变质量Chetaev型非完整系统Appell方程Mei对称性的共形不变性与守恒量*

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研究了变质量Chetaev型非完整系统Appell方程Mei对称性的共形不变性和守恒量. 在群的无限小变换下, 定义了变质量Chetaev型非完整系统Appell方程Mei对称性和共形不变性, 给出了该系统Mei对称性的共形不变性确定方程, 并推导出系统相应的守恒量表达式. 最后, 给出了应用算例.

关键词: 变质量, Chetaev型非完整系统, Appell方程, 共形不变性

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1 引 言

随着人们对Noether 理论[1]认识的逐渐加深, 动力学系统的Noether 对称性、Lie 对称性和Mei对称性及其守恒量的研究取得了很大的进展[2−12]. 共形不变性是研究约束力学系统对称性的一种重要的方法. 1997年, Galiullin等[13]研究了特殊无限小变换下Birkhoff系统的共形不变性. 近几年, 国内学者在研究Lagrange系统、Hamilton系统、机电系统等系统的共形不变性方面已做了很多工作[14−25].

为了适应空间技术和其他工业技术的进一步发展, 研究火箭、喷气式飞机等变质量系统的动力学理论显得日益重要[26]. 郑世旺等[27]研究了变质量非完整系统Tzénoff方程的Lie对称性及其导出的守恒量; 徐超和李元成[28]研究了奇异变质量单面非完整系统Nielsen方程的Noether-Lie对称性与守恒量; 贾利群等[29]研究了相对运动变质量力学系统Appell方程的广义Lie对称性导致的广义Hojman守恒量; 张斌等[30]研究了变质量非完整系统的Lagrange对称性与守恒量. 但是作者目前尚未见到关于变质量Chetaev型非完整系统Appell方程Mei对称性的共形不变性的报道. 本文给出了变质量Chetaev型非完整系统Appell方程Mei对称性的共形不变性定义, 得到了该系统Mei对称性的共形不变性确定方程, 并推导出系统的Mei守恒量表达式.

2 变质量Chetaev型非完整系统的运动微分方程

研究由N个质点组成的变质量力学系统. 由n个广义坐标$q_s(s=1,2,\cdots,n)$确定系统的位形, 且系统的运动受到g个彼此相容且独立的双面理想Chetaev型非完整约束,

$$f_\beta(t,\boldsymbol{q},\dot{\boldsymbol{q}})=0 \quad (\beta=1,2,\cdots,g). \tag{1}$$

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约束方程(1)加在虚位移δq_s上的Chetaev条件为

$$\frac{\partial f_\beta}{\partial \dot{q}_s}\delta q_s = 0 \quad (\beta = 1, 2, \cdots, g). \tag{2}$$

变质量Chetaev型非完整系统的运动微分方程可表示为Appell方程,

$$\begin{aligned}\frac{\partial S}{\partial \ddot{q}_s} &= Q_s + P_s + \lambda_\beta \frac{\partial f_\beta}{\partial \dot{q}_s}\\ &= Q_s + P_s + \varGamma_s \quad (s = 1, 2, \cdots, n),\end{aligned} \tag{3}$$

其中, S是系统的Appell函数, S是关于$t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$的函数; λ_β是与第β个约束对应的约束乘子, λ_β是关于$t, \boldsymbol{q}, \dot{\boldsymbol{q}}$的函数; Q_s是与第s个广义坐标q_s对应的非势广义力, Q_s是关于$t, \boldsymbol{q}, \dot{\boldsymbol{q}}$的函数; $\varGamma_s$和P_s分别是与第s个广义坐标q_s对应的约束力和广义反推力,

$$\varGamma_s = \varGamma_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \lambda_\beta \frac{\partial f_\beta}{\partial \dot{q}_s}, \tag{4}$$

$$P_s = \dot{m}_i(\boldsymbol{u}_i + \dot{\boldsymbol{r}}_i)\cdot\frac{\partial \boldsymbol{r}_i}{\partial q_s} - \frac{1}{2}\dot{\boldsymbol{r}}_i \cdot \dot{\boldsymbol{r}}_i \frac{\partial m_i}{\partial q_s}. \tag{5}$$

这里, $s = 1, 2, \cdots, n$; $m_i = m_i(t, \boldsymbol{q})$和$\dot{\boldsymbol{r}}_i$分别为$t$时刻第$i$个质点的质量和速度; $\boldsymbol{u}_i$为微粒相对于第i个质点的相对速度.

在积分运动微分方程之前, 可由方程(1)和(3)求出约束乘子. 令

$$\varLambda_s = \varLambda_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = Q_s + \varGamma_s \quad (s = 1, 2, \cdots, n),$$

则方程(3)改写为

$$\frac{\partial S}{\partial \ddot{q}_s} = \varLambda_s + P_s \quad (s = 1, 2, \cdots, n). \tag{6}$$

方程(6)称为变质量Chetaev型非完整系统的Appell方程,其中$\varLambda_s$为与第s个广义坐标q_s对应的广义合力.

如果系统所受的约束满足方程(1), 那么方程(6)的解则给出了变质量Chetaev型非完整系统的运动规律. 利用方程(6)可求出所有广义加速度$\ddot{q}_s$,

$$\ddot{q}_s = \alpha_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) \quad (s = 1, 2, \cdots, n). \tag{7}$$

3 变质量Chetaev型非完整系统Appell方程Mei对称性的共形不变性

引入时间t和广义坐标q_s的无限小变换的展开式,

$$\begin{aligned}t^* &= t + \varepsilon\xi_0(t, \boldsymbol{q}, \dot{\boldsymbol{q}}),\\ q_s^*(t^*) &= q_s(t) + \varepsilon\xi_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}})\\ &\qquad (s = 1, 2, \cdots, n),\end{aligned} \tag{8}$$

其中, ε为无限小参数; ξ_0, ξ_s为无限小变换生成元. 由(8)式得

$$\begin{aligned}\frac{\mathrm{d}q_s^*}{\mathrm{d}t^*} &= \frac{\mathrm{d}q_s + \varepsilon\mathrm{d}\xi_s}{\mathrm{d}t + \varepsilon\mathrm{d}\xi_0}\\ &= \dot{q}_s + \varepsilon(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0) + O(\varepsilon^2),\\ \frac{\mathrm{d}^2 q_s^*}{\mathrm{d}t^{*2}} &= \ddot{q}_s + \varepsilon[(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0)^{\cdot} - \ddot{q}_s\dot{\xi}_0] + O(\varepsilon^2).\end{aligned} \tag{9}$$

假设系统的动力学函数S, $\varLambda_s$, P_s和f_β经历无限小变换后分别变为S^*, $\varLambda_s^*$, P_s^*和f_β^*, 将S^*, $\varLambda_s^*$, P_s^*和f_β^*在$(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$处做Taylor级数展开得

$$S^* = S(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) + \varepsilon\tilde{X}^{(2)}(S) + O(\varepsilon^2), \tag{10}$$

$$\begin{aligned}\varLambda_s^* &= \varLambda_s^*\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\ &= \varLambda_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) + \varepsilon\tilde{X}^{(1)}(\varLambda_s)\\ &\quad + O(\varepsilon^2) \quad (s = 1, 2, \cdots, n),\end{aligned} \tag{11}$$

$$\begin{aligned}P_s^* &= P_s\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\ &= P_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) + \varepsilon\tilde{X}^{(1)}(P_s)\\ &\quad + O(\varepsilon^2) \quad (s = 1, 2, \cdots, n),\end{aligned} \tag{12}$$

$$\begin{aligned}f_\beta^* &= f_\beta\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right)\\ &= f_\beta\left(t, \boldsymbol{q}, \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t}\right) + \varepsilon\tilde{X}^{(1)}(f_\beta)\\ &\quad + O(\varepsilon^2) \quad (\beta = 1, 2, \cdots, g).\end{aligned} \tag{13}$$

引进无限小变换生成元向量

$$\tilde{X}^{(0)} = \xi_0\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q_s}, \tag{14}$$

以及它的一次扩展$\tilde{X}^{(1)}$和二次扩展$\tilde{X}^{(2)}$,

$$\tilde{X}^{(1)} = \tilde{X}^{(0)} + \left[\frac{\bar{\mathrm{d}}\xi_s}{\mathrm{d}t} - \dot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\right]\frac{\partial}{\partial \dot{q}_s}, \tag{15}$$

$$\begin{aligned}\tilde{X}^{(2)} = \tilde{X}^{(1)} + \bigg[&\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\bar{\mathrm{d}}\xi_s}{\mathrm{d}t} - \dot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\right)\\ &- \ddot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\bigg]\frac{\partial}{\partial \ddot{q}_s},\end{aligned} \tag{16}$$

其中

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \dot{q}_s\frac{\partial}{\partial q_s} + \alpha_s\frac{\partial}{\partial \dot{q}_s} + \dot{\alpha}_s\frac{\partial}{\partial \ddot{q}_s}. \tag{17}$$

定义1 用无限小变换后的动力学函数S^*, P_s^*, $\varLambda_s^*$, f_β^*代替变换前的动力学函数S, P_s, $\varLambda_s$, f_β

时, 如果系统运动微分方程的形式保持不变, 则称这种对称性为Mei对称性.

根据Mei对称性的定义, 方程(6)的Mei对称性可表示为

$$\frac{\partial S^*}{\partial \ddot{q}_s} = \Lambda_s^* + P_s^* \quad (s=1,2,\cdots,n), \tag{18}$$

方程(1)的Mei对称性可表示为

$$\begin{aligned} f_\beta^* &= f_\beta\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}q^*}{\mathrm{d}t^*}\right) \\ &= 0 \quad (\beta=1,2,\cdots,g). \end{aligned} \tag{19}$$

将(10), (11), (12)式代入(18)式, 并忽略ε^2以上的高阶项, 利用方程(6)可得

$$\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s) = 0. \tag{20}$$

方程(20)称为变质量Chetaev型非完整系统Appell方程Mei对称性的判据方程.

将(13)式的高阶项略去, 利用方程(1)和(19)式可得到变质量Chetaev型非完整约束方程(1)在无限小变换(8)式下不变性的限制方程为

$$\tilde{X}^{(1)}[f_\beta(t, \boldsymbol{q}, \dot{\boldsymbol{q}})] = 0. \tag{21}$$

考虑Chetaev约束条件(2)对无限小生成元ξ_0, ξ_s的限制, (2)式可改写为

$$\begin{aligned} &\frac{\partial f_\beta}{\partial \dot{q}_s}(\xi_s - \dot{q}_s \xi_0) \\ &= 0 \quad (\beta=1,2,\cdots,g;\ s=1,2,\cdots,n). \end{aligned} \tag{22}$$

方程(22)称为附加限制方程.

定义2 如果无限小变换(8)式的无限小生成元ξ_0, ξ_s使判据方程(20)和约束限制方程(21)成立, 则称这种对称性为变质量Chetaev型非完整系统Appell方程(6)的弱Mei对称性. 如果无限小生成元ξ_0, ξ_s使判据方程(20)、约束限制方程(21)和附加限制方程(22)成立, 则称这种对称性为变质量Chetaev型非完整系统Appell方程(6)的强Mei对称性.

定义3 对于变质量Chetaev型非完整系统的Appell方程(6), 如果存在矩阵$\boldsymbol{M}_s^k$满足

$$\begin{aligned} &\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s) \\ &= \boldsymbol{M}_s^k\left(\frac{\partial S}{\partial \ddot{q}_k} - \Lambda_k - P_k\right) \\ &\quad (s,k=1,2,\cdots,n), \end{aligned} \tag{23}$$

则在无限小变换(8)式作用下, 方程(6)可具有Mei对称性的共形不变性. 方程(23)是满足Mei对称性共形不变性的确定方程, 其中$\boldsymbol{M}_s^k$为共形因子.

命题1 如果方程(6)在无限小变换(8)式作用下具有Mei对称性, 且存在矩阵$\boldsymbol{F}_s^k$满足

$$\begin{aligned} &\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s) \\ &\quad - \left\{\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s)\right\}\bigg|_{\frac{\partial S}{\partial \ddot{q}_s} = \Lambda_s + P_s} \\ &= \boldsymbol{F}_s^k\left(\frac{\partial S}{\partial \ddot{q}_k} - \Lambda_k - P_k\right) \\ &\quad (s,k=1,2,\cdots,n), \end{aligned} \tag{24}$$

则方程(6)在无限小变换(8)式作用下具有共形不变性, 且同时具有Mei对称性的充分必要条件为$\boldsymbol{M}_s^k = \boldsymbol{F}_s^k$.

证明 由于方程(6)的Mei对称性满足(24)式, 如果存在一个矩阵$\boldsymbol{F}_s^k$满足(24)式, 则(24)式改写为

$$\begin{aligned} &\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s) \\ &= \boldsymbol{F}_s^k\left(\frac{\partial S}{\partial \ddot{q}_k} - \Lambda_k - P_k\right) \\ &\quad (s,k=1,2,\cdots,n). \end{aligned} \tag{25}$$

由方程(23)可知, 系统的共形因子$\boldsymbol{M}_s^k = \boldsymbol{F}_s^k$.

同时, 由方程(23)和(24)式可知,

$$\begin{aligned} &(\boldsymbol{M}_s^k - \boldsymbol{F}_s^k)\left(\frac{\partial S}{\partial \ddot{q}_k} - \Lambda_k - P_k\right) \\ &= \left\{\frac{\partial}{\partial \ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\Lambda_s + P_s)\right\}\bigg|_{\frac{\partial S}{\partial \ddot{q}_s} = \Lambda_s + P_s} \\ &\quad (s,k=1,2,\cdots,n). \end{aligned} \tag{26}$$

如果$\boldsymbol{M}_s^k = \boldsymbol{F}_s^k$, 则可得到(20)式. 因此, 变质量Chetaev型非完整系统Appell方程具有Mei对称性.

定义4 如果无限小变换(8)式的无限小生成元ξ_0, ξ_s使Mei对称性共性不变形的确定方程(23)和约束限制方程(21)成立, 则这种共形不变性为变质量Chetaev 型非完整系统Appell方程(6)的弱Mei对称性的共形不变性. 如果无限小生成元ξ_0, ξ_s使Mei对称性共性不变形的确定方程(23)、约束限制方程(21)和附加限制方程(22)成立, 则这种共形不变性为变质量Chetaev型非完整系统Appell方程(6)的强Mei对称性的共形不变性.

4 变质量Chetaev型非完整系统的结构方程和守恒量

命题2 对于满足Mei对称性判据方程(20)的无限小生成元ξ_0, ξ_s, 如果存在规范函数$G_{\rm M} = G_{\rm M}(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$满足如下结构方程:

$$\begin{aligned}&\tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} + \tilde{X}^{(1)}[\tilde{X}^{(2)}(S)]\\&+ (\xi_s - \dot{q}_s\xi_0)\tilde{E}_s[\tilde{X}^{(2)}(S)]\\&+ \xi_0[\tilde{X}^{(2)}(\varLambda_s + P_s)]\frac{\bar{\rm d}\alpha_s}{{\rm d}t} + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t} = 0,\end{aligned}\tag{27}$$

则变质量Chetaev型非完整系统Appell方程(6)的Mei对称性、弱Mei对称性、强Mei对称性的共形不变性导致的Mei守恒量为

$$\begin{aligned}I_{\rm M} = {}&\xi_0\tilde{X}^{(2)}(S) + \frac{\partial \tilde{X}^{(2)}(S)}{\partial \dot{q}_s}(\xi_s - \dot{q}_s\xi_0) + G_{\rm M}\\={}&\text{const}.\end{aligned}\tag{28}$$

5 算 例

变质量Chetaev型非完整约束系统的Appell函数为

$$S = \frac{1}{2}m(t)(\ddot{q}_1^2 + \ddot{q}_2^2) + \dot{m}(t)q_1,\tag{29}$$

其中

$$m(t) = m_0 t \quad (m_0\text{为常数}),\tag{30}$$

非势广义力

$$\begin{aligned}Q_1 &= -\frac{1}{t(1+t^2)}m(t)\dot{q}_1,\\Q_2 &= -\frac{1}{t(1+t^2)}m(t)\dot{q}_1.\end{aligned}\tag{31}$$

此系统所受的Chetaev型非完整约束方程为

$$f = \dot{q}_2 - t\dot{q}_1 = 0.\tag{32}$$

设微粒并入的绝对速度为零. 下面研究此系统的Mei对称性、共形不变性和Mei守恒量.

由方程(5)可知,

$$\begin{aligned}P_1 &= 0,\\P_2 &= 0.\end{aligned}\tag{33}$$

由方程(6)和(33)式得

$$m(t)\ddot{q}_1 = -\frac{1}{t(1+t^2)}m(t)\dot{q}_1 - \lambda t,$$

$$m(t)\ddot{q}_2 = -\frac{1}{(1+t^2)}m(t)\dot{q}_1 + \lambda.\tag{34}$$

于是有

$$\begin{aligned}\ddot{q}_1 &= -\frac{\lambda t}{m(t)} - \frac{1}{t(1+t^2)}\dot{q}_1,\\\ddot{q}_2 &= -\frac{1}{(1+t^2)}\dot{q}_1 + \frac{\lambda}{m(t)}.\end{aligned}\tag{35}$$

由(32)式得

$$\ddot{q}_2 - \dot{q}_1 - t\ddot{q}_1 = 0.\tag{36}$$

将(35)式代入(36)式得

$$\lambda = \frac{m(t)\dot{q}_1}{1+t^2},\tag{37}$$

再将(37)式代入(35)式得

$$\begin{aligned}\ddot{q}_1 &= \alpha_1 = -\frac{\dot{q}_1}{t},\\\ddot{q}_2 &= \alpha_2 = 0.\end{aligned}\tag{38}$$

取无限小变换生成元

$$\begin{aligned}\xi_0 &= 0,\\\xi_1 &= t\dot{q}_1 + \dot{q}_2,\\\xi_2 &= -t\dot{q}_1 + t\dot{q}_2 - q_2,\end{aligned}\tag{39}$$

则

$$\begin{aligned}&\frac{\partial}{\partial \ddot{q}_1}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\varLambda_1 + P_1) = \ddot{q}_1 + \frac{\dot{q}_1}{t},\\&\frac{\partial}{\partial \ddot{q}_2}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\varLambda_2 + P_2) = \ddot{q}_2.\end{aligned}\tag{40}$$

由方程(23)和(40)式可得到共形因子

$$\boldsymbol{M} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}.\tag{41}$$

将(38)式代入(40)式得

$$\begin{aligned}&\frac{\partial}{\partial \ddot{q}_1}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\varLambda_1 + P_1) = 0,\\&\frac{\partial}{\partial \ddot{q}_2}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(2)}(\varLambda_2 + P_2) = 0.\end{aligned}\tag{42}$$

容易证明, 无限小变换生成元(39)式满足约束限制方程(21), 但不满足附加限制方程(22). 因此, 相应的共形不变性既是变质量Chetaev型非完整系统Appell方程的Mei对称性的共形不变性, 也是变质量Chetaev型非完整系统Appell方程的弱Mei对称性的共形不变性. 又因为

$$\tilde{X}^{(2)}(S) = m_0\xi_1,\tag{43}$$

$$\tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] = m_0(t\dot{\xi}_1 + \dot{\xi}_2),\tag{44}$$

$$\tilde{E}_1[\tilde{X}^{(2)}(S)] = m_0,\tag{45}$$

$$\tilde{E}_2[\tilde{X}^{(2)}(S)] = 0, \tag{46}$$

将 (39), (43)—(46) 式代入结构方程 (27) 可得

$$G_{\mathrm{M}} = -m_0 t(t\dot{q}_1 + \dot{q}_2). \tag{47}$$

再由 (28) 式可得

$$I_{\mathrm{M}} = m_0(t\dot{q}_2 - t\dot{q}_1 - q_2) = \mathrm{const}. \tag{48}$$

6 结 论

在群的无限小变换下, 变质量 Chetaev 型非完整系统 Appell 方程存在 Mei 对称性的共形不变性. 如果无限小变换生成元满足系统的 Mei 对称性共形不变性的确定方程、限制方程 (或确定方程、限制方程及附加限制方程) 时, 可得到系统弱 (强)Mei 对称性的共形不变性, 并且当满足一定条件时系统 Mei 对称性的共形不变性可导致相应的 Mei 守恒量.

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Conformal invariance and conserved quantity of Mei symmetry for Appell equations in nonholonomic systems of Chetaev's type with variable mass*

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Abstract

Conformal invariance and conserved quantity of Mei symmetry for Appell equations of nonholonomic system of Chetaev's type with variable mass are studied. The conformal invariance and Mei symmetry for Appell equations of nonholonomic systems of Chetaev's type with variable mass are defined under the infinitesimal transformation of group, and the determining equations of conformal invariance of Mei symmetry for the system are given. Then, the expression of the corresponding conserved quantity of the system is derived. Finally, an example is given to illustrate the application of the results.

Keywords: variable mass, nonholonomic systems of Chetaev's type, Appell equation, conformal invariance

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