \exp(\lambda_1 t) \\ &\quad + \left\{ T(0) - \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \right\} \exp(\lambda_2 t) \\ &\quad + \left[ \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \left\{ bDg(T_0(t_1), q_0(t_1)) + c(a - \lambda_2) \left[ \frac{dT_0(t_1)}{dt_1} - Cf(T_0(t_1), q_0(t_1)) \right] \right\} \right. \\ &\quad \left. \times \exp(\lambda_1(t - t_1)) dt_1 \right] \delta \\ &\quad + \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \sum_{k=2}^m \left[ \int_0^t \{ bDg(T_0(t_1), q_0(t_1)) + c(a - \lambda_2) [F_{1k}(t_1) + F_{2(k-1)}(t_1)] \} \right. \\ &\quad \left. \times \exp(\lambda_1(t - t_1)) dt_1 \right] \delta^k + O(\delta^{m+1}) \quad (0 \leq t \leq \bar{T}, 0 < \delta \ll 1), \end{aligned} \quad (26)$$

$$\begin{aligned} q(t) &= \sum_{i=0}^{\infty} q_i(t) \delta^i + O(\delta^{m+1}) \\ &= \frac{(a - \lambda_1)c}{b \sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \exp(\lambda_1 t) \\ &\quad + \frac{a - \lambda_2}{b} \left\{ T(0) - \frac{c}{\sqrt{(ac + R_q)^2 + 4bcd}} [(a - \lambda_2(0))T(0) - bq(0)] \right\} \exp(\lambda_2 t) \\ &\quad + \left[ \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \int_0^t \left\{ -bDg(T_0(t_1), q_0(t_1)) - c(a - \lambda_1) \left[ \frac{dT_0(t_1)}{dt_1} - Cf(T_0(t_1), q_0(t_1)) \right] \right\} \right. \\ &\quad \left. \times \exp(\lambda_2(t - t_1)) dt_1 \right] \delta \\ &\quad + \frac{1}{\sqrt{(ac + R_q)^2 + 4bcd}} \sum_{k=2}^m \left[ \int_0^t \{ -bDg(T_0(t_1), q_0(t_1)) - c(a - \lambda_1) G_{i-1}(t_1) \} \exp(\lambda_2(t - t_1)) dt_1 \right] \delta^k \\ &\quad + O(\delta^{m+1}) \quad (0 \leq t \leq \bar{T}, 0 < \delta \ll 1), \end{aligned} \quad (27)$$

其中  $\lambda_j, F_{ji}$  和  $G_i (j = 1, 2; i = 1, 2, \dots, m)$  分别由(10), (21)–(23)式表示,且  $\bar{T}$  为足够大的正常数.

4. 解的一致有效性

下面说明(26)和(27)式是在  $t \in [0, \bar{T}]$  上对于  $0 < \delta \ll 1$  为一致有效的渐近展开式. 令

$$T(t, \delta) = \sum_{i=0}^n T_i(t) \delta^i + R_1(t, \delta), \quad (28)$$

$$q(t, \delta) = \sum_{i=0}^n q_i(t) \delta^i + R_2(t, \delta). \quad (29)$$

将(28)和(29)式代入(1)–(3)式, 有

$$\begin{aligned} \frac{dR_1}{dt} - aR_1 + bR_2(t - \delta) + \varepsilon_1 f(R_1, R_2) &= \frac{dT}{dt} - aT + bq(t - \delta) + \varepsilon_1 f(T - \sum_{i=0}^n T_i(t) \delta^i, q - \sum_{i=0}^n q_i(t) \delta^i) \\ &\quad - \frac{d}{dt} \sum_{i=0}^n T_i(t) \delta^i - a \sum_{i=0}^n T_i(t) \delta^i + b \sum_{i=0}^n q_i(t - \delta) \delta^i \\ &= - \frac{d}{dt} \sum_{i=0}^n T_i(t) \delta^i - a \sum_{i=0}^n T_i(t) \delta^i + b \sum_{i=0}^n q_i(t - \delta) \delta^i \\ &\quad + \varepsilon_1 f(\sum_{i=0}^n T_i(t) \delta^i, \sum_{i=0}^n q_i(t) \delta^i) \\ &= - \frac{dT_0}{dt} + aT_0 - bq_0 + \left[ - \frac{dT_1}{dt} + aT_1 - bq_1 + b \frac{dq_0}{dt} - Cf(T_0, q_0) \right] \delta \\ &\quad + \sum_{i=2}^m \left[ - \frac{dT_i}{dt} + aT_i - bq_i + F_{1i}(t) + F_{2(i-1)}(t) \right] \delta^i + O(\delta^{m+1}) \\ &= O(\delta^{m+1}) \quad (t \in [0, \bar{T}], 0 < \delta \ll 1), \end{aligned} \quad (30)$$

$$\begin{aligned} c \frac{dR_2}{dt} - dR_1 + R_q R_2 + \varepsilon_2 g(R_1, R_2) &= c \frac{dq}{dt} - dT + R_q q + \varepsilon_2 g(T - \sum_{i=0}^n T_i(t) \delta^i, q - \sum_{i=0}^n q_i(t) \delta^i) \\ &\quad - c \frac{d}{dt} \sum_{i=0}^n q_i(t) \delta^i - d \sum_{i=0}^n T_i(t) \delta^i + R_q \sum_{i=0}^n q_i(t) \delta^i \\ &= - c \frac{d}{dt} \sum_{i=0}^n q_i(t) \delta^i - d \sum_{i=0}^n T_i(t) \delta^i + R_q \sum_{i=0}^n q_i(t) \delta^i \\ &\quad + \varepsilon_2 g(\sum_{i=0}^n T_i(t) \delta^i, \sum_{i=0}^n q_i(t) \delta^i) \\ &= - c \frac{dq_0}{dt} + dT_0 - R_q q_0 + \left[ - c \frac{dq_1}{dt} + dT_1 - R_q q_1 - Dg(T_0, q_0) \right] \delta \\ &\quad + \sum_{i=2}^m \left[ c \frac{dq_i}{dt} - dT_i + R_q q_i + G_{(i-1)}(t) \right] \delta^i + O(\delta^{m+1}) \\ &= O(\delta^{m+1}) \quad (t \in [0, \bar{T}], 0 < \delta \ll 1). \end{aligned} \quad (31)$$

我们还能得到

$$\begin{aligned} R_1(t) &= T(t) - \sum_{i=0}^n T_i(t) \delta^i \\ &= r - r = 0 \quad (-\delta \leq t \leq 0), \end{aligned} \quad (32)$$

$$\begin{aligned} R_2(0) &= q(0) - \sum_{i=0}^n q_i(0) \delta^i \\ &= s - s = 0 \quad (-\delta \leq t \leq 0). \end{aligned} \quad (33)$$

于是由(30)–(33)式知, 在  $t \in [0, \bar{T}]$  上对于

$0 < \delta \ll 1$ , (26)和(27)式为厄尔尼诺/拉尼娜和南方涛动模型(1)–(3)式解的一致有效的渐近展开式<sup>[21,22]</sup>.

5. 举 例

为了解释得到摄动解的一次渐近展开式(26)和(27)的精度, 现对扰动时滞模型(1)–(3)式的

相关系数取值如下:  $a = 0.97 \times 10^{-2} \text{ } ^\circ\text{C m}^2 \cdot \text{N}^{-1} \text{ a}^{-1}$ ,  $b = 1.5 \times 10^2 \text{ } ^\circ\text{C m}^2 \cdot \text{N}^{-1} \text{ a}^{-1}$ ,  $d = 1.5 \times 10^{-2} \text{ } ^\circ\text{C}^{-1} \text{ N} \cdot \text{m}^{-2} \text{ a}^{-1}$ ,  $R_q = 0.5 \text{ a}^{-1}$ ,  $r = 1 \text{ } ^\circ\text{C}$ ,  $s = 1 \times 10^{-2} \text{ N} \cdot \text{m}^{-2}$ , 并设  $c = 0, f = 0, g = T^3$ . 这时, 我们可分别得到海表温度  $T$  和信风强度  $q$  在  $\delta = 0.1$  和  $\delta = 0.05$  时的模拟解和摄动解, 结果如图 1—图 4 所示.

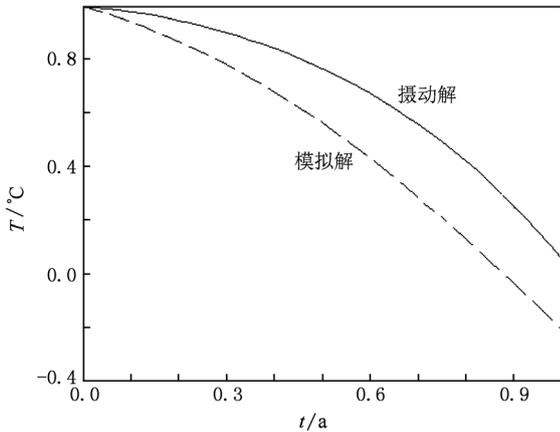


图 1  $\delta = 0.1$  时海表温度  $T$  的模拟解与摄动解

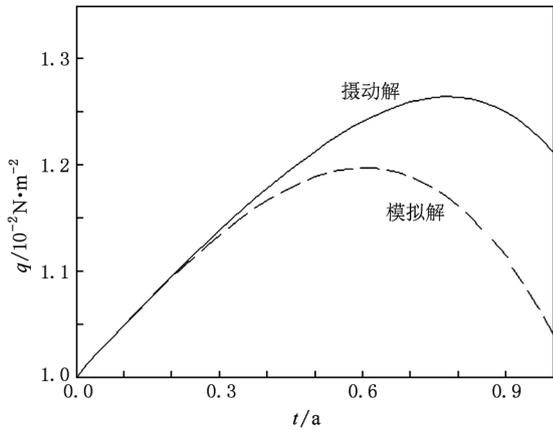


图 2  $\delta = 0.1$  时信风强度  $q$  的模拟解与摄动解

由图 1—图 4 可以看出, 用摄动方法得到的海表温度  $T$  和信风强度  $q$  具有较好的精度, 并且参数  $\delta > 0$  的值越小精度越高.

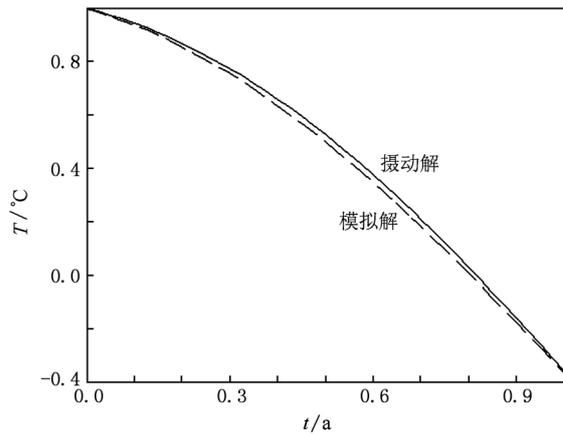


图 3  $\delta = 0.05$  时海表温度  $T$  的模拟解与摄动解

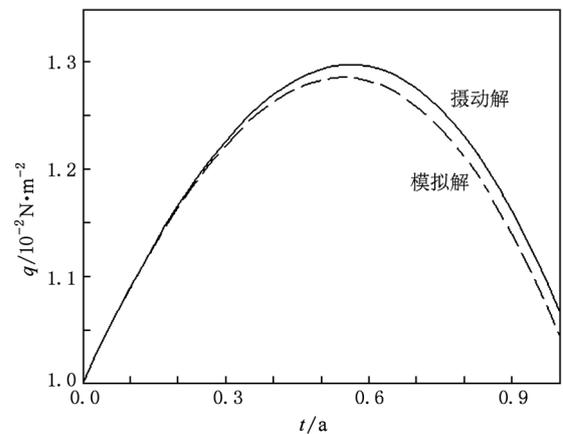


图 4  $\delta = 0.05$  时信风强度  $q$  的模拟解与摄动解

## 6. 结 论

本文研究了在赤道东太平洋海-气振子模型 (1)—(3) 式的海表温度和信风强度. 研究表明, 它们的性态在相平面上具有不稳定的鞍点. 利用解析运算对赤道东太平洋的海表温度异常和信风强度异常可作进一步定性、定量方面的性态分析和预报.

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## Asymptotic solution to the delay sea-sir oscillator for El Niño/La Niña-southern oscillation mechanism\*

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### Abstract

A class of coupled system of the El Niño/La Niña-southern oscillation mechanism is studied. Using the asymptotic analytic perturbation method and the simple and valid technique, the asymptotic expansions of solution to the El Niño/La Niña-southern oscillation model are obtained and the asymptotic behavior of solution to the corresponding problem is considered.

**Keywords:** nonlinear, asymptotic behavior, El Niño/La Niña-southern oscillation model

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