相空间中相对运动完整力学系统的共形不变性与守恒量*

王廷志[1)†]　孙现亭[2)]　韩月林[1)]

1) (江南大学理学院, 无锡　214122)

2) (平顶山学院电气信息工程学院, 平顶山　467000)

(2013 年 12 月 20 日收到; 2014 年 1 月 8 日收到修改稿)

研究了相空间中相对运动完整力学系统的共形不变性与守恒量. 给出了该系统共形不变性的定义, 并推导出相空间中相对运动完整力学系统的运动微分方程具有共形不变性并且是Lie对称性的充分必要条件. 利用规范函数满足的结构方程导出该系统相应的守恒量, 并给出应用算例.

关键词: 相空间, 相对运动, 共形不变性, 守恒量

PACS: 45.20.Jj, 02.20.Sv, 11.30.–j　　**DOI:** 10.7498/aps.63.104502

1 引　言

对称性广泛存在于自然界中, 在分析力学中也称为不变性. 利用对称性寻求守恒量是认识世界和解决问题的有效途径之一. 自1918年Noether定理[1]被提出以来, 在对称性与守恒量方面的研究取得了很多重要成果[2−7]. 近些年, 人们主要通过Lie 对称性[8−15]、Mei对称性[16−20]和共形不变性[21−30]来寻求守恒量. 分析力学可分为位形空间中的Lagrange力学和相空间中的Hamilton 力学, 与前者相比, 后者具有结构简单、形式对称、应用广泛等优点. 文献[31]研究了Hamilton系统的共形不变性与守恒量. 文献[32]研究了相空间中变质量力学系统的Hojman守恒量. 文献[33]研究了非保守Hamilton系统中Lie 对称性直接导致的Mei守恒量. 本文研究相空间中相对运动完整力学系统的共形不变性与守恒量, 在给出共形不变性定义的基础上, 推导出相空间中相对运动完整力学系统的运动微分方程具有共形不变性并且是Lie 对称性的充分必要条件, 利用规范函数满足的结构方程导出系统相应的守恒量, 最后给出应用算例.

2 系统的运动微分方程

设载体的角速度为$\boldsymbol{\omega}$, 载体极点O的速度为$\boldsymbol{v}_0$, 系统由N个质点组成, 在t时刻, 第i个质点的质量为m_i $(i=1,2,\cdots,N)$, 在相空间中相对运动完整力学系统的微分方程可表示为

$$\dot{q}_s=\frac{\partial H}{\partial p_s},$$
$$\dot{p}_s=-\frac{\partial H}{\partial q_s}+Q_s+Q_s^{\dot{\omega}}+\varGamma_s. \tag{1}$$

这里, $H=H(t,\boldsymbol{q},\boldsymbol{p})$是系统的Hamilton函数; q_s为广义坐标, $s=1,2,\cdots,n$; p_s为广义动量; Q_s为广义非势力, $Q_s=Q_s(t,\boldsymbol{q},\boldsymbol{p})$; 广义回转惯性力$Q_s^{\dot{\omega}}$为

$$Q_s^{\dot{\omega}}=-(\dot{\boldsymbol{\omega}}\times m_i\boldsymbol{r}_i')\cdot\frac{\partial \boldsymbol{r}_i'}{\partial q_s}, \tag{2}$$

式中$\boldsymbol{r}_i'$为第i个质点的相对矢径; 广义陀螺力$\varGamma_s$为

$$\varGamma_s=\gamma_{sk}\dot{q}_k,$$
$$\gamma_{sk}=2\boldsymbol{\omega}\cdot\left(m_i\frac{\partial \boldsymbol{r}_i'}{\partial q_s}\times\frac{\partial \boldsymbol{r}_i'}{\partial q_k}\right). \tag{3}$$

---

* 国家自然科学基金 (批准号: 11142014) 资助的课题.

† 通讯作者. E-mail: wangtizh@qq.com

　http://wulixb.iphy.ac.cn

方程 (1) 可表示为

$$\begin{aligned}\dot{q}_s &= g_s(t,\boldsymbol{q},\boldsymbol{p}),\\ \dot{p}_s &= h_s(t,\boldsymbol{q},\boldsymbol{p})\end{aligned}\tag{4}$$

或

$$\boldsymbol{F}_s=\begin{bmatrix}F_s^q\\ F_s^p\end{bmatrix}=\begin{bmatrix}\dot{q}_s-g_s(t,\boldsymbol{q},\boldsymbol{p})\\ \dot{p}_s-h_s(t,\boldsymbol{q},\boldsymbol{p})\end{bmatrix}=0.\tag{5}$$

3 系统的共形不变性与共形因子

引入时间 t、广义坐标 q_s 和广义动量 p_s 的无限小变换,

$$\begin{aligned}t^* &= t+\varepsilon\xi_0(t,\boldsymbol{q},\boldsymbol{p}),\\ q_s^*(t^*) &= q_s+\varepsilon\xi_s(t,\boldsymbol{q},\boldsymbol{p}),\\ p_s^*(t^*) &= p_s+\varepsilon\eta_s(t,\boldsymbol{q},\boldsymbol{p}).\end{aligned}\tag{6}$$

方程 (6) 中, ε 为无限小参数, ξ_0,ξ_s,η_s 为无限小变换生成元. 在无限小变换方程 (6) 下, q_s, p_s 彼此独立.

引入无限小变换生成元向量 $X^{(0)}$ 及其一次展式 $X^{(1)}$,

$$X^{(0)}=\xi_0\frac{\partial}{\partial t}+\xi_s\frac{\partial}{\partial q_s}+\eta_s\frac{\partial}{\partial p_s},\tag{7}$$

$$\begin{aligned}X^{(1)} &= X^{(0)}+(\dot{\xi}_s-\dot{q}_s\dot{\xi}_0)\frac{\partial}{\partial\dot{q}_s}\\ &\quad+(\dot{\eta}_s-\dot{p}_s\dot{\xi}_0)\frac{\partial}{\partial\dot{p}_s}.\end{aligned}\tag{8}$$

定义 1 在无限小变换方程 (6) 下, 若非退化矩阵 $\boldsymbol{B}_s^k$ 满足

$$X^{(1)}(\boldsymbol{F}_s)=\boldsymbol{B}_s^k\boldsymbol{F}_k\quad(s,k=1,2,\cdots,n),\tag{9}$$

则称这种不变性为 Hamilton 方程 (5) 的共形不变性, (9) 式是方程 (5) 共形不变的确定方程, $\boldsymbol{B}_s^k$ 称为共形因子.

在方程 (5) 中, 如无限小生成元 ξ_0,ξ_s,η_s 满足确定方程

$$X^{(1)}(\boldsymbol{F}_s)|_{\boldsymbol{F}_s=0}=0,\tag{10}$$

则称这种对称性为系统的 Lie 对称性.

下面求共形因子 $\boldsymbol{B}_s^k$. 因为

$$\begin{aligned}X^{(1)}(\boldsymbol{F}_s^q) &= X^{(1)}[\dot{q}_s-g_s(t,\boldsymbol{q},\boldsymbol{p})]\\ &= \dot{\xi}_s-\dot{q}_s\dot{\xi}_0-\xi_0\frac{\partial g_s}{\partial t}\\ &\quad-\xi_k\frac{\partial g_s}{\partial q_k}-\eta_k\frac{\partial g_s}{\partial p_k},\end{aligned}\tag{11}$$

$$\begin{aligned}X^{(1)}(\boldsymbol{F}_s^p) &= X^{(1)}[\dot{p}_s-h_s(t,\boldsymbol{q},\boldsymbol{p})]\\ &= \dot{\eta}_s-\dot{p}_s\dot{\xi}_0-\xi_0\frac{\partial h_s}{\partial t}-\xi_k\frac{\partial h_s}{\partial q_k}\\ &\quad-\eta_k\frac{\partial h_s}{\partial p_k},\end{aligned}\tag{12}$$

又有

$$\begin{aligned}\dot{\xi}_s &= \frac{\partial\xi_s}{\partial t}+\frac{\partial\xi_s}{\partial q_k}\dot{q}_k\\ &\quad+\frac{\partial\xi_s}{\partial p_k}\dot{p}_k\quad(s,k=1,2,\cdots,n),\end{aligned}\tag{13}$$

$$\begin{aligned}\dot{\eta}_s &= \frac{\partial\eta_s}{\partial t}+\frac{\partial\eta_s}{\partial q_k}\dot{q}_k\\ &\quad+\frac{\partial\eta_s}{\partial p_k}\dot{p}_k\quad(s,k=1,2,\cdots,n),\end{aligned}\tag{14}$$

将 (13) 式代入 (11) 式并计算差值得

$$\begin{aligned}&X^{(1)}(F_s^q)-X^{(1)}(F_s^q)|_{F_s^q=0}\\ &=\left(\frac{\partial\xi_s}{\partial q_k}-\delta_s^k\dot{\xi}_0\right)(\dot{q}_k-g_k)\\ &=\left(\frac{\partial\xi_s}{\partial q_k}-\delta_s^k\dot{\xi}_0\right)(F_k^q)\\ &=\boldsymbol{Q}_s^k(F_k^q),\end{aligned}\tag{15}$$

将 (14) 式代入 (12) 式并计算差值得

$$\begin{aligned}&X^{(1)}(F_s^p)-X^{(1)}(F_s^p)|_{F_s^p=0}\\ &=\left(\frac{\partial\eta_s}{\partial p_k}-\delta_s^k\dot{\xi}_0\right)(\dot{p}_k-h_k)\\ &=\left(\frac{\partial\eta_s}{\partial p_k}-\delta_s^k\dot{\xi}_0\right)(F_k^p)\\ &=\boldsymbol{P}_s^k(F_k^p).\end{aligned}\tag{16}$$

考虑到

$$\boldsymbol{F}_s=\begin{pmatrix}F_s^q\\ F_s^p\end{pmatrix},$$

$$\boldsymbol{\alpha}_s^k=\begin{pmatrix}\boldsymbol{Q}_s^k & 0\\ 0 & \boldsymbol{P}_s^k\end{pmatrix},$$

则 (15), (16) 式可表示为

$$\begin{aligned}&X^{(1)}(\boldsymbol{F}_s)-X^{(1)}(\boldsymbol{F}_s)|_{\boldsymbol{F}_s=0}\\ &=\boldsymbol{\alpha}_s^k(\boldsymbol{F}_k)\quad(s,k=1,2,\cdots,n).\end{aligned}\tag{17}$$

这里,

$$\boldsymbol{Q}_s^k=\left(\frac{\partial\xi_s}{\partial q_k}-\delta_s^k\dot{\xi}_0\right),$$

$$\boldsymbol{P}_s^k=\left(\frac{\partial\eta_s}{\partial p_k}-\delta_s^k\dot{\xi}_0\right).$$

如系统具有共形不变性且是Lie对称性, 由(9), (10)和(17)式可得

$$\boldsymbol{B}_s^k \boldsymbol{F}_k - \boldsymbol{\alpha}_s^k \boldsymbol{F}_k = X^{(1)}(\boldsymbol{F}_s)|_{F_s=0} = 0, \tag{18}$$

即

$$\boldsymbol{B}_s^k = \boldsymbol{\alpha}_s^k \quad (s, k = 1, 2, \cdots, n). \tag{19}$$

由此易得到命题1.

命题1 在无限小变换方程(6)作用下, 如果相空间中相对运动完整力学系统(5)是Lie对称性的, 且存在矩阵$\boldsymbol{\alpha}_s^k$满足

$$\begin{aligned} &X^{(1)}(\boldsymbol{F}_s) - X^{(1)}(\boldsymbol{F}_s)|_{F_s=0} \\ &= \boldsymbol{\alpha}_s^k(\boldsymbol{F}_k) \quad (s, k = 1, 2, \cdots, n), \end{aligned}$$

则在无限小变换方程(6)作用下, 相空间中相对运动完整力学系统(5)既是共形不变性又是Lie对称性的充分必要条件是

$$\begin{aligned} \boldsymbol{B}_s^k &= \begin{pmatrix} \boldsymbol{Q}_s^k & 0 \\ 0 & \boldsymbol{P}_s^k \end{pmatrix}, \\ \boldsymbol{Q}_s^k &= \left(\frac{\partial \xi_s}{\partial q_k} - \delta_s^k \dot{\xi}_0\right), \\ \boldsymbol{P}_s^k &= \left(\frac{\partial \eta_s}{\partial p_k} - \delta_s^k \dot{\xi}_0\right), \end{aligned} \tag{20}$$

其中$s, k = 1, 2, \cdots, n$.

4 共形不变性与守恒量

根据相空间中相对运动完整力学系统的共形不变性, 由Lie对称性可求出相应的守恒量, 可得到命题2.

命题2 对于相空间中相对运动完整力学系统(5), 若共形不变性的无限小生成元ξ_0, ξ_s, η_s和规范函数G满足Lie对称性结构方程

$$\begin{aligned} &p_s \dot{\xi}_s - H\dot{\xi}_0 - \frac{\partial H}{\partial t}\xi_0 - \frac{\partial H}{\partial q_s}\xi_s \\ &+ (Q_s + Q_s^{\dot{\omega}} + \varGamma_s)(\xi_s - \dot{q}_s \xi_0) + \dot{G} \\ &= 0, \end{aligned} \tag{21}$$

则共形不变性存在守恒量I, 即

$$I = p_s \xi_s - H\xi_0 + G = \text{const.} \tag{22}$$

证明 对(22)式求导并利用(1)和(21)式得

$$\begin{aligned} \frac{\mathrm{d}I}{\mathrm{d}t} &= \dot{p}_s \xi_s + p_s \dot{\xi}_s - \dot{H}\xi_0 - H\dot{\xi}_0 + \dot{G} \\ &= \dot{p}_s \xi_s + p_s \dot{\xi}_s - \dot{H}\xi_0 - H\dot{\xi}_0 + H\dot{\xi}_0 \\ &\quad + \frac{\partial H}{\partial t}\xi_0 + \frac{\partial H}{\partial q_s}\xi_s \\ &\quad - p_s \dot{\xi}_s - (Q_s + Q_s^{\dot{\omega}} + \varGamma_s)(\xi_s - \dot{q}_s \xi_0) \\ &= \left(\frac{\partial H}{\partial q_s} - Q_s - Q_s^{\dot{\omega}} - \varGamma_s\right)(\xi_s - \dot{q}_s \xi_0) \\ &\quad + \dot{p}_s\left(\xi_s - \frac{\partial H}{\partial p_s}\xi_0\right) \\ &= 0. \end{aligned}$$

5 算 例

二自由度相空间中相对运动完整力学系统为

$$\begin{aligned} H &= \frac{1}{2}(p_1^2 + p_2^2) - \frac{1}{2}\omega^2 q_1^2, \\ Q_1 &= q_1, \\ Q_2 &= 0, \\ Q_1^{\dot{\omega}} &= Q_2^{\dot{\omega}} = 0, \\ \varGamma_1 &= \dot{q}_1, \\ \varGamma_2 &= -\dot{q}_2. \end{aligned} \tag{23}$$

这里ω为常数. 下面研究该系统的共形不变性与守恒量.

将(23)式代入方程(1)可得

$$\begin{aligned} \dot{q}_1 &= p_1, \\ \dot{q}_2 &= p_2; \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{p}_1 &= \omega^2 q_1 + q_1 + \dot{q}_1, \\ \dot{p}_2 &= -\dot{q}_2. \end{aligned} \tag{25}$$

取无限小变换生成元函数为

$$\begin{aligned} \xi_0 &= 0, \\ \xi_1 &= q_1, \\ \xi_2 &= q_2, \\ \eta_1 &= p_1, \\ \eta_2 &= p_2, \end{aligned} \tag{26}$$

则

$$\begin{aligned} &X^{(1)}\begin{pmatrix} F_1^q \\ F_2^q \end{pmatrix} \\ &= \left(\xi_s \frac{\partial}{\partial q_s} + \eta_s \frac{\partial}{\partial p_s} + \dot{\xi}_s \frac{\partial}{\partial \dot{q}_s} + \dot{\eta}_s \frac{\partial}{\partial \dot{p}_s}\right)\begin{pmatrix} \dot{q}_1 - p_1 \\ \dot{q}_2 - p_2 \end{pmatrix} \\ &= \begin{pmatrix} \dot{\xi}_1 - \eta_1 \\ \dot{\xi}_2 - \eta_2 \end{pmatrix} \end{aligned}$$

$$=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}\dot{q}_1-p_1\\\dot{q}_2-p_2\end{pmatrix},\tag{27}$$

$$X^{(1)}\begin{pmatrix}F_1^p\\F_2^p\end{pmatrix}$$
$$=\left(\xi_s\frac{\partial}{\partial q_s}+\eta_s\frac{\partial}{\partial p_s}+\dot{\xi}_s\frac{\partial}{\partial \dot{q}_s}+\dot{\eta}_s\frac{\partial}{\partial \dot{p}_s}\right)$$
$$\times\begin{pmatrix}\dot{p}_1-\omega^2q_1-q_1-\dot{q}_1\\\dot{p}_2+\dot{q}_2\end{pmatrix}$$
$$=\begin{pmatrix}\dot{\eta}_1-(\omega^2+1)\xi_1-\dot{\xi}_1\\\dot{\eta}_2+\dot{\xi}_2\end{pmatrix}$$
$$=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}\dot{p}_1-(\omega^2+1)q_1-\dot{q}_1\\\dot{p}_2+\dot{q}_2\end{pmatrix}.\tag{28}$$

因此, 共形因子为

$$\boldsymbol{Q}_s^k=\boldsymbol{P}_s^k=\begin{pmatrix}1&0\\0&1\end{pmatrix}.\tag{29}$$

也可从方程(20)求出共形因子

$$\boldsymbol{Q}_s^k=\left(\frac{\partial\xi_s}{\partial q_k}-\delta_s^k\dot{\xi}_0\right)=\frac{\partial\xi_s}{\partial q_k},\tag{30}$$

$$\boldsymbol{P}_s^k=\left(\frac{\partial\eta_s}{\partial p_k}-\delta_s^k\dot{\xi}_0\right)=\frac{\partial\eta_s}{\partial p_k}.\tag{31}$$

(30)和(31)式的计算结果与(29)式的计算结果相同, 共形不变性的确定方程可表示为

$$X^{(1)}\begin{pmatrix}F_s^q\\F_s^p\end{pmatrix}=\begin{pmatrix}\boldsymbol{Q}_s^k&0\\0&\boldsymbol{P}_s^k\end{pmatrix}\begin{pmatrix}F_k^q\\F_k^p\end{pmatrix}.\tag{32}$$

这时, 相空间中相对运动完整力学系统(5)不仅是共形不变性的, 同时又是Lie对称性的.

将(23), (26)式代入(21)式并积分得

$$G=-p_1q_1-p_2q_2+p_2\,\mathrm{e}^t.\tag{33}$$

将(33)和(26)式代入(22)式可得守恒量

$$I=p_2\,\mathrm{e}^t=\mathrm{const}.\tag{34}$$

6 结 论

本文研究了相空间中相对运动完整力学系统的共形不变性, 得到了相空间中相对运动完整力学系统的共形不变性导致的守恒量及其满足的条件. 由于是在相空间中研究相对运动完整力学系统的共形不变性与守恒量, 守恒量表达式(22)比原来的守恒量表达式[7]更为简洁且易计算.

参考文献

[1] Noether A E 1918 *Nachr. Akad. Wiss. Göttingen Math. Phys.* **2** 235

[2] Djukic D S, Vujanovi B D 1975 *Acta Mech.* **23** 17

[3] Lutzky M 1979 *J. Phys. A: Math. Gen.* **12** 973

[4] Hojman S A 1992 *J. Phys. A: Math. Gen.* **25** L291

[5] Mei F X 2004 *Symmetries and Conserved Quantities of Constrained Mechanical Systems* (Beijing: Beijing Institute of Technology Press) p43 (in Chinese) [梅凤翔 2004 约束力学系统的对称性与守恒量 (北京: 北京理工大学出版社) 第43页]

[6] Galiullin A S, Gafarov G G, Malaishka R P, Khwan A M 1997 *Analytical Dynamics of Helmholtz Birkhoff and Nambu Systems* (Moscow: UFN) p183 (in Russian)

[7] Mei F X 1999 *Applications of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing: Science Press) p366 (in Chinese) [梅凤翔 1999 李群和李代数对约束力学系统的应用 (北京: 科学出版社) 第366页]

[8] Zhang M L, Wang X X, Han Y L, Jia L Q 2012 *Chin. Phys. B* **21** 100203

[9] Wang X X, Han Y L, Zhang M L, Jia L Q 2013 *Chin. Phys. B* **22** 020201

[10] Luo S K, Li Z J, Li L 2012 *Acta Mech.* **223** 2621

[11] Li Z J, Luo S K 2012 *Nonlinear Dyn.* **70** 1117

[12] Han Y L, Wang X X, Zhang M L, Jia L Q 2013 *Nonlinear Dyn.* **71** 401

[13] Han Y L, Wang X X, Zhang M L, Jia L Q 2013 *Nonlinear Dyn.* **73** 357

[14] Lou Z M, Mei F X, Chen Z D 2012 *Acta Phys. Sin.* **61** 110204 (in Chinese) [楼智美, 梅凤翔, 陈子栋 2012 物理学报 **61** 110204]

[15] Xu C, Li Y C 2013 *Acta Phys. Sin.* **62** 171101 (in Chinese) [徐超, 李元成 2013 物理学报 **62** 171101]

[16] Yang X F, Sun X T, Wang X X, Zhang M L, Jia L Q 2011 *Acta Phys. Sin.* **60** 111101 (in Chinese) [杨新芳, 孙现亭, 王肖肖, 张美玲, 贾利群 2011 物理学报 **60** 111101]

[17] Wang X X, Zhang M L, Han Y L, Jia L Q 2012 *Acta Phys. Sin.* **61** 200203 (in Chinese) [王肖肖, 张美玲, 韩月林, 贾利群 2012 物理学报 **61** 200203]

[18] Sun X T, Han Y L, Wang X X, Zhang M L, Jia L Q 2012 *Acta Phys. Sin.* **61** 200204 (in Chinese) [孙现亭, 韩月林, 王肖肖, 张美玲, 贾利群 2012 物理学报 **61** 200204]

[19] Zhang M L, Wang X X, Han Y L, Jia L Q 2012 *Chin. Phys. B* **21** 100203

[20] Jia L Q, Wang X X, Zhang M L, Han Y L 2012 *Nonlinear Dyn.* **69** 1807

[21] Zhang Y, Xue Y 2009 *Chin. Q. Mech.* **30** 216 (in Chinese) [张毅, 薛纭 2009 力学季刊 **30** 216]

[22] Cai J L 2009 *Acta Phys. Sin.* **58** 22 (in Chinese) [蔡建乐 2009 物理学报 **58** 22]

[23] Han Y L, Sun X T, Zhang Y Y, Jia L Q 2013 *Acta Phys. Sin.* **62** 160201 (in Chinese) [韩月林, 孙现亭, 张耀宇, 贾利群 2013 物理学报 **62** 160201]

[24] Chen X W, Zhao Y H, Liu C 2009 *Acta Phys. Sin.* **58** 5150 (in Chinese) [陈向炜, 赵永红, 刘畅 2009 物理学报 **58** 5150]

[25] Chen R, Xu X J 2012 *Acta Phys. Sin.* **61** 021102 (in Chinese) [陈蓉, 许学军 2012 物理学报 **61** 021102]

[26] Chen X W, Zhao Y H, Li Y M 2009 *Chin. Phys. B* **18** 3139

[27] Cai J L, Shi S S, Fang H J, Xu J 2012 *Mechanica* **47** 63

[28] Huang W L, Cai J L 2012 *Acta Mech.* **223** 433

[29] Cai J L 2012 *Nonlinear Dyn.* **69** 487

[30] Wang T Z, Sun X T, Han Y L 2013 *Acta Phys. Sin.* **62** 231101 (in Chinese) [王廷志, 孙现亭, 韩月林 2013 物理学报 **62** 231101]

[31] Cai J L, Luo S K, Mei F X 2008 *Chin. Phys. B* **17** 3170

[32] Fang J H, Zhang P Y 2004 *Acta Phys. Sin.* **53** 4041 (in Chinese) [方建会, 张鹏玉 2004 物理学报 **53** 4041]

[33] Fang J H, Zhang B, Zhang W W, Xu R L 2012 *Chin. Phys. B* **21** 050202

Conformal invariance and conserved quantity of relative motion holonomic dynamical system in phase space*

Wang Ting-Zhi[1)†] Sun Xian-Ting[2)] Han Yue-Lin[1)]

1) (*School of Science, Jiangnan University, Wuxi 214122, China*)

2) (*School of Electric and Information Engineering, Pingdingshan University, Pingdingshan 467000, China*)

(Received 20 December 2013; revised manuscript received 8 January 2014)

Abstract

Conformal invariance and conserved quantity of relative motion holonomic dynamical system in phase space are studied. The definition of conformal invariance of relative motion holonomic dynamical system in phase space is provided. The necessary and sufficient conditions that conformal invariance of the system would be Lie symmetry are deduced. By use of a structure equation that the gauge function satisfies, the corresponding conserved quantity of the system is derived. Finally an illustrative example is given to verify the results.

Keywords: phase space, relative motion, conformal invariance, conserved quantity

PACS: 45.20.Jj, 02.20.Sv, 11.30.–j **DOI:** 10.7498/aps.63.104502

* Project supported by the National Natural Science Foundation of China (Grant No. 11142014).

† Corresponding author. E-mail: wangtizh@qq.com