1:2 内共振及主共振情况下高架索的面内振动分析*

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考虑了由货物自重引起高架索静态构型的分段特性、高架索的倾斜角以及发送端水平运动等多种因素的影响, 建立了海上航行干货补给高架索系统面内振动的三自由度分段动力学模型. 将惯性项解耦后, 得到了 1:2 内共振与主共振同时作用时的常微分方程, 并利用多尺度法对其进行了摄动分析和数值分析, 为后续开展相关的非线性动力学研究奠定了一定的理论基础.

关键词: 分段, 高架索系统, 摆动, 多尺度法

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1 引 言

由于高架索索端运动以及集中质量位置的影响, 高架索的动力学特性非常复杂, 然而, 目前高架索系统的动力学研究非常有限. 卢永锦 [1] 分析了横向补给系统高架索的线性动力学响应特性, 并将理论结果与实验结果进行对比分析. 何学军和张良欣等 [2,3] 通过引入 δ 方程表示集中质量对系统的影响, 建立了高架索系统的连续体的非线性动力学模型, 分析了高架索系统在参数激励下的非线性动力学响应特性. 任爱娣 [4] 不考虑集中质量摆动的影响, 建立了横向补给高架索系统的三自由度非线性动力学模型, 结合 Galerkin 截断与多尺度方法对动力学模型进行了渐近分析.

本文综合文献 [4—7], 考虑了由货物自重引起高架索静态构型的分段特性、高架索与水平面之间形成的倾斜角以及发送端水平运动等多种因素的影响, 建立了集中质量摆动时三自由度分段高架索动力学模型, 三自由度分别为集中质量及其两侧分段高架索离开平衡位置的位移. 将惯性项解耦后得到了三自由度常微分方程, 并利用多尺度法对其进行了摄动分析和数值分析, 为后续开展相关的非线性动力学研究奠定了一定的理论基础.

2 控制方程的建立

货物作用在高架索的面积有限, 可以假设为一个集中质量作用于高架索. 考虑集中质量对高架索静态构型的影响, 可以将高架索系统视为由两段索以及集中质量组成. 系统的几何变形如图 1 所示.

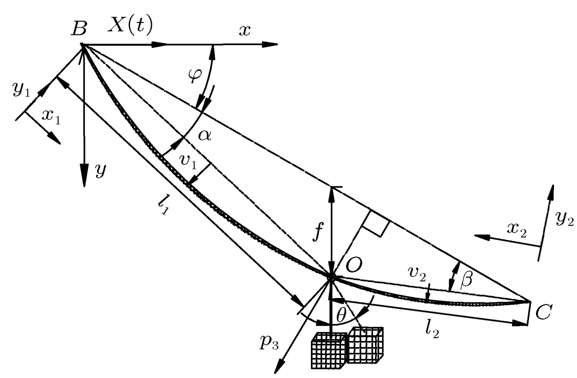

图 1　高架索系统示意图

图 1 中 B 为高架索的发送端, C 点为高架索的接收端, O 点为集中质量所在位置; l_i 为各段高架索的长度, x_i, y_i 为高架索 BO, CO 的局部坐标, φ 为高架索 BC 与水平线的夹角, α, β 分别表示由集中质量引起的高架索 BO, CO 段的附加倾角, θ 为集中质量的摆动幅角, ω 为货物的摆动角速度,

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$X(t)=x_0\sin\Omega t$ 为高架索发送端的水平位移, Ω 为激励频率, v_i 为各段高架索偏离平衡状态的动位移, p_3 为集中质量垂直于 BC 方向的动位移, f 为集中质量点沿竖直方向的静挠度. 综合考虑货物摆动角速度和摆动幅角对f的影响, f由 (1) 式确定:

$$\begin{aligned}f=&\big[\rho Agl_1l_2(l_1+l_2)\cos\varphi+2Mgl_1l_2\cos\varphi\\&+2Mgl_1l_2\tan(\theta\sin\omega t)\sin\varphi\big]\\&\times[2H(l_1+l_2)]^{-1}.\end{aligned}\tag{1}$$

由上式可见, 集中载荷的摆动将改变系统的静挠度.

基于 Rega[5] 以及 Jensen 等 [6] 的研究, 高架索系统的控制方程为

$$\begin{aligned}&\rho A\ddot v_1+\rho A\mu_1\dot v_1-N_1v''_1\\&-EA\left[\frac{1}{l_1}\int_0^{l_1}\left(y'_1v'_1+\frac12v'^2_1\right)\mathrm dx_1+\frac{X\cos\varphi}{l_1+l_2}\right]\\&\times v''_1=0,\end{aligned}\tag{2a}$$

$$\begin{aligned}&\rho A\ddot v_2+\rho A\mu_2\dot v_2-N_2v''_2\\&-EA\left[\frac{1}{l_2}\int_0^{l_2}\left(y'_2v'_2+\frac12v'^2_2\right)\mathrm dx_2+\frac{X\cos\varphi}{l_1+l_2}\right]\\&\times v''_2=0,\end{aligned}\tag{2b}$$

$$\begin{aligned}&M\ddot p_3+M\mu_3\dot p_3+\int_0^{l_1}\frac{\rho Ax_1\ddot v_1}{l_1}\mathrm dx_1\\&+EA\bigg[\frac{1}{l_1}\int_0^{l_1}\left(y'_1v'_1+\frac12v'^2_1\right)\mathrm dx_1+\frac{X\cos\varphi}{l_1+l_2}\\&\quad+\frac{N_1}{EA}\bigg]\frac{f\cos\eta_1+p_3-X\sin\eta_1}{l_1}\\&+\int_0^{l_2}\frac{\rho Ax_2\ddot v_2}{l_2}\mathrm dx_2\\&+EA\bigg[\frac{1}{l_2}\int_0^{l_2}\left(y'_2v'_2+\frac12v'^2_2\right)\mathrm dx_2+\frac{X\cos\varphi}{l_1+l_2}\\&\quad+\frac{N_2}{EA}\bigg]\frac{f\cos\eta_2+p_3}{l_1}=0.\end{aligned}\tag{2c}$$

方程 (2a),(2b) 以及 (2c) 分别为高架索BO段、CO段以及集中质量的动力学方程. 方程 (1) 和 (2) 中, ρ 为高架索的密度, A 为高架索截面面积, "$'$" 表示对坐标 x_i 的偏微分, "$\cdot$" 表示对时间的偏微分, E 为高架索的杨氏模量, μ_i 为各段高架索的阻尼系数, y_i 各段高架索在局部坐标下的静力构型, $y_i=\dfrac{\rho Ag}{2N_i}(l_ix_i-x_i^2)$, N_i 为各段高架索的初始静张力, g 为重力加速度, $H=N_1\cos(\varphi+\alpha)=N_2\cos(\varphi-\beta)$, $\dfrac{\sin\alpha}{f}=\dfrac{\cos\varphi}{l_1}$, $\dfrac{\sin\beta}{f}=\dfrac{\cos\varphi}{l_2}$, H 为高架索的水平张力, M 为系统输送货物的集中质量.

为了便于分析, 令 $\eta_1=\varphi+\alpha$, $\eta_2=\varphi-\beta$.

为了得到非线性系统的常微分方程, 利用 Galerkin 截断将连续状态变量 v_1, v_2 离散, 各段高架索的 1 阶振动模态为

$$\begin{aligned}v_1(x_1,t)=&p_1\sin\frac{\pi x_1}{l_1}+\frac{x_1}{l_1}p_3\\&+\left(1-\frac{x_1}{l_1}\right)X\sin\eta_1,\\v_2(x_2,t)=&p_2\sin\frac{\pi x_2}{l_2}+\frac{x_2}{l_2}p_3.\end{aligned}\tag{3}$$

将 (3) 式代入 (2a) 和 (2b) 式, 利用 Galerkin 积分, 再将其结果代入 (3c) 式, 解耦其中的惯性项 $\ddot v_1$ 和 $\ddot v_2$ 得到:

$$\begin{aligned}&\ddot p_1+\frac{2\ddot p_3}{\pi}+\mu_1\dot p_1+\frac{2\sin\eta\ddot X}{\pi}\\&+\frac{N_1\pi^2p_1}{\rho Al_1^2}+\frac{E\pi^4p_1^3}{4\rho l_1^4}\\&+\frac{2EA\pi gp_1^2}{N_1l_1^2}+\frac{E\pi^2\cos\varphi}{(l_1+l_2)\rho l_1^2}Xp_1\\&+\frac{E\pi^2(p_3-\sin\eta_1X)^2p_1}{2\rho l_1^4}=0,\end{aligned}\tag{4a}$$

$$\begin{aligned}&\ddot p_2+\frac{2\ddot p_3}{\pi}+\mu_2\dot p_2+\frac{2\mu_2\dot p_3}{\pi}\\&+\frac{E\pi^4p_2^3}{4\rho l_2^4}+\frac{2EA\pi gp_2^2}{N_2l_2^2}+\frac{N_2\pi^2p_2}{\rho Al_2^2}\\&+\frac{E\pi^2\cos\varphi}{(l_1+l_2)\rho l_2^2}Xp_2+\frac{E\pi^2p_2p_3^2}{2\rho l_2^4}=0,\end{aligned}\tag{4b}$$

$$\begin{aligned}&M\ddot p_3+M\mu_3\dot p_3+\int_0^{l_1}\frac{\rho Ax_1\ddot v_1}{l_1}\mathrm dx_1\\&+EA\bigg[\frac{\cos\varphi X}{l_1+l_2}+\frac{1}{l_1}\int_0^{l_1}\left(y'_1v'_1+\frac12v'^2_1\right)\mathrm dx_1\\&\quad+\frac{N_1}{EA}\bigg]\frac{f\cos\eta+p_3-X\sin\eta}{l_1}\\&+\int_0^{l_2}\frac{\rho Ax_2\ddot v_2}{l_2}\mathrm dx_2+EA\bigg[\frac{\cos\varphi X}{l_1+l_2}\\&\quad+\frac{1}{l_2}\int_0^{l_2}(y'_2v'_2+\frac12v'^2_2)\mathrm dx_2+\frac{N_2}{EA}\bigg]\\&\times\frac{f\cos\xi+p_3}{l_1}=0.\end{aligned}\tag{4c}$$

将方程 (4c) 代入方程 (4a) 及 (4b), 将惯性项 $\ddot p_3$ 解耦, 可得高架索系统的三自由度常微分方程:

$$\begin{aligned}&\ddot p_1+\mu_1\dot p_1+\sum_{i=1}^{3}\alpha_{1i}p_i+\frac{2\sin\eta\ddot X}{\pi}\\&+\frac{2EA\pi gp_1^2}{N_1l_1^2}+\frac{E\pi^2}{2\rho l_1^2}\bigg[\frac{\pi^2p_1^3}{2l_1^2}+\frac{2\cos\varphi}{l_1+l_2}Xp_1\end{aligned}$$

$$
\begin{aligned}
&+\frac{(p_3-\sin\eta_1 X)^2 p_1}{l_1^2}\Bigg]-\frac{2F_3}{\pi}=0,\\
&\ddot{p}_2+\mu_2\dot{p}_2+\sum_{i=1}^{3}\alpha_{2i}p_i+\frac{2EA\pi g p_2^2}{N_2 l_2^2}\\
&+\frac{E\pi^2}{2\rho l_2^4}\left(\frac{\pi^2 p_2^3}{2}+p_2 p_3^2\right)+\frac{E\pi^2\cos\varphi}{(l_1+l_2)\rho l_2^2}Xp_2\\
&-\frac{2F_3}{\pi}=0,\\
&\ddot{p}_3+\mu_3\dot{p}_3+\sum_{i=1}^{3}\alpha_{3i}p_i+F_3=0. \qquad (5)
\end{aligned}
$$

其中

$$
\begin{aligned}
\alpha_{11}=&\frac{N_1\pi^2}{\rho A l_1^2}-\frac{2\alpha_{31}}{\pi},\ \alpha_{12}=\frac{2\alpha_{32}}{\pi},\ \alpha_{21}=\frac{2\alpha_{31}}{\pi},\\
\alpha_{22}=&\frac{N_2\pi^2}{\rho A l_2^2}-\frac{2\alpha_{32}}{\pi},\quad \alpha_{13}=\alpha_{23}=\frac{2\alpha_{33}}{\pi},\\
\alpha_{31}=&\frac{2E\rho A^2 g f\cos\eta_1}{\pi}-\frac{\pi N_1}{M l_1},\\
\alpha_{32}=&\frac{2E\rho A^2 g f\cos\eta_2}{\pi}-\frac{\pi N_2}{M l_2},\\
\alpha_{33}=&\frac{N_1}{M l_1}+\frac{N_2}{M l_2},\\
F_3=&\frac{EA}{M}\sum_{i=1}^{2}\Bigg\{\Bigg[\frac{fX\cos\eta_i\cos\varphi}{l_i(l_1+l_2)}\\
&+\frac{X\cos\varphi(p_3-\pi p_i)}{l_i(l_1+l_2)}+\frac{2\rho A g p_i(p_3-\pi p_i)}{\pi l_i N_i}\\
&+\frac{f\cos\eta_i}{4l_i^2}(2p_3^2+\pi^2 p_i^2)\Bigg]\\
&+\sum_{i=1}^{2}\frac{fN_i\cos\eta_i}{EAl_i}\\
&+\sum_{i=1}^{2}\left(\frac{p_3^2}{2l_i^3}-\frac{\pi p_i p_3^2}{2l_i^3}+\frac{\pi^2 p_i^2 p_3}{4l_i^3}-\frac{\pi^3 p_i^3}{4l_i^3}\right)\\
&+\frac{\sin^2\eta_1 X^2}{2l_1^3}(f\cos\eta_1-\pi p_1+3p_3-X\sin\eta_1)\\
&-\frac{X\sin\eta_1}{l_1}\left(\frac{fp_3\cos\eta_1}{l_1^2}+\frac{X\cos\varphi}{l_1+l_2}\right)\\
&-X\sin\eta_1\Bigg(\frac{N_1}{EAl_1}+\frac{2\rho A g p_1}{\pi l_1 N_1}+\frac{\pi^2 p_1^2}{4l_1^3}\\
&-\frac{\pi p_1 p_3}{l_1^3}+\frac{3p_3^2}{2l_1^3}\Bigg)\Bigg\}.
\end{aligned}
$$

3 摄动分析

方程 (5) 为具有刚度项耦合的常微分方程, 由多尺度法 [8], 设方程的一阶解为

$$
p_n=p_{n0}(T_0,T_1)+\varepsilon p_{n1}(T_0,T_1)\ (n=1,2,3), \qquad (6)
$$

其中 $T_0=t, T_1=\varepsilon t$, ε 为小量标志.

考虑 1 : 2 内共振 ($\omega_1\approx 2\omega_3$) 及外激励频率 Ω 接近于 ω_1 的情况.

令 $\omega_1=2\omega_3+\varepsilon\sigma_3$.

引入解谐参数 σ_3, 并令

$$
2\omega_1 T_0=\omega_3 T_0-\varepsilon\sigma_3 T_0=\omega_3 T_0-\sigma_3 T_1.
$$

引入解谐参数 σ_1, 再令 $\Omega=\omega_1+\varepsilon\sigma_1$.

将方程 (6) 代入方程 (5), 由方程 ε^0, ε^1 项系数相等, 可得

$$
D_0^2 p_{n0}+\alpha_{n1}p_{10}+\alpha_{n2}p_{20}+\alpha_{n3}p_{30}=0, \qquad (7)
$$

$$
\begin{aligned}
&D_0^2 p_{n1}+\alpha_{n1}p_{11}+\alpha_{n2}p_{21}+\alpha_{n3}p_{31}\\
=&F_{n1}(p_{10},p_{20},p_{30}), \qquad (8)
\end{aligned}
$$

其中 $n=1, 2, 3$.

方程 (7) 的解可以表示为

$$
\begin{aligned}
p_{10}=&\sum_{n=1}^{2}c_n A_{2n-1}(T_1)\exp(\mathrm{i}\omega_{2n-1}T_0)+\mathrm{cc},\\
p_{20}=&\sum_{n=2}^{3}c_{n+1}A_n(T_1)\exp(\mathrm{i}\omega_n T_0)+\mathrm{cc},\\
p_{30}=&\sum_{n=1}^{3}A_n(T_1)\exp(\mathrm{i}\omega_n T_0)+\mathrm{cc}, \qquad (9)
\end{aligned}
$$

其中

$$
\begin{aligned}
c_1=&\frac{\alpha_{13}}{\omega_1^2-\alpha_{11}},\\
c_2=&\frac{\omega_3^2\alpha_{13}-\alpha_{13}\alpha_{22}+\alpha_{12}\alpha_{23}}{\omega_3^4-\omega_3^2\alpha_{11}-\omega_3^2\alpha_{22}-\alpha_{12}\alpha_{21}+\alpha_{11}\alpha_{22}},\\
c_3=&\frac{\alpha_{23}}{\omega_2^2-\alpha_{22}},\\
c_4=&\frac{\omega_3^2\alpha_{23}-\alpha_{11}\alpha_{23}+\alpha_{13}\alpha_{21}}{\omega_3^4-\omega_3^2\alpha_{11}-\omega_3^2\alpha_{22}-\alpha_{12}\alpha_{21}+\alpha_{11}\alpha_{22}}.
\end{aligned}
$$

ω_n^2 为如下方程的解

$$
\begin{aligned}
&\omega_n^6-(\alpha_{11}+\alpha_{22}+\alpha_{33})\omega_n^4-(\alpha_{13}\alpha_{31}\\
&-\alpha_{11}\alpha_{22}+\alpha_{23}\alpha_{32}-\alpha_{11}\alpha_{33}-\alpha_{22}\alpha_{33}\\
&+\alpha_{12}\alpha_{21})\omega_n^2+\alpha_{13}\alpha_{22}\alpha_{31}+\alpha_{11}\alpha_{23}\alpha_{32}\\
&-\alpha_{11}\alpha_{22}\alpha_{33}-\alpha_{12}\alpha_{23}\alpha_{31}-\alpha_{13}\alpha_{21}\alpha_{32}\\
&+\alpha_{12}\alpha_{21}\alpha_{33}=0. \qquad (10)
\end{aligned}
$$

将方程 (9) 代入方程 (8), 其特性形式可以表示为

$$p_{n1}=\sum_{k=1}^{3}q_{nk}\exp(\mathrm{i}\omega_k T_0)+\mathrm{cc}\quad(n=1,2,3). \tag{11}$$

将方程 (11) 代入方程 (8), 方程两侧 $\exp(\mathrm{i}\omega_1T_0)$, $\exp(\mathrm{i}\omega_2T_0)$, $\exp(\mathrm{i}\omega_3T_0)$ 项系数相等, 可得

$$\begin{aligned}&\left(\alpha_{11}-\omega_n^2\right)q_{1n}+\alpha_{12}q_{2n}+\alpha_{13}q_{3n}=R_{1n},\\&\alpha_{21}q_{1n}+\left(\alpha_{22}-\omega_n^2\right)q_{2n}+\alpha_{23}q_{3n}=R_{2n},\\&\alpha_{31}q_{1n}+\alpha_{32}q_{2n}+\left(\alpha_{33}-\omega_n^2\right)q_{3n}=R_{3n},\end{aligned}\tag{12}$$

其中 $n=1,2,3$.

$$\begin{aligned}R_{11}=&\mathrm{i}c_1\omega_1\left(A'_1+\mu_1A_1\right)-\frac{2}{\pi}R_{31}\\&+\frac{\mathrm{i}\omega_1^2\sin\eta\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}x_0}{\pi}\\&+\frac{E\pi^2}{4\rho l_1^4}\Big[c_1\sin^2\eta x_0^2\left(A_1-\frac{\mathrm{e}^{2\mathrm{i}T_0\sigma_1}\bar{A}_1}{2}\right)\\&+4\mathrm{i}c_1\sin\eta x_0\Big(\mathrm{e}^{\mathrm{i}T_0\sigma_1}A_1\bar{A}_1-\frac{\mathrm{e}^{-\mathrm{i}T_0\sigma_1}}{2}A_1^2\Big)\\&+4\mathrm{i}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}\sin\eta x_0c_2A_3\bar{A}_3+4c_1A_1A_2\bar{A}_2\\&+4\Big(c_1+2c_2+\frac{3\pi^2c_1c_2^2}{2}\Big)A_1A_3\bar{A}_3\\&+6\Big(c_1+\frac{\pi^2c_1^3}{2}\Big)\bar{A}_1A_1^2\Big]\\&+\frac{2EAg\pi c_2^2\,\mathrm{e}^{-\mathrm{i}T_0\sigma_3}}{l_1^2N_1}A_3^2\\&+\frac{i\omega_1^2x_0\sin\eta\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}}{\pi}\omega_1^2,\\R_{13}=&\mathrm{i}c_2\omega_3\left(A'_3+\mu_1A_3\right)-\frac{2}{\pi}R_{33}\\&-\frac{\mathrm{i}E\pi^2c_2\cos\varphi\,\mathrm{e}^{\mathrm{i}T_0(\sigma_1+\sigma_3)}x_0}{2\rho l_1^2(l_1+l_2)}\bar{A}_3\\&+\frac{E\pi^2}{4\rho l_1^4}\Big[2\mathrm{i}\sin\eta x_0(c_1+c_2)(\mathrm{e}^{\mathrm{i}T_0\sigma_1}A_3\bar{A}_1\\&-\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_1A_3)+c_2\sin^2\eta x_0^2A_3\\&+4\left(2c_1+c_2+\frac{3\pi^2c_1^2c_2}{2}\right)A_1\bar{A}_1A_3\\&+4c_2A_2\bar{A}_2A_3+6\left(c_2+\frac{\pi^2c_2^3}{2}\right)\bar{A}_3A_3^2\\&+\frac{4EAg\pi c_1c_2\,\mathrm{e}^{-\mathrm{i}T_0\sigma_3}}{l_1^2N_1}A_1\bar{A}_3\Big];\end{aligned}\tag{13a}$$

$$\begin{aligned}R_{22}=&\mathrm{i}c_3\omega_2(A'_2+\mu_2A_2)-\frac{2R_{32}}{\pi}\\&+\frac{E\pi^2}{\rho l_2^4}\Big[c_3A_1\bar{A}_1A_2+\Big(c_3+2c_4\\&+\frac{3\pi^2c_3c_4^2}{2}\Big)A_3\bar{A}_3A_2+\frac{3}{4}\left(2c_3+\pi^2c_3^3\right)A_2^2\bar{A}_2,\\R_{23}=&\mathrm{i}c_4\omega_3(A'_3+\mu_2A_3)-\frac{2R_{33}}{\pi}\\&-\frac{\mathrm{i}E\pi^2c_4\cos\varphi x_0\bar{A}_3}{2\rho l_2^2(l_1+l_2)}\,\mathrm{e}^{\mathrm{i}T_0(\sigma_1+\sigma_3)}\\&+\frac{E\pi^2}{\rho l_2^4}\Big[c_4A_1\bar{A}_1A_3+\Big(2c_3+c_4\\&+\frac{3\pi^2c_3^2c_4}{2}\Big)A_2\bar{A}_2A_3\\&+\frac{3}{4}\left(2c_4+\pi^2c_4^3\right)A_3^2\bar{A}_3\Big];\end{aligned}\tag{13b}$$

$$\begin{aligned}R_{31}=&\mathrm{i}\omega_1(A'_1+\mu_3A_1)+r_{11}\left(A_1-\frac{\bar{A}_1}{2}\,\mathrm{e}^{2\mathrm{i}T_0\sigma_1}\right)\\&+\mathrm{i}r_{21}\left(A_1\bar{A}_1\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}-\frac{A_1^2}{2}\,\mathrm{e}^{-\mathrm{i}T_0\sigma_1}\right)\\&+\mathrm{i}r_{31}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}A_3\bar{A}_3+\frac{\mathrm{i}EA\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}x_0}{2M}\\&\times\left(\frac{\sin\eta_1N_1}{EAl_1}-\sum_{i=1}^{2}\frac{f\cos\varphi\cos\eta_i}{l_i(l_1+l_2)}\right)\\&+\frac{3\mathrm{i}EA\sin^3\eta_1x_0^3}{16Ml_1^3}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}+r_{41}\,\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_3^2\\&+\frac{3\mathrm{i}EA\sin\eta_1x_0A_2\bar{A}_2}{2Ml_1^3}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}+r_{51}A_1^2\bar{A}_1\\&+r_{61}A_1A_2\bar{A}_2+r_{71}A_1A_3\bar{A}_3,\\R_{32}=&\mathrm{i}\omega_2(A'_2+\mu_3A_2)+\frac{3EA\sin^2\eta_1}{4Ml_1^3}x_0^2A_2\\&+\frac{\mathrm{i}EA(\pi c_1-3)\sin\eta_1x_0}{2Ml_1^3}\Big(A_1A_2\,\mathrm{e}^{-\mathrm{i}T_0\sigma_1}\\&-A_2\bar{A}_1\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}\Big)+r_{52}A_2^2\bar{A}+r_{62}A_1\bar{A}_1A_2\\&+r_{72}A_3\bar{A}_3A_2,\\R_{33}=&\mathrm{i}\omega_3(A'_3+\mu_3A_3)+r_{12}A_3\\&+\mathrm{i}r_{13}\,\mathrm{e}^{\mathrm{i}T_0(\sigma_1+\sigma_3)}\bar{A}_3\\&+\mathrm{i}r_{22}\left(\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_1A_3-\mathrm{e}^{\mathrm{i}T_0\sigma_1}A_3\bar{A}_1\right)\\&+r_{32}\,\mathrm{e}^{\mathrm{i}T_0\sigma_3}A_1\bar{A}_3+r_{81}A_1\bar{A}_1A_3\\&+r_{82}A_2\bar{A}_2A_3+r_{91}A_3^2\bar{A}_3.\end{aligned}\tag{13c}$$

其中

$$\begin{aligned}r_{1i}=&\frac{EA\sin^2\eta_1x_0^2(3-\pi c_i)}{4Ml_1^3}\quad(i=1,2);\\r_{13}=&\frac{EAx_0}{2M}\left[\frac{\cos\varphi}{l_1+l_2}\sum_{i=1}^{2}\frac{\pi c_{2i}-1}{l_i}\right.\\&\left.+\sin\eta_1\left(\frac{f\cos\eta_1}{l_1^3}+\frac{2\rho Agc_2}{\pi l_1N_1}\right)\right];\end{aligned}$$

$$r_{2k}=\frac{EA\sin\eta_1x_0}{2Ml_1^3}\Big[\pi(3-k)c_1+\pi(k-1)c_2-3-\frac{\pi^2c_1^{3-k}c_2^{k-1}}{2}\Big]\ (k=1,2);$$

$$r_{31}=\frac{EA\sin\eta_1x_0}{2Ml_1^3}\left(3-2\pi c_2+\frac{\pi^2c_2^2}{2}\right);$$

$$r_{32}=\frac{EA}{M}\left[\sum_{i=1}^{2}\frac{f\cos\eta_i}{l_i^3}+\frac{f\cos\eta_1\pi^2c_1c_2}{2l_1^3}+2\rho Ag\left(\frac{c_1+c_2}{\pi l_1N_1}+\frac{c_4}{\pi l_2N_2}-\frac{2c_1c_2}{l_1N_1}\right)\right];$$

$$r_{41}=\frac{EA}{M}\sum_{i=1}^{2}\left[\frac{f\cos\eta_i(2+\pi^2c_{2i}^2)}{4l_i^3}+\frac{2\rho Ag(c_{2i}-\pi c_{2i}^2)}{\pi l_iN_i}\right];$$

$$r_{5i}=\frac{3EA}{2M}\left(\sum_{k=1}^{2}\frac{1}{l_k^3}-\frac{\pi c_{2i-1}}{l_i^3}+\frac{\pi^2c_{2i-1}^2}{2l_i^3}-\frac{\pi^3c_{2i-1}^3}{2l_i^3}\right);$$

$$r_{6i}=\frac{EA}{M}\left(\sum_{k=1}^{2}\frac{3}{l_k^3}-\frac{\mathrm{i}\pi c_1}{l_1^3}-\frac{(3-\mathrm{i})\pi c_3}{2l_2^3}+\frac{\pi^2c_{5-2i}^2}{2l_{3-\mathrm{i}}^3}\right);$$

$$r_{7i}=\frac{EA}{M}\left(\sum_{k=1}^{2}\frac{6-4\pi c_{2k}+\pi^2c_{2k}^2}{2l_k^3}-\frac{\pi c_{2i-1}}{l_i^3}+\frac{\pi^2c_{2i-1}c_{2i}}{l_i^3}-\frac{3\pi^3c_{2i-1}c_{2i}^2}{2l_i^3}\right);$$

$$r_{8i}=\frac{EA}{M}\left[\sum_{k=1}^{2}\frac{3-\pi c_{2k}}{l_k^3}-\frac{2\pi c_{2i-1}}{l_i^3}+\frac{\pi^2}{l_i^3}\left(\frac{c_{2i-1}^2}{2}+c_{2i-1}c_{2i}-\frac{3\pi c_{2i-1}^2c_{2i}}{2}\right)\right];$$

$$r_{9i}=\frac{3EA}{2M}\sum_{k=1}^{2}\frac{1-\pi c_{2k}+\pi^2c_{2i}^2-\pi^3c_{2i}^3}{l_k^3}.$$

方程 (12) 的可解性条件 [8] 为

$$B_{1n}R_{1n}+B_{2n}R_{2n}+B_{3n}R_{3n}=0, \tag{14}$$

其中 $B_{1n}=\alpha_{22}\alpha_{31}-\alpha_{21}\alpha_{32}-\alpha_{31}\omega_n^2$, $B_{2n}=\alpha_{11}\alpha_{32}-\alpha_{12}\alpha_{31}-\alpha_{32}\omega_n^2$, $B_{3n}=\alpha_{12}\alpha_{21}-(\omega_n^2-\alpha_{11})(\omega_n^2-\alpha_{22})$ $(n=1, 2, 3)$. 将方程 (13a), (13b) 和 (13c) 代入方程 (14), 可得系统的平均方程:

$$\begin{aligned}
&\mathrm{i}\omega_1\left(b_{11}A'_1+b_{21}A_1\right)+b_{31}\left(A_1-\frac{\mathrm{e}^{2\mathrm{i}T_0\sigma_1}}{2}\bar{A}_1\right)\\
&+\mathrm{i}b_{41}\left(\mathrm{e}^{\mathrm{i}T_0\sigma_1}\bar{A}_1A_1-\frac{\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_1^2}{2}\right)\\
&+\mathrm{i}b_{51}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}\bar{A}_3A_3+b_{61}\bar{A}_1A_1^2+b_{71}A_2\bar{A}_2A_1\\
&+b_{81}A_3\bar{A}_3A_1+\mathrm{i}\bar{b}_1\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}\bar{A}_2A_2+\bar{b}_2\,\mathrm{e}^{-\mathrm{i}T_0\sigma_3}A_3^2\\
&+\mathrm{i}b_{91}\,\mathrm{e}^{\mathrm{i}T_0\sigma_1}=0,\\
&\mathrm{i}\omega_2\left(b_{12}A'_2+b_{22}A_2\right)+b_{32}A_2\\
&+\mathrm{i}b_{42}\left(\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_1-\mathrm{e}^{\mathrm{i}T_0\sigma_1}\bar{A}_1\right)A_2\\
&+b_{52}A_1\bar{A}_1A_2+b_{62}\bar{A}_2A_2^2+b_{72}A_3\bar{A}_3A_2=0,\\
&\mathrm{i}\omega_3\big(b_{13}A'_3+b_{23}A_3\big)+b_{33}A_3\\
&+\mathrm{i}b_{43}\,\mathrm{e}^{\mathrm{i}T_0(\sigma_1+\sigma_3)}\bar{A}_3\\
&+b_{53}\,\mathrm{e}^{\mathrm{i}T_0\sigma_3}A_1\bar{A}_3+\mathrm{i}b_{63}\big(\mathrm{e}^{\mathrm{i}T_0\sigma_1}\bar{A}_1A_3\\
&-\mathrm{e}^{-\mathrm{i}T_0\sigma_1}A_1A_3\big)+b_{73}A_1\bar{A}_1A_3\\
&+b_{83}A_2\bar{A}_2A_3+b_{93}\bar{A}_3A_3^2=0.
\end{aligned}\tag{15}$$

令 $A_n=\dfrac{1}{2}a_n\exp(\mathrm{i}\theta_n)\quad(n=1,2,3)$, 代入方程 (15), 将实虚部分离, 得到平均方程的极坐标形式:

$$\begin{aligned}
&-b_{11}\omega_1a'_1\\
=&\left(b_{21}\omega_1+\frac{b_{31}}{2}\sin2\gamma_1\right)a_1+\frac{b_{41}}{4}\cos\gamma_1a_1^2\\
&+\frac{1}{2}\cos\gamma_1\bar{b}_1a_2^2+\frac{1}{2}\left(b_{51}\cos\gamma_1-\bar{b}_2\sin\gamma_2\right)a_3^2\\
&+2b_{91}\cos\gamma_1,\\
&b_{11}\omega_1a_1\theta'_1\\
=&b_{31}\left(1-\frac{1}{2}\cos2\gamma_1\right)a_1+\frac{3b_{41}}{4}\sin\gamma_1a_1^2\\
&+\frac{1}{2}\left(b_{51}\sin\gamma_1+\bar{b}_2\cos\gamma_2\right)a_3^2+\frac{b_{61}}{4}a_1^3\\
&+\frac{b_{71}}{4}a_2^2a_1+\frac{b_{81}}{4}a_3^2a_1+\bar{b}_1\sin\gamma_1a_2^2+2b_{91}\sin\gamma_1,\\
&b_{12}\omega_2a'_2=-b_{22}\omega_2a_2,\\
&b_{12}\omega_2a_2\theta'_2=b_{32}a_2-b_{42}\sin\gamma_1a_1a_2+\frac{b_{52}}{4}a_1^2a_2\\
&\qquad+\frac{b_{62}}{4}a_2^3+\frac{b_{72}}{4}a_3^2a_2,\\
&-b_{13}\omega_3a'_3=b_{23}\omega_3a_3+b_{43}a_3\cos(\gamma_2-\gamma_1)\\
&\qquad+\frac{b_{53}}{2}\sin\gamma_1a_1a_3,\\
&b_{13}\omega_3a_3\theta'_3=\left[b_{33}-b_{43}\sin(\gamma_2-\gamma_1)\right]a_3\\
&\qquad+\left(\frac{b_{53}}{2}\cos\gamma_2+b_{63}\sin\gamma_1\right)a_1a_3\\
&\qquad+\frac{b_{73}}{4}a_1^2a_3+\frac{b_{83}}{4}a_2^2a_3+\frac{b_{93}}{4}a_3^3,
\end{aligned}\tag{16}$$

其中 $\gamma_1=\theta_1-\sigma_1T_1$, $\gamma_2=\theta_1-2\theta_3+\sigma_3T_1$.

由系统稳态解存在条件 $a'_1 = a'_2 = a'_3 = \gamma'_1 = \gamma'_2 = 0$, 可得系统的频响方程:

$$\begin{aligned}
&\left(b_{21}\omega_1 + \frac{b_{31}}{2}\sin 2\gamma_1\right)a_1 + \frac{b_{41}}{4}\cos\gamma_1 a_1^2 \\
&+ \frac{1}{2}\cos\gamma_1 \bar{b}_1 a_2^2 + \frac{1}{2}\left(b_{51}\cos\gamma_1 - \bar{b}_2\sin\gamma_2\right)a_3^2 \\
&+ 2b_{91}\cos\gamma_1 = 0, \\
&b_{23}\omega_3 a_3 + b_{43}a_3\cos(\gamma_2-\gamma_1) + \frac{b_{53}}{2}\sin\gamma_2 a_1 a_3 = 0, \\
&b_{11}\omega_1 a_1\sigma_1 = b_{31}\left(1 - \frac{1}{2}\cos 2\gamma_1\right)a_1 \\
&+ \frac{3b_{41}}{4}\sin\gamma_1 a_1^2 + \frac{1}{2}\left(b_{51}\sin\gamma_1 + \bar{b}_2\cos\gamma_2\right)a_3^2 \\
&+ \frac{b_{61}}{4}a_1^3 + \frac{b_{71}}{4}a_2^2 a_1 + \frac{b_{81}}{4}a_3^2 a_1 \\
&+ \bar{b}_1\sin\gamma_1 a_2^2 + 2b_{91}\sin\gamma_1, \\
&b_{13}\omega_3 a_3(\sigma_1+\sigma_3) = 2\Big\{\left[b_{33} - b_{43}\sin(\gamma_2-\gamma_1)\right]a_3 \\
&+ \left(\frac{b_{53}}{2}\cos\gamma_2 + b_{63}\sin\gamma_1\right)a_1 a_3 + \frac{b_{73}}{4}a_1^2 a_3 \\
&+ \frac{b_{83}}{4}a_2^2 a_3 + \frac{b_{93}}{4}a_3^3\Big\}.
\end{aligned} \tag{17}$$

其中

$$\begin{aligned}
b_{11} &= \left(c_1 - \frac{2}{\pi}\right)B_{11} + B_{31}; \\
b_{21} &= B_{11}c_1\mu_1 + \left(B_{31} - \frac{2}{\pi}B_{11}\right)\mu_3; \\
b_{31} &= \frac{B_{11}E\pi^2 c_1\sin^2\eta_1}{4\rho l_1^4}x_0^2 + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{11}; \\
b_{41} &= \frac{B_{11}E\pi^2 c_1\sin\eta_1}{\rho l_1^4}x_0 + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{21}; \\
b_{51} &= \frac{B_{11}E\pi^2 c_2\sin\eta_1}{\rho l_1^4}x_0 + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{31}; \\
b_{61} &= \frac{3B_{11}E\pi^2(2c_1 + \pi^2 c_1^3)}{4\rho l_1^4} \\
&\quad + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{51}; \\
b_{71} &= \frac{B_{11}E\pi^2 c_1}{\rho l_1^4} + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{61}; \\
b_{81} &= \frac{B_{11}E\pi^2}{\rho l_1^4}\left(c_1 + 2c_2 + \frac{3\pi^2 c_1 c_2^2}{2}\right) \\
&\quad + \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{71}; \\
b_{91} &= \frac{B_{11}\omega_1^2\sin\eta_1 x_0}{\pi} + \left(B_{31} - \frac{2}{\pi}B_{11}\right) \\
&\quad \times\left\{\frac{3EA\sin^3\eta_1 x_0^3}{16Ml_1^3} + \frac{EAx_0}{2M}\left(\frac{\sin\eta_1 N_1}{EAl_1}\right.\right. \\
&\quad \left.\left. - \sum_{i=1}^{2}\frac{f\cos\varphi\cos\eta_i}{l_i(l_1+l_2)}\right)\right\}; \\
\bar{b}_1 &= \left(B_{31} - \frac{2}{\pi}B_{11}\right)\frac{3EA\sin\eta_1 x_0}{2Ml_1^3}; \\
\bar{b}_2 &= \left(B_{31} - \frac{2}{\pi}B_{11}\right)r_{41} + \frac{2B_{11}EAg\pi c_2^2}{l_1^2 N_1}; \\
b_{12} &= \left(c_1 - \frac{2}{\pi}\right)B_{22} + B_{32}; \\
b_{22} &= B_{22}c_3\mu_2 + \left(B_{32} - \frac{2}{\pi}B_{22}\right)\mu_3; \\
b_{32} &= \left(B_{32} - \frac{2}{\pi}B_{22}\right)\frac{3EA\sin^2\eta_1}{4Ml_1^3}x_0^2; \\
b_{42} &= \left(B_{32} - \frac{2}{\pi}B_{22}\right)\frac{EA(\pi c_1 - 3)\sin\eta_1}{2Ml_1^3}x_0; \\
b_{52} &= \frac{B_{11}E\pi^2 c_3}{\rho l_2^4} + \left(B_{32} - \frac{2}{\pi}B_{22}\right)r_{62}; \\
b_{62} &= \frac{3B_{11}E\pi^2(2c_3 + \pi^2 c_3^3)}{4\rho l_2^4} \\
&\quad + \left(B_{32} - \frac{2}{\pi}B_{22}\right)r_{52}; \\
b_{72} &= \frac{B_{11}E\pi^2}{\rho l_2^4}\left(c_3 + 2c_4 + \frac{3\pi^2 c_3 c_4^2}{2}\right) \\
&\quad + \left(B_{32} - \frac{2}{\pi}B_{22}\right)r_{72}; \\
b_{13} &= B_{13}c_2 + B_{23}c_4 + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]; \\
b_{23} &= B_{13}c_2\mu_1 + B_{23}c_4\mu_2 \\
&\quad + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]\mu_3; \\
b_{33} &= B_{13}c_2\sin^2\eta_1 x_0^2 + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]r_{12}; \\
b_{43} &= -\sum_{i=1}^{2}\frac{B_{i3}E\pi^2 c_{2i}\cos\varphi x_0}{2\rho l_i^2(l_1+l_2)} \\
&\quad + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]r_{13}; \\
b_{53} &= \frac{4B_{13}EAg\pi c_1 c_2}{l_1^2 N_1} + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]r_{32}; \\
b_{63} &= \frac{B_{13}E\pi^2 x_0\sin\eta_1(c_1+c_2)}{2\rho l_1^4} \\
&\quad + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]r_{22}; \\
b_{73} &= \frac{B_{13}E\pi^2}{\rho l_1^4}\left(2c_1 + c_2 + \frac{3}{2}\pi^2 c_1^2 c_2\right) \\
&\quad + \frac{B_{23}E\pi^2 c_4}{\rho l_2^4} + \left[B_{33} - \frac{2}{\pi}(B_{13}+B_{23})\right]r_{81};
\end{aligned}$$

$$
\begin{aligned}
b_{83} =& \frac{B_{13}E\pi^2 c_2}{\rho l_1^4} + \frac{B_{23}E\pi^2}{\rho l_2^4}\left(2c_3 + c_4 + \frac{3}{2}\pi^2 c_3^2 c_4\right) \\
& + \left[B_{33} - \frac{2}{\pi}(B_{13} + B_{23})\right] r_{82}; \\
b_{93} =& \frac{3B_{13}E\pi^2}{4\rho l_1^4}\left(2c_2 + \pi^2 c_2^3\right) \\
& + \frac{3B_{23}E\pi^2}{4\rho l_2^4}\left(2c_4 + \pi^2 c_4^3\right) \\
& + \left[B_{33} - \frac{2}{\pi}(B_{13} + B_{23})\right] r_{91}.
\end{aligned}
$$

由于 (17) 式中既有平方项, 又有立方项, 多个非线性项使得这个方程组非常复杂, 因此我们采取数值方法对频响曲线进行分析.

4 频响曲线数值分析

数值计算时, 我们选取 $E = 200$ GPa, $\rho = 7800$ kg/m^3, $x_0 = 0.2$ m, $d = 25$ mm, $\mu_1 = \mu_2 = 5$, $\mu_3 = 0.01$, $\omega_1 = 5.609$, $\omega_2 = 39.4$, $\omega_3 = 2.85$, $l_1 = 44$ m, $l_2 = 6$ m, $H = 20$ kN, $\varphi = \pi/6$, $\theta = \pi/6$.

(17) 式中的非线性项很多, 为了简化计算, 可略去式中的小量, 将上述参数值代入该式. 经过初步数值计算可知, b_{31}, b_{23}, b_{63} 的值与式中其他系数相比, 相差的量级很大, 这几个系数可忽略不计. 同时短索的振动幅值 a_2 与长索和集中荷载的振动幅值相比很小, 可视为不振动, 即 $a_2 = 0$. 则 (17) 式可改写为

$$
\begin{aligned}
& b_{21}\omega_1 a_1 + \frac{1}{4}b_{41}\cos\gamma_1 a_1^2 + \frac{1}{2}a_3^2\big(b_{51}\cos\gamma_1 \\
& \quad -\bar{b}_2\sin\gamma_2\big) + 2b_{91}\cos\gamma_1 = 0, \qquad (18a)
\end{aligned}
$$

$$
b_{43}a_3\cos(\gamma_2 - \gamma_1) + \frac{b_{53}}{2}\sin\gamma_2 a_1 a_3 = 0, \qquad (18b)
$$

$$
\begin{aligned}
& b_{11}\omega_1 a_1\sigma_1 \\
=& \frac{3}{4}b_{41}a_1^2\sin\gamma_1 + \frac{1}{2}\left(\bar{b}_2\cos\gamma_2 + b_{51}\sin\gamma_1\right)a_3^2 \\
& + \frac{b_{61}}{4}a_1^3 + \frac{b_{81}}{4}a_3^2 a_1 + 2b_{91}\sin\gamma_1, \qquad (18c)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}b_{13}\omega_3 a_3(\sigma_1 + \sigma_3) \\
=& b_{33} - b_{43}\sin(\gamma_2 - \gamma_1)a_3 + \frac{b_{53}}{2}\cos\gamma_2 a_1 a_3 \\
& + \frac{b_{73}}{4}a_1^2 a_3 + \frac{b_{93}}{4}a_3^3. \qquad (18d)
\end{aligned}
$$

为了获得系统的频响方程, 需消去 (18) 式中的 γ_1, γ_2. 首先消去 (18b) 和 (18d) 式中的 $\gamma_2 - \gamma_1$, 可得 $\cos\gamma_2$ 的表达式

$$
\cos\gamma_2 = f_1/f_2, \qquad (19)
$$

其中

$$
\begin{aligned}
f_1 =& -4b_{33}^2 + 4b_{43}^2 - b_{53}a_1 - 2b_{33}b_{73}a_1 - b_{73}^2 a_1^3 \\
& - 2b_{33}b_{93}a_3^2 - b_{73}b_{93}a_1 a_3^2 - b_{93}^2 a_3^4 \\
& + b_{13}\omega_3\left(4b_{33} + b_{73}a_1 + b_{93}a_3^2\right)(\sigma_1 + \sigma_3) \\
& - b_{13}^2\omega_3^2(\sigma_1 + \sigma_3)^2, \\
f_2 =& b_{53}a_1\big(4b_{33} + b_{73}a_1^2 + b_{93}^2 a_3^2 \\
& - 2b_{13}\omega_3(\sigma_1 + \sigma_3)\big).
\end{aligned}
$$

将 (19) 式代入 (18c) 式中, 得到 $\sin\gamma_1$ 的表达式:

$$
\begin{aligned}
\sin\gamma_1 =& -\frac{1}{f_3}\Big(\frac{1}{4}b_{61}a_1^3 + \frac{1}{4}b_{81}a_1 a_3^2 - b_{11}a_1\sigma_1\omega_1 \\
& + \frac{1}{2}\bar{b}_2 a_3^2\frac{f_1}{f_2}\Big), \qquad (20)
\end{aligned}
$$

其中

$$
f_3 = 2b_{91} + \frac{3}{4}b_{41}a_1^2 + \frac{1}{2}b_{53}a_3^2.
$$

再将 (19), (20) 式代入 (18a) 和 (18b) 式, 可分别得到 $\cos\gamma_1, \sin\gamma_2$ 的表达式:

$$
\begin{aligned}
\cos\gamma_1 =& -\frac{b_{23}a_1\omega_1}{f_3} + \left(b_{21}b_{43}\bar{b}_2 a_1 a_3^2\omega_1\frac{f_1}{f_2}\right) \\
& \times\left\{2f_3\left[\frac{1}{2}b_{43}\bar{b}_2 a_3^2\frac{f_1}{f_2} + \frac{f_3}{2}b_{53}a_1\right.\right. \\
& - b_{43}\left(\frac{b_{61}}{4}a_1^3 + \frac{b_{81}}{4}a_1 a_3^2 - b_{11}a_1\sigma_1\omega_1\right. \\
& \left.\left.\left. + \frac{1}{2}\bar{b}_2 a_3^2\frac{f_1}{f_2}\right)\right]\right\}, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
\sin\gamma_2 =& \left(b_{21}b_{43}a_1\omega_1\frac{f_1}{f_2}\right)\times\left[\frac{1}{2}b_{43}\bar{b}_2 a_3^2\frac{f_1}{f_2}\right. \\
& + \frac{f_3}{2}b_{53}a_1 - b_{43}\left(\frac{b_{61}}{4}a_1^3 + \frac{b_{81}}{4}a_1 a_3^2\right. \\
& \left.\left. - b_{11}a_1\sigma_1\omega_1 + \frac{1}{2}\bar{b}_2 a_3^2\frac{f_1}{f_2}\right)\right]^{-1}. \qquad (22)
\end{aligned}
$$

将 (19)—(22) 式代入 (18d) 式, 可得 a_1,a_3 关于 σ_1, σ_3 的频响方程 1; 由 (18a) 和 (18c) 式中消去 γ_2 后, 将 (20) 和 (21) 式代入, 可得 a_1, a_3 关于 σ_1, σ_3 的频响方程 2. 令 $\sigma_3 = 0$, 消去两频响方程式中的 a_3, 可得 a_1-σ_1 的频响方程; 同理, 令 $\sigma_1 = 0$, 消去两频响方程式中的 a_1, 可得 a_3-σ_3 的频响方程.

利用 Mathematica 程序对系统的频响方程进行数值分析, 可得 a_1-σ_1 的频响曲线如图 2 所示,

σ_3-a_3 的频响曲线如图 3 所示. 图中实线表示稳定的摄动解, 虚线表示不稳定的摄动解.

从图中可以看出, 集中荷载作用下的高架索在激励频率接近系统的固有频率时, 系统将发生主共振, 系统存在多个稳定和不稳定的解, 出现了比较复杂的分叉现象. 解谐参数越接近零点, 系统就越接近主共振状态, 系统解的情况越复杂. 以 σ_3-a_3 频响曲线图为例, 在零点附近出现了 3 个稳态解和 1 个非稳态解, 在远离零点处则只有 1 个稳态解. 由于非线性因素的影响, 不管是幅值的变化程度, 还是分叉点的个数, 解谐参数 σ_3 对 a_3 的影响都远远大于解谐参数 σ_1 对 a_1 的影响.

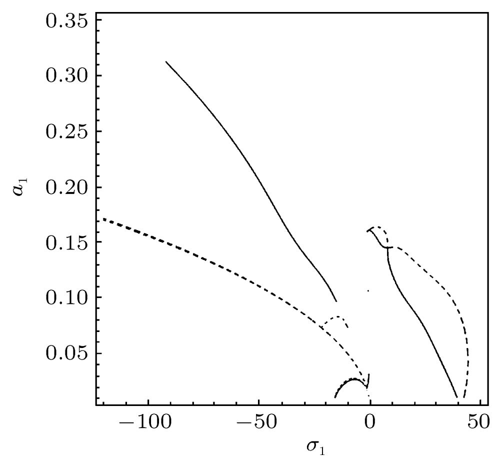

图 2　频率响应曲线 ($\sigma_3 = 0$)

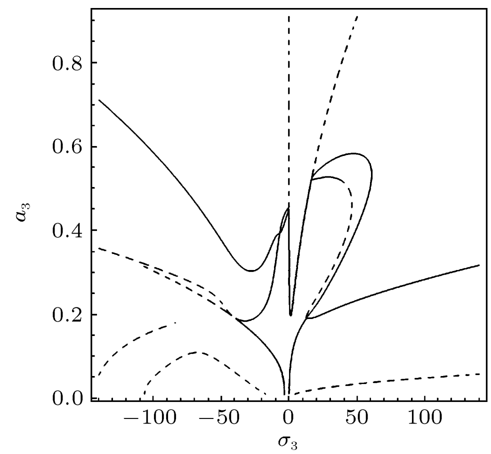

图 3　频率响应曲线 ($\sigma_1 = 0$)

5 结 论

本文综合考虑了由货物自重引起高架索静态构型的分段特性、高架索的倾斜角以及发送端水平运动等多种因素的影响, 建立了海上航行补给过程中分段高架索的三自由度非线性动力学模型. 将惯性项解耦后得到了三自由度常微分方程, 并利用多尺度法对其进行了摄动分析和频响数值分析, 为后续开展相关的非线性动力学研究奠定了理论基础.

[1] Lu Y J 1996 *Shipbuliding of China* **37** 17 (in Chinese) [卢永锦 1996 中国造船 **37** 17]

[2] He X J, Zhang L X, Ren A D 2010 *Acta Phys. Sin.* **59** 3088 (in Chinese) [何学军, 张良欣, 任爱娣 2010 物理学报 **59** 3088]

[3] Zhang L X, He X J, Ren A D 2010 *Proceedings of the 2010 International Conference on Application of Mathematics and Physics V2: Advances on Applied Mathematics and Computation Mathematics* Nanjing, China, May 8–9, 2010 p104

[4] Ren A D, He X J, Zhang L X 2010 *The International Conference on E-Product, E-Service and E-Entertainment* Henan, China, November 4–6, 2010 p4662

[5] Rega G 2004 *Am. Soc. Mech. Eng.* **57** 443

[6] Jensen C N, Nielsen S R K, Svrensen J D 2002 *J. Sound Vib.* **256** 499

[7] Chen L Q, Yang X D 2005 *J. Sound Vib.* **2** 879

[8] Nayfeh A H, Mook D T 1979 *Nonlinear Osciallations* (New York: John Wiley and Sons) p72

Inplane oscillation analysis of the highline system with 1 : 2 internal resonance and dominate resonance*

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Abstract

A three-degree-of-freedom nonlinear dynamic model of piecewise highline system during underway replenishment is investigated when the lumped mass swings in which the influences of piecewise characteristic of highline cable's static profile due to cargo deadweight, the incline angle and the horizontal motion of the sending end are considered. Considering 1:2 internal resonance and primary resonance, ordinary differential equations are gained by decoupling the inertial terms. Perturbation analysis and numerical analysis are carried out by adopting the multiple-scale method, which will be the theoretical basis of the further research of nonlinear dynamics.

Keywords: piecewise, the highline system, swing, multiple-scale method

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