单层石墨烯片的非线性板模型*

黄坤[1)] 殷雅俊[2)†] 吴继业[3)]

1) (昆明理工大学土木工程学院工程力学系, 昆明 650500)

2) (清华大学航天航空学院工程力学系, 北京 100084)

3) (南京工业大学力学部, 南京 211816)

(2013 年 12 月 25 日收到; 2014 年 4 月 21 日收到修改稿)

基于实验得到的非线性本构关系和板理论, 本文建立了包含三次及五次非线性项的单层石墨烯片的板动力学模型. 针对四边简支矩形板, 使用 Ritz 法研究了在板中点作用集中力时的静力弯曲, 以及边界均匀受力时的静力屈曲问题. 结果显示, 基于非线性本构关系的板模型能很好的描述单层石墨烯片的力学行为, 而且模型中的五次非线性项对结构的弯曲变形有显著影响.

关键词: 单层石墨烯, 非线性本构关系, 板理论, Ritz 法

PACS: 62.20.Dc, 62.25.+g, 81.40.Jj, 81.40.Lm **DOI:** 10.7498/aps.63.156201

1 引 言

石墨烯独特的力学、热学及电磁学性能, 使其具有广阔的应用前景, 成为近年的研究热点[1−6]. 石墨烯的晶格结构近乎完美, 澄清其力学性质可为研究其他结构复杂的纳米材料的力学性质提供参考. 因此, 研究者从不同侧面对石墨烯的力学性质进行了研究[7−12]. Yakobson 等在研究碳纳米管时, 把其看成是石墨烯片卷成的圆柱壳, 并研究了其非线性静力屈曲[7]. Ouyang 等通过对碳-碳共价键的连续化处理, 得到了石墨烯片比拟为板壳时的弹性参数[8−10]. 尽管连续介质理论和分子动力学模拟是目前研究石墨烯的力学性质主要方法, 但通过实验来验证理论及计算的结果是必要的. 在文献 [13] 中, Lee 等通过实验得到了单层石墨烯片在单轴拉伸时的非线性本构关系, 之后 Cadelano 等通过连续介质弹性理论与原子模拟对比, 得到了 Lee 的本构方程中的非线性弹性系数[14]. Lee 及 Cadelano 的研究表明, 单层石墨烯片的弹性行为是物理非线性的, 因此有必要在石墨烯片的板壳理论中考虑此非线性的影响. 本文中, 以文献 [13] 提出的二次非线性本构关系为基础, 建立了单层石墨烯片的非线性板模型, 并通过该模型研究了矩形单层石墨烯片在边界均匀受力时的屈曲.

2 模型建立

单层石墨烯是碳原子通过共价键构成的二维结构 (图 1 (a)), 但通过 Cauchy-Born 原理可以把离散晶格结构的石墨烯片近似为一薄板 (图 1 (b)), 并应用宏观连续介质力学来研究其力学性质[15].

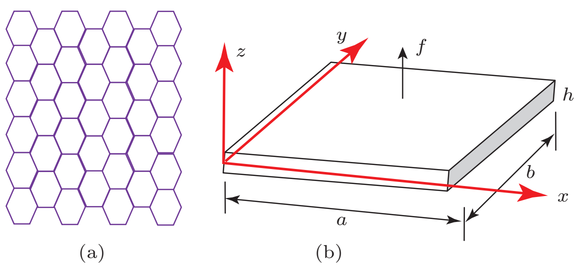

图 1 单层石墨烯片的结构模型 (a) 离散模型; (b) 连续模型

* 国家自然科学基金 (批准号: 11272175, 11072125) 和江苏省自然科学基金 (批准号: BK20130910) 资助的课题.

† 通讯作者. E-mail: yinyj@tsinghua.edu.cn

此外还可以通过非局部连续介质理论来考虑尺度效应对石墨烯片力学性质的影响[16]. 近期的理论及实验均表明, 用连续介质理论来研究石墨烯片是可行的, 但其应力应变关系是非线性的[13,14]. 本节, 基于实验得到的非线性本构方程来建立单层石墨烯片的动力学方程.

Lee通过实验, 提出了单层石墨烯片的非线性本构关系[13]

$$\sigma = C\varepsilon + D\varepsilon^2. \tag{1}$$

通过本构方程可得石墨烯片的势能密度为

$$U = \frac{1}{2}\left(C\varepsilon^2 + D\varepsilon^3\right), \tag{2}$$

其中C是二阶线弹性系数, D是三阶弹性系数. 注意到单层石墨烯的二维结构, 其势能密度(2)可表示为[14]

$$\begin{aligned} 2U = \Big[&\bar{E}\left(1+\nu\right)^{-1}\varepsilon_{\alpha\alpha}\varepsilon_{\beta\beta} + \bar{E}\nu\left(1-\nu^2\right)^{-1}\varepsilon_{\alpha\beta}^2 \\ &+ \bar{\Lambda}_1\left(\varepsilon_{\alpha\alpha}^3 + \varepsilon_{\beta\beta}^3\right) + \bar{\Lambda}_2\varepsilon_{\alpha\alpha}\varepsilon_{\beta\beta}\varepsilon_{\alpha\beta} \\ &+ \bar{\Lambda}_3\varepsilon_{\alpha\beta}^3\Big]. \end{aligned} \tag{3}$$

在此$\alpha, \beta = x, y$, (3)式满足Einstein求和约定, $\bar{\Lambda}_j$, $j = 1, 2, 3$与石墨烯晶格弹性常数的关系为

$$\begin{aligned} \bar{\Lambda}_1 &= \frac{1}{12}\left(C_{111} - C_{222}\right), \\ \bar{\Lambda}_2 &= \frac{1}{4}\left(C_{222} - C_{112}\right), \\ \bar{\Lambda}_3 &= \frac{1}{12}\left(2C_{111} - C_{222} - 3C_{112}\right). \end{aligned}$$

晶格的弹性系数可以通过实验或原子计算的方法得到, 可取为[14] $C_{111} = -2724.7\ \mathrm{N{\cdot}m^{-1}}$, $C_{112} = -519.1\ \mathrm{N{\cdot}m^{-1}}$, $C_{222} = -2523.2\ \mathrm{N{\cdot}m^{-1}}$. 把石墨烯片看成无厚度膜时, 弹性模量和Poisson比为[14] $\bar{E} = 350\ \mathrm{N{\cdot}m^{-1}}$, $\nu = 0.186$. 在使用板壳理论来研究单层石墨烯片的力学行为时, 需要把二维的弹性系数转换为体积弹性系数. 在此把平面弹性参数在石墨烯片厚度上的平均作为体积弹性系数, 则有$E = \bar{E}h^{-1}$, $\Lambda_j = \bar{\Lambda}_j h^{-1}$, $j = 1, 2, 3$. 将单层石墨烯的厚度取为多层石墨烯的层间距[14], 即$h = 0.335$ nm, 有$E \approx 1.04$ TPa. 设石墨烯片x, y, z方向的位移分别为u, v, w, 考虑到一般情况下u, v远小于w, 可把位移场写为[16,17]

$$u = -z\frac{\partial w}{\partial x}, \quad v = -z\frac{\partial w}{\partial y}, \quad w = w.$$

则应变张量的分量为

$$\begin{aligned} \varepsilon_{xx} &= -z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \\ \varepsilon_{xy} &= -2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \\ \varepsilon_{yy} &= -z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2. \end{aligned} \tag{4}$$

把(4)式代入(3)式, 用体积弹性参数代替二维弹性参数, 忽略六次以上以及和h^3有关非线性项, 之后在结构的体积上积分, 得

$$\begin{aligned} U = &\iint_A \left\{k_1\left[(2H)^2 + k_2 K\right]\right\}\mathrm{d}x\mathrm{d}y \\ &+ \iint_A \left\{k_3\left[\left(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right)^2\right.\right. \\ &\left.\left.+ k_2\left(\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)^2 - \frac{1}{4}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^2\right)\right]\right\} \\ &\times \mathrm{d}x\mathrm{d}y + h\iint_A \left\{\frac{C_{111}}{48}\left(\frac{\partial w}{\partial x}\right)^6\right. \\ &+ \frac{C_{222}}{48}\left(\frac{\partial w}{\partial y}\right)^6 + \frac{3C_{222} - C_{111}}{16}\left(\frac{\partial w}{\partial x}\right)^4 \\ &\times \left(\frac{\partial w}{\partial y}\right)^2 + \frac{3C_{111} - 2C_{222}}{16}\left(\frac{\partial w}{\partial x}\right)^2 \\ &\left.\times \left(\frac{\partial w}{\partial y}\right)^4\right\}\mathrm{d}x\mathrm{d}y, \end{aligned} \tag{5}$$

其中A为板中面面积,

$$\begin{aligned} k_1 &= \frac{Eh^3}{24\left(1-\nu^2\right)}, \quad k_2 = 2\left(1-\nu\right), \\ k_3 &= \frac{Eh}{2\left(1-\nu^2\right)}; \end{aligned}$$

H, K分别为板中面的平均曲率和Gauss曲率[9]:

$$\begin{aligned} H &= \frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right), \\ K &= \left(\frac{\partial^2 w}{\partial y \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}. \end{aligned} \tag{6}$$

根据位移场, 结构的动能可表示为

$$\begin{aligned} V = &\iint_A \frac{m}{2}\left[h\left(\frac{\partial w}{\partial t}\right)^2\right. \\ &\left.+ \frac{h^3}{12}\left(\left(\frac{\partial^2 w}{\partial x \partial t}\right)^2 + \left(\frac{\partial^2 w}{\partial y \partial t}\right)^2\right)\right]\mathrm{d}A. \end{aligned} \tag{7}$$

假设载荷除作用于板面的$f\left(x, y, t\right)$外, 还有作用在板边上的轴向力, 如图2, 则外力势能为[18]

$$\begin{aligned} W = \frac{1}{2}\iint_A &\left[2fw + N_x\left(\frac{\partial w}{\partial x}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right. \\ &\left.+ N_y\left(\frac{\partial w}{\partial y}\right)^2\right]\mathrm{d}A. \end{aligned} \tag{8}$$

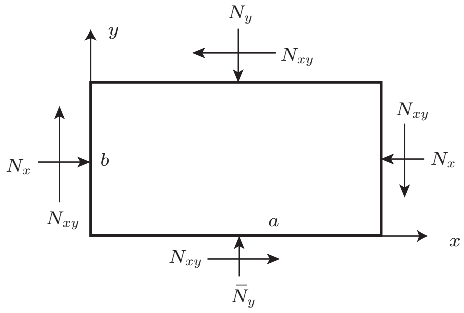

图 2　边界上的载荷

使用动能和势能, 可构造 Hamilton 量 $L = V - U + W$. 根据 Hamilton 原理[18],

$$\delta\int_{t_0}^{t_1} L\mathrm{d}t = 0, \tag{9}$$

得系统的动力学方程为

$$\begin{aligned}
& m\frac{\partial^2 w}{\partial t^2} + 2k_1\nabla^4 w + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} \\
& + N_y\frac{\partial^2 w}{\partial y^2} + \frac{\partial}{\partial x}\Bigg\{k_3\Bigg[2\Bigg(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\Bigg) \\
& \times\left(\frac{\partial w}{\partial x}\right) + \frac{k_2}{2}\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2\Bigg]\Bigg\} + \frac{\partial}{\partial y}\Bigg\{k_3\Bigg[2\Bigg(\left(\frac{\partial w}{\partial x}\right)^2 \\
& + \left(\frac{\partial w}{\partial y}\right)^2\Bigg)\frac{\partial w}{\partial y} + \frac{k_2}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial w}{\partial y}\Bigg]\Bigg\} \\
& + h\frac{\partial}{\partial x}\Bigg\{\frac{C_{111}}{8}\left(\frac{\partial w}{\partial x}\right)^5 + \frac{3C_{222} - C_{111}}{4}\left(\frac{\partial w}{\partial x}\right)^3 \\
& \times\left(\frac{\partial w}{\partial y}\right)^2 + \frac{3C_{111} - 2C_{222}}{8}\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial y}\right)^4\Bigg\} \\
& + h\frac{\partial}{\partial y}\Bigg\{\frac{C_{222}}{6}\left(\frac{\partial w}{\partial y}\right)^5 + \frac{3C_{222} - C_{111}}{8}\left(\frac{\partial w}{\partial x}\right)^4\frac{\partial w}{\partial y} \\
& + \frac{3C_{111} - 2C_{222}}{4}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^3\Bigg\} \\
& = f(x, y, t),
\end{aligned} \tag{10}$$

式中,

$$\nabla^4 = \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4}\right).$$

因为 h 是小量, 方程 (10) 中省略了包含 h^3 的惯性项. 对于边缘简支的矩形薄板, 方程 (10) 的边界条件为

$$w = 0,\quad \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0. \tag{11}$$

从方程 (10) 可知, 考虑到单层石墨烯的物理非线性效应后, 结构的动力学方程包含立方及五次非线性项. 对包含奇次非线性的系统, 结构的变形会影响到振动频率, 并出现 1:3 内共振等现象[19], 其动力学行为需要专门讨论. 本文中仅考虑静力学行为.

3　静力弯曲与边界受力的屈曲稳定性

3.1　静力弯曲

省略动力学方程中的惯性项, 可得单层石墨烯片的静力学控制方程为

$$\begin{aligned}
& 2k_1\nabla^4 w + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x\partial y} + N_y\frac{\partial^2 w}{\partial y^2} \\
& + \frac{\partial}{\partial x}\Bigg\{k_3\Bigg[2\Bigg(\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\Bigg)\left(\frac{\partial w}{\partial x}\right) \\
& + \frac{k_2}{2}\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2\Bigg]\Bigg\} + \frac{\partial}{\partial y}\Bigg\{k_3\Bigg[2\Bigg(\left(\frac{\partial w}{\partial x}\right)^2 \\
& + \left(\frac{\partial w}{\partial y}\right)^2\Bigg)\frac{\partial w}{\partial y} + \frac{k_2}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial w}{\partial y}\Bigg]\Bigg\} \\
& + h\frac{\partial}{\partial x}\Bigg[\frac{C_{111}}{8}\left(\frac{\partial w}{\partial x}\right)^5 + \frac{3C_{222} - C_{111}}{4}\left(\frac{\partial w}{\partial x}\right)^3 \\
& \times\left(\frac{\partial w}{\partial y}\right)^2 + \frac{3C_{111} - 2C_{222}}{8}\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial y}\right)^4\Bigg] \\
& + h\frac{\partial}{\partial y}\Bigg[\frac{C_{222}}{6}\left(\frac{\partial w}{\partial y}\right)^5 + \frac{3C_{222} - C_{111}}{8}\left(\frac{\partial w}{\partial x}\right)^4\frac{\partial w}{\partial y} \\
& + \frac{3C_{111} - 2C_{222}}{4}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^3\Bigg] \\
& = f(x, y, t).
\end{aligned} \tag{12}$$

对应方程 (12) 的势能量泛函为, $\varPi = U - W$. 方程 (12) 可通过势能泛函取极小得到, 即

$$\delta\varPi = \delta(U - W) = 0. \tag{13}$$

直接求解非线性方程 (12) 是困难的. 本文使用 Ritz 法近似求解泛函 (13), 来得到方程 (12) 的近似解. 设 N_x, N_{xy}, N_y 为常数, 并取忽略非线性项时方程 (12) 的解为基函数, 则泛函 (13) 的解表示为[17,20]

$$w(x, y) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} A_{ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}.$$

一次近似时, 取 $i = j = 1$, 则上式简化为

$$w(x, y) = A_{11}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}. \tag{14}$$

把 (14) 式代入 (13) 式, 积分后可得

$$\begin{aligned}
\varPi = {} & \left[\frac{9\pi^4 k_1 ab}{64}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)^2 - \frac{N_x\pi^2 b}{8a} - \frac{N_y\pi^2 a}{8b}\right]A_{11}^2 \\
& + \frac{k_3\pi^4}{64}\left[\frac{9b}{a^3} + \frac{9a}{b^3} + \left(\frac{3k_2}{4} + 2\right)\frac{1}{ab}\right]A_{11}^4 \\
& + \frac{\pi^6}{256}\Bigg[\frac{25C_{111}b}{48a^5} + \frac{25C_{222}a}{48b^5} + \frac{(3C_{222} - C_{111})}{16a^3b}
\end{aligned}$$

$$+\frac{(3C_{111}-2C_{222})}{16ab^3}\Bigg]A_{11}^6-\int_0^a\int_0^b A_{11}f(x,y)\times\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\mathrm{d}x\mathrm{d}y. \quad (15)$$

若 $f(x,y)$ 为作用在板中点的集中载荷时, 可把 $f(x,y)$ 展成Fourier级数

$$f=\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}f_{ij}\sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b},$$

Fourier系数可表示为[17]

$$f_{ij}=\frac{4f}{ab}\sin\frac{i\pi}{2}\sin\frac{j\pi}{2}.$$

此时有

$$\int_0^a\int_0^b A_{11}f(x,y)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)\mathrm{d}x\mathrm{d}y=A_{11}f.$$

根据Ritz法, 从 $\dfrac{\partial\Pi}{\partial A_{11}}=0$, 可得确定弯曲幅值的代数方程

$$\lambda_1A_{11}^5+\lambda_2A_{11}^3+\lambda_3A_{11}+\lambda_4=0, \quad (16)$$

其中

$$\lambda_1=\frac{6\pi^6}{256a^4}\left[\frac{25C_{111}\xi}{48}+\frac{25C_{222}}{48\xi^5}+\frac{(3C_{222}-C_{111})}{16\xi}+\frac{(3C_{111}-2C_{222})}{16\xi^3}\right],$$

$$\lambda_2=\frac{k_3\pi^4}{24}\left[\frac{9b}{a^3}+\frac{9a}{b^3}+\left(\frac{3k_2}{4}+2\right)\frac{1}{ab}\right],$$

$$\lambda_3=\frac{\pi^4k_1ab}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2-\frac{N_x\pi^2b}{4a}-\frac{N_y\pi^2a}{4b},$$

$$\lambda_4=-f.$$

方程(16)为五次代数方程, 没有解析表达, 需要通过数值方法来求解.

3.2 边界均匀受力的稳定性

本节研究结构边界双向均匀受力时的屈曲失稳问题. 令 $f=0$, 则(16)式简化为

$$\lambda_1A_{11}^5+\lambda_2A_{11}^3+\lambda_3A_{11}=0, \quad (17)$$

则在仅有边界轴力作用时的解为

$$A_{11}=0 \quad \text{或} \quad A_{11}^2=\frac{-\lambda_2\pm\sqrt{\lambda_2^2-4\lambda_1\lambda_3}}{2\lambda_1}. \quad (18)$$

从(18)式可知, 考虑物理非线性时板屈曲失稳后的弯曲幅值和轴向载荷成非线性关系. 结构失稳的临界载荷可从 $\lambda_3=0$ 得到

$$\frac{\pi^2k_1ab}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2=\frac{N_x^{\mathrm{cr}}b}{4a}+\frac{N_y^{\mathrm{cr}}a}{4b}. \quad (19)$$

在 $N_y=0$ 或 $N_x=0$ 时的临界载荷分别为

$$\frac{\pi^2k_1ab}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2=\frac{N_x^{\mathrm{cr}}b}{4a}, \quad (20\mathrm{a})$$

$$\frac{\pi^2k_1ab}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2=\frac{N_y^{\mathrm{cr}}a}{4b}. \quad (20\mathrm{b})$$

从临界载荷的表达式可知, 临界载荷和结构的物理非线性无关.

4 算例及讨论

根据前面的讨论, 有

$$k_1=\frac{Eh^3}{24(1-\nu^2)}\approx1.6875\times10^{-18},$$

$$k_2=2(1-\nu)=1.628,$$

$$k_3=\frac{Eh}{2(1-\nu^2)}=1.8044\times10^2.$$

若令 $a=1$ μm, $b=\xi a$, 有

$$\lambda_1=\frac{150hA_1\pi^6}{4096a^4}\left(\frac{1+\xi^6}{\xi^5}\right),$$

$$\lambda_2=\frac{k_3\pi^4}{24a^2}\left[9\xi+\frac{9}{\xi^3}+\left(\frac{3k_2}{4}+2\right)\frac{1}{\xi}\right],$$

$$\lambda_3=\frac{\pi^4k_1}{2a^2}\left(\xi+\frac{1}{\xi}\right)^2-\frac{N_x\pi^2\xi}{4}-\frac{N_y\pi^2}{4\xi},$$

$$\lambda_4=-\frac{f}{\xi a^2}.$$

当板面在垂向(z方向)作用集中载荷时, 石墨烯片的弯曲幅值可由(16)式确定. 令 $\xi=1$, 即取为正方形板, 在 $N_x=0$时, 有如下挠曲幅值和载荷幅值的关系(图3).

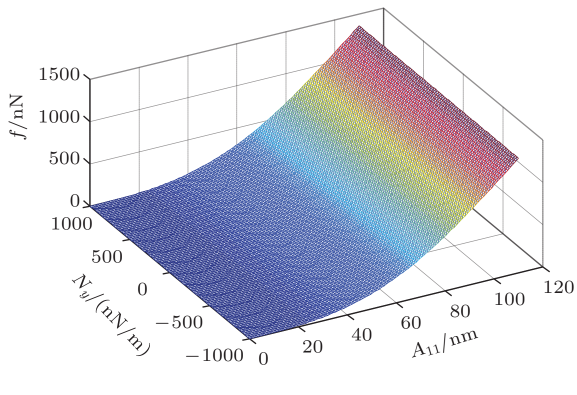

图3 弯曲幅值 A_{11} 随载荷 f 与 N_y 的变化

此外, N_x 对石墨烯片的弯曲变形也有影响. 当 N_x 大于临界为压力时结构将出现屈曲, 此时即使 z 轴方向的集中载荷为零也会产生垂向变形, 如图4中的 A_{11}^{cr}, 且 A_{11}^{cr} 可通过(18)式确定. 从图4还

可看出, 本构方程中物理非线性造成五次非线性项增大了结构变形, 起到刚度软化的作用, 而且随着变形的增加影响越显著. 文献[13]中采用直径1 μm的圆形单层石墨烯片进行实验测量, 当板中点作用1000 nN时板中点位移约110 nm, 和本文的结果接近.

从方程(19)可得结构在边界均匀受力时的临界载荷为

$$N_x^{\mathrm{cr}}\xi+\frac{N_y^{\mathrm{cr}}}{\xi}=\frac{2k_1\pi^2\xi^2}{a^2}\left(1+\frac{1}{\xi^2}\right)^2. \tag{21}$$

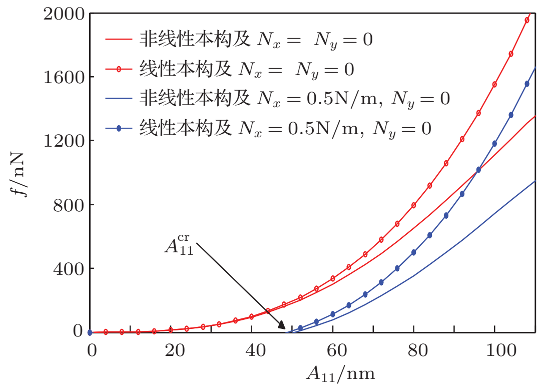

图4 边界均匀压力对石墨烯片弯曲变形的影响

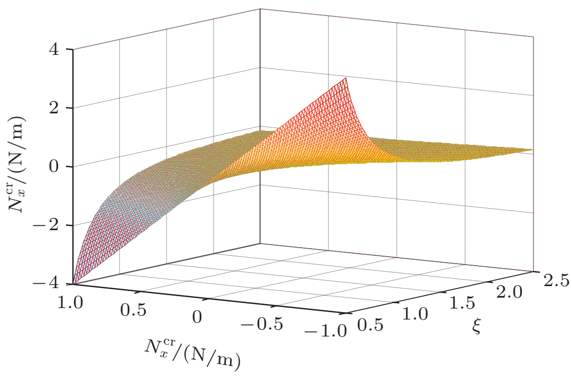

图5 边界临界载荷与长宽比 ξ 的关系

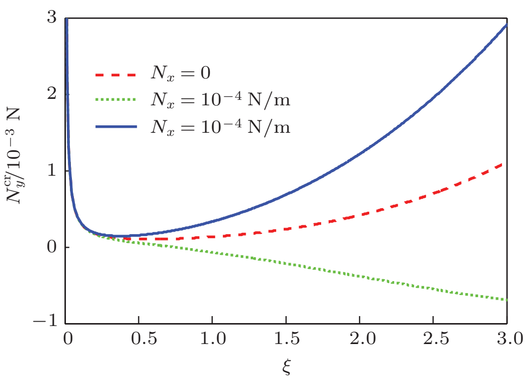

图6 边界临界载荷 N_y^{cr} 与长宽比 ξ 的关系

取 a = 1 μm, 则从(21)可得临界载荷与板长宽比的关系(图5). 对特定的 N_x 可得图6.

图5、图6表明, 单层石墨烯片的几何尺寸对结构屈曲有重要影响. 当 $a/b < 0.05$ 时临界载荷快速增加; 而当 $a/b > 0.05$ 时, 临界载荷的变化将趋缓. 而且当 ξ 较大, 平行两边受压时, 相邻平行两边受拉才能保证结构不发生屈曲, 例如图6中 $N_x = 10^{-4}$ N的情况.

5 结 论

本文通过平方非线性的本构关系和Hamilton原理, 建立了单层石墨烯片的非线性板动力学模型, 并通过Ritz法求解了四边简支矩形板的静力弯曲和边界均匀受力时的静力屈曲问题. 结果显示, 基于非线性本构关系的板模型能很好的描述单层石墨烯片的力学行为.

参考文献

[1] Singh V, Joung D, Zhai L, Das S, Khondaker S I, Seal S 2011 *Progress in Materials Science* **56** 1178

[2] Geim A K, Novoselov K S 2007 *Nature Materials* **6** 183

[3] Han T W, He P F 2010 *Acta Phys. Sin.* **59** 3408 (in Chinese) [韩同伟, 贺鹏飞 2010 物理学报 **59** 3408]

[4] Ouyang Y, Peng J C, Wang H, Yi S P 2008 *Acta Phys. Sin.* **57** 615 (in Chinese) [欧阳玉, 彭景翠, 王慧, 易双萍 2008 物理学报 **57** 615]

[5] Zhu G B, Zhang P 2013 *Chin. Phys. B* **22** 017303

[6] Sun L F, Dong L M, Fang C 2013 *Chin. Phys. B* **22** 047203

[7] Yakobson B I, Brabec C J, Bernholc J 1996 *Phys. Rev. Lett.* **76** 2511

[8] Ouyang Z C, Su Z B, Wang C L 1997 *Phys. Rev. Lett.* **78** 4055

[9] Tu Z, Ouyang Z C 2002 *Phy. Rev. B* **65** 233407

[10] Zhou X, Zhou J J, Ouyang Z C 2000 *Phys. Rev. B* **62** 13692

[11] Dong J I, Gao X, Kong X Y, Li J M 2007 *Chin. Phys. Lett.* **24** 165

[12] Kong Y, Li F S, Yang S J K 2007 *Chin. Phys. Lett.* **24** 2036

[13] Lee C, Wei X, Kysar J W, Hone J 2008 *Science* **321** 385

[14] Cadelano E, Palla P L, Giordano S, Colombo L 2009 *Phys. Rev. Lett.* **102** 235502

[15] Wu J, Hwang K C, Huang Y 2008 *J. Mech. Phys. Solids* **56** 279

[16] Pradhan S C, Phadikar J K 2009 *Phys. Lett. A* **373** 1062

[17] Hwang K C, Xia Z X 1987 *Theory of plate and shell* (Beijing: Tsinghua University Press) p6 (in Chinese) [黄克智, 夏之熙 1987 板壳理论(北京: 清华大学出版社)第6页]

[18] Washizu K 1975 *Variational methods in elasticity and plasticity* (Oxford: Pergamon Press) p5

[19] Nayfeh A H, Mook D T 2008 *Nonlinear oscillations* (New York: Wiley. Com) p1

[20] Wu L Y 1989 *Theory of plate and shell* (Shanghai: Shanghai Jiaotong University Press) p5 (in Chinese) [吴连元 1989 板壳理论 (上海: 上海交通大学出版社) 第 5 页]

A nonlinear plate theory for the monolayer graphene*

Huang Kun[1)] Yin Ya-Jun[2)†] Wu Ji-Ye[3)]

1) (*Department of Engineering Mechanics, Faculty of Civil Engineering and Mechanics, Kunming University of Science and Technology, Kunming 650500, China*)

2) (*Department of Engineering Mechanics, School of Aerospace, Tsinghua University, Beijing 100084, China*)

3) (*Division of Mechanics, Nanjing University of Technology, Nanjing 211816, China*)

(Received 25 December 2013; revised manuscript received 21 April 2014)

Abstract

In the present paper, the kinematic equation of a monolayer graphene is proposed based on a plate theory, and the nonlinear elasticity stress-strain relations are obtained from experiments. The equation includes cubic and quintic nonlinearities. The bending produced when subjected to a concentrated force at the center of the plate and the static buckling arising from edge in-plane axial uniform loads are investigated using Ritz methods for a simply-supported rectangular plate. Results suggest that the plate theory with nonlinear constitutive equation may characterize the mechanical property of a monolayer graphene appropriately, and the quintic nonlinearities have a significant effect on the bending deformations of the graphene.

Keywords: monolayer graphene, nonlinear constitutive equation, plate theory, Ritz methods

PACS: 62.20.Dc, 62.25.+g, 81.40.Jj, 81.40.Lm **DOI:** 10.7498/aps.63.156201

* Project supported by the National Natural Science Foundation of China (Grant Nos. 11272175, 11072125), and the Natural Science Foundation of Jiangsu province, China (Grant No. BK20130910).

† Corresponding author. E-mail: yinyj@tsinghua.edu.cn