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Thermo-vacuum state in a negative binomial optical field and its application

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对应负二项式光场的热真空态及其应用*

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有限温度下的光场理论的核心是引入热真空态, 它也是利用量子统计手段全面研究电磁场的基础. 本文在 Takahashi 和 Umezawa 的热场动力学理论基础上, 首次采用有序算符内的积分方法对负二项式光场 $\rho_s=\sum_{n=0}^{\infty}\binom{n+s}{n}\gamma^{s+1}(1-\gamma)^n\left|n\right\rangle\left\langle n\right|$, 寻找相应的热真空态. 发现该热真空态是基于在混沌光场所对应的热真空态上的虚模激发, 或取负二项式纯态的形式 $\sum_{n=0}^{\infty}\sqrt{\binom{n+s}{n}\gamma^{s+1}(1-\gamma)^n}\left|n,\tilde{s}+\tilde{n}\right\rangle$, 其中 "$\tilde{s}$" 代表虚模自由度. 对此热真空态求纯态平均可方便地得到负二项式光场的 Wigner 函数和光子数涨落.

关键词: 负二项式光场, 热真空态, 有序算符内的积分方法, 虚模激发

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1 引 言

由于电磁场是具有无穷多模式的系统, 并且每个模式需用其所处的量子态作统计描述, 所以要想全面地描述电磁场就必需用量子统计手段, 即以密度算符为研究对象. 一般而言, 密度算符是混合态. 在量子光学和量子统计理论中常把求力学量 A 在混合态 (由密度算符 $\rho=\mathrm{e}^{-\beta H}/Z$ 表示, H 是系统的哈密顿量, $Z=\mathrm{tr}\mathrm{e}^{-\beta H}$ 是配分函数) 的系综平均

$$\langle A\rangle=\mathrm{tr}\left(\rho A\right),\tag{1}$$

简化为对某个纯态的平均值的计算, 即寻找 $\left|\psi\left(\beta\right)\right\rangle$ 使得

$$\langle A\rangle=\left\langle\psi\left(\beta\right)\right|A\left|\psi\left(\beta\right)\right\rangle,\tag{2}$$

这里 $\beta=1/kT$, k 是 Boltzmann 常数. 这样的计算有其便利之处, 但由于 $\left|\psi\left(\beta\right)\right\rangle$ 是在实-虚双模空间中定义的, 因此其代价是需要对系统空间扩增一个虚空间 [1,2]. 例如对于有限温度 T 下的混沌光场

$$\rho_{\mathrm{c}}=\sum_{n=0}^{\infty}\gamma(1-\gamma)^n\left|n\right\rangle\left\langle n\right|,\quad\mathrm{Tr}\rho_{\mathrm{c}}=1,$$
$$\left|n\right\rangle=a^{\dagger n}\left|0\right\rangle/\sqrt{n!},\ \gamma=1-\mathrm{e}^{-\frac{\hbar\omega}{kT}}.\tag{3}$$

平均光子数 $\mathrm{Tr}\left(\rho_{\mathrm{c}}a^{\dagger}a\right)=1/\gamma-1\equiv n_{\mathrm{c}}$, 文献 [3, 4] 已经给出相应的热真空态 (纯态) 是

$$\left|0\left(\beta\right)\right\rangle=\mathrm{sech}\theta\exp\left[a^{\dagger}\tilde{a}^{\dagger}\tanh\theta\right]\left|0,\tilde{0}\right\rangle,\tag{4}$$

其中 θ 满足

$$\tanh\theta=\exp\left(-\frac{\hbar\omega}{2kT}\right)=\sqrt{1-\gamma},\tag{5}$$

$\left|0,\tilde{0}\right\rangle$ 是零温度下的真空态, $a\left|0,\tilde{0}\right\rangle=0$, $\tilde{a}\left|0,\tilde{0}\right\rangle=0$, a 和 $a^{\dagger}$ 是实模湮没算符和产生算符, $\tilde{a}$ 和 $\tilde{a}^{\dagger}$ 是虚模湮没算符和产生算符, 它们满足 $\left[a,a^{\dagger}\right]=\left[\tilde{a},\tilde{a}^{\dagger}\right]=1$. 当我们进一步用 $\left|n,\tilde{n}\right\rangle=\left(a^{\dagger}\tilde{a}^{\dagger}\right)^n\left|0,\tilde{0}\right\rangle/n!$ 将 $\left|0\left(\beta\right)\right\rangle$ 改写为

$$\left|0\left(\beta\right)\right\rangle=\mathrm{sech}\theta\sum_{n=0}^{\infty}\tanh^n\theta\left|n,\tilde{n}\right\rangle$$

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$$= \sum_{n=0}^{\infty} \sqrt{\gamma(1-\gamma)^n} \left| n, \tilde{n} \right\rangle , \qquad (6)$$

就可以觉察到(6)式中的因子 $\sqrt{\gamma(1-\gamma)^n}$ 恰好是(3)式中的因子的开根号.

热真空态理论可以简化统计物理的若干计算, 例如要计算处于混沌光场的平均光子数, 只需算

$$\begin{aligned}
&\langle 0(\beta)| a^{\dagger}a |0(\beta)\rangle \\
&= \sum_{n'=0}^{\infty} \sqrt{\gamma(1-\gamma)^{n'}} \langle n', \tilde{n}'| a^{\dagger}a \\
&\quad \times \sum_{n=0}^{\infty} \sqrt{\gamma(1-\gamma)^{n}} |n, \tilde{n}\rangle \\
&= \gamma \sum_{n=0}^{\infty} n(1-\gamma)^n = \sinh^2\theta \\
&= \frac{1}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1}, \qquad (7)
\end{aligned}$$

而要算混合态的Wigner函数 $\mathrm{tr}(\varDelta(\alpha)\rho)$, 也只需算 $\langle\psi(\beta)|\varDelta(\alpha)|\psi(\beta)\rangle$, 这里 $\varDelta(\alpha)$ 是Wigner算符. 热真空态理论不但简化了计算, 而且给予光场以新的物理解释. 例如(4)式可以看作是双模压缩真空态[5], 其中一个虚模代表热库的影响. 从另一角度看, $|0(\beta)\rangle$ 又是一个纠缠态, 而对纯纠缠态 $|0(\beta)\rangle\langle 0(\beta)|$ 的部分求迹(见(14)式中的推导)[6], 可以导致混合态的出现.

由于热真空态理论已经成为量子光学领域中研究光场特性的重要方法之一. 例如Xu等利用热真空态理论计算了有限温度下双耦合谐振子的热真空态[7], Hu等研究了光子增(减)压缩热态[8], 本文将研究负二项式光场的热真空态及其应用. 一个有趣的问题是对于光场负二项式态(它可以产生于热光子束被原子吸收 s 个光子的过程中)[9]相应的热真空态是什么? 光场负二项式分布的应用很广泛, 例如单模粒子数态 $|s,0\rangle$ 经过双模压缩算符 $\exp\left[\zeta\left(a_1^{\dagger}a_2^{\dagger} - a_1a_2\right)\right]$ 作用后变成

$$\begin{aligned}
|\zeta\rangle_s &= \exp\left[\zeta\left(a_1^{\dagger}a_2^{\dagger} - a_1a_2\right)\right]|s,0\rangle \\
&= (\mathrm{sech}\zeta)^{1+s} \sum_{n=0}^{\infty} \sqrt{\frac{(n+s)!}{n!s!}} (\tanh\zeta)^n \\
&\quad \times |n+s, n\rangle , \qquad (8)
\end{aligned}$$

就呈现负二项式分布

$$(\mathrm{sech}^2\zeta)^{1+s} \frac{(n+s)!}{n!s!} (\tanh^2\zeta)^n. \qquad (9)$$

(8)式也是一个纠缠态. 又如, 涉及Laguerre多项式的负二项式定理给寻找新的光场提供了数理基础[10]. 所以求负二项式态所对应的热真空态问题有其物理价值, 此态可以提供关于负二项式态的新的物理看法. 但是, 由于缺乏有效的理论方法, 长期以来未见有这方面的文献报道. 我们将采用有序算符内的积分方法[11,12]解此问题, 结果发现该热真空态是在混沌光场所对应的热真空态上的虚模激发, 并取负二项纯态的形式(见(24)式). 对此热真空态求纯态平均可方便地得到负二项式光场的Wigner函数和光子数涨落.

2 热真空态所应满足的条件

对于给定的现实的光场如何求热真空态 $|\psi(\beta)\rangle$ 呢? 换言之, $|\psi(\beta)\rangle$ 应该满足什么条件呢? 定义 $\mathrm{Tr} = \mathrm{tr}\,\tilde{\mathrm{tr}}$ 为对实-虚两个空间都求迹的记号, tr只对实空间求迹, $\tilde{\mathrm{tr}}$ 只对虚空间求迹, 那么

$$\begin{aligned}
\langle A\rangle &= \langle\psi(\beta)| A |\psi(\beta)\rangle \\
&= \mathrm{Tr}\left[A|\psi(\beta)\rangle\langle\psi(\beta)|\right] \\
&= \mathrm{tr}\left\{A\left[\tilde{\mathrm{tr}}\,|\psi(\beta)\rangle\langle\psi(\beta)|\right]\right\}. \qquad (10)
\end{aligned}$$

鉴于 $|\psi(\beta)\rangle$ 涉及实-虚两个空间, 因此

$$\tilde{\mathrm{tr}}\,|\psi(\beta)\rangle\langle\psi(\beta)| \neq \langle\psi(\beta)|\,\psi(\beta)\rangle. \qquad (11)$$

对照(1)和(11)式可见待求的 $|\psi(\beta)\rangle$ 应该满足如下公式:

$$\tilde{\mathrm{tr}}\,|\psi(\beta)\rangle\langle\psi(\beta)| = \rho. \qquad (12)$$

我们考察一下(4)式(混沌光场的热真空)是否满足 $\tilde{\mathrm{tr}}\,|0(\beta)\rangle\langle 0(\beta)| = \rho_{\mathrm{c}}$. 为此目的, 用虚模相干态的完备性

$$\begin{aligned}
&\int \frac{\mathrm{d}^2 z}{\pi} |\tilde{z}\rangle\langle\tilde{z}| = 1, \quad |\tilde{z}\rangle = \exp\left[-\frac{|z|^2}{2} + z\tilde{a}^{\dagger}\right]|\tilde{0}\rangle, \\
&\tilde{a}|\tilde{z}\rangle = z|\tilde{z}\rangle \qquad (13)
\end{aligned}$$

看出

$$\begin{aligned}
&\tilde{\mathrm{tr}}\left[|0(\beta)\rangle\langle 0(\beta)|\right] \\
&= \tilde{\mathrm{tr}}\left[\int \frac{\mathrm{d}^2 z}{\pi} |\tilde{z}\rangle\langle\tilde{z}|\,0(\beta)\rangle\langle 0(\beta)|\right] \\
&= \int \frac{\mathrm{d}^2 z}{\pi} \langle\tilde{z}|\,0(\beta)\rangle\langle 0(\beta)\,|\tilde{z}\rangle \\
&= \mathrm{sech}^2\theta \int \frac{\mathrm{d}^2 z}{\pi} \langle\tilde{z}|\,\mathrm{e}^{a^{\dagger}\tilde{a}^{\dagger}\tanh\theta}|0,\tilde{0}\rangle
\end{aligned}$$

$$
\begin{aligned}
&\times \langle 0,\tilde{0}|\, \mathrm{e}^{a\tilde{a}\tanh\theta}\,|\tilde{z}\rangle \\
&= \operatorname{sech}^2\theta \int \frac{\mathrm{d}^2 z}{\pi} \langle \tilde{z}|\, \mathrm{e}^{a^{\dagger} z^{*}\tanh\theta}\,|0,\tilde{0}\rangle \\
&\quad \times \langle 0,\tilde{0}|\, \mathrm{e}^{az\tanh\theta}\,|\tilde{z}\rangle \\
&= \operatorname{sech}^2\theta \int \frac{\mathrm{d}^2 z}{\pi}\, \mathrm{e}^{a^{\dagger} z^{*}\tanh\theta}\,|0\rangle\langle 0|\, \mathrm{e}^{az\tanh\theta}\,\mathrm{e}^{-|z|^2} \\
&= \operatorname{sech}^2\theta \int \frac{\mathrm{d}^2 z}{\pi} :\mathrm{e}^{-|z|^2+a^{\dagger} z^{*}\tanh\theta+az\tanh\theta-a^{\dagger}a}: \\
&= \operatorname{sech}^2\theta :\mathrm{e}^{a^{\dagger}a(\tanh^2\theta-1)}: \\
&\equiv \gamma : \exp\left[-\gamma a^{\dagger} a\right]: \\
&= \operatorname{sech}^2\theta\, \mathrm{e}^{a^{\dagger}a\ln\tanh^2\theta} \overset{\tanh\theta=\exp\left(-\frac{\hbar\omega}{2kT}\right)}{=\!=\!=\!=\!=} \\
&\quad \times \left(1-\mathrm{e}^{-\frac{\hbar\omega}{kT}}\right) \mathrm{e}^{-\frac{\hbar\omega}{kT}a^{\dagger}a} = \rho_{\mathrm{c}},
\end{aligned} \tag{14}
$$

即热真空态 $|0(\beta)\rangle\langle 0(\beta)|$ 对虚模的求迹确实是混沌光场.

3 光场负二项式态对应的热真空

光场负二项式态的原始定义是

$$
\rho_s = \sum_{n=0}^{\infty} \binom{n+s}{n} \gamma^{s+1}(1-\gamma)^n |n\rangle\langle n|, \tag{15}
$$

从负二项式分布

$$
\binom{n+s}{n} \gamma^{s+1}(1-\gamma)^n,\quad 0<\gamma<1,\quad s\geqslant 0, \tag{16}
$$

和负二项式定理

$$
(1+x)^{-(s+1)} = \sum_{n=0}^{\infty} \frac{(n+s)!}{n!s!}(-x)^n, \tag{17}
$$

可知

$$
\operatorname{tr}\rho_s = \gamma^{s+1}\sum_{n=0}^{\infty}\frac{(n+s)!}{n!s!}(1-\gamma)^n = 1. \tag{18}
$$

为了求出光场负二项式态对应的热真空, 我们首先要对其密度矩阵变形, 由上式及真空投影算符的正规乘积表达式 $|0\rangle\langle 0| =: \exp\left(-a^{\dagger}a\right):$, 和 $a|n\rangle = \sqrt{n}\,|n-1\rangle$, 可得

$$
\begin{aligned}
\rho_s &= \frac{\gamma^{s+1}}{s!(1-\gamma)^s} a^s \sum_{n=0}^{\infty}(1-\gamma)^n |n\rangle\langle n|\, a^{\dagger s} \\
&= \frac{\gamma}{s!n_{\mathrm{c}}^s} a^s : \exp\left\{[(1-\gamma)-1]\,a^{\dagger}a\right\} : a^{\dagger s} \\
&= \frac{\gamma}{s!n_{\mathrm{c}}^s} a^s \mathrm{e}^{\lambda a^{\dagger}a} a^{\dagger s},
\end{aligned} \tag{19}
$$

其中

$$
n_{\mathrm{c}} = \frac{1-\gamma}{\gamma},\quad \lambda = \ln(1-\gamma). \tag{20}
$$

(19) 式说明: 当光子计数器检测到一束混沌光的一些光子时也会产生负二项式态. 引入实模相干态

$$
\begin{aligned}
|z\rangle &= \exp\left(-\frac{1}{2}|z|^2 + z a^{\dagger}\right)|0\rangle \\
&= \exp\left(-\frac{1}{2}|z|^2\right)\|z\rangle, \\
\|z\rangle &= \exp\left(z a^{\dagger}\right)|0\rangle,
\end{aligned} \tag{21}
$$

其完备性用有序算符内的积分方法表示为

$$
\begin{aligned}
&\int \frac{\mathrm{d}^2 z}{\pi} |z\rangle\langle z| \\
&= \int \frac{\mathrm{d}^2 z}{\pi} : \exp\left(-|z|^2 + z a^{\dagger} + z^{*} a - a^{\dagger} a\right) : \\
&= 1.
\end{aligned} \tag{22}
$$

再用 $\mathrm{e}^{\lambda a^{\dagger}a} =: \exp\left\{\left[\mathrm{e}^{\lambda}-1\right]a^{\dagger}a\right\}:$ 和有序算符内的积分方法, 有

$$
\begin{aligned}
& a^s\, \mathrm{e}^{\lambda a^{\dagger}a} a^{\dagger s} \\
&= \int \frac{\mathrm{d}^2 z}{\pi} a^s : \exp\Big(-|z|^2 + z^{*} a^{\dagger}\,\mathrm{e}^{\lambda/2} \\
&\quad + z a\, \mathrm{e}^{\lambda/2} - a^{\dagger} a\Big) : a^{\dagger s} \\
&= \int \frac{\mathrm{d}^2 z}{\pi} \mathrm{e}^{-|z|^2} a^s \big\|z^{*}\,\mathrm{e}^{\lambda/2}\big\rangle\big\langle z\,\mathrm{e}^{\lambda/2}\big\| a^{\dagger s}.
\end{aligned} \tag{23}
$$

注意到 $\langle\tilde{0}\,|\tilde{z}\rangle = \mathrm{e}^{-|z|^2/2}$ 和 (20) 式, 将 (23) 式化为

$$
\begin{aligned}
& a^s\, \mathrm{e}^{\lambda a^{\dagger}a} a^{\dagger s} \\
&= \int \frac{\mathrm{d}^2 z}{\pi} z^s z^{*s}\, \mathrm{e}^{\lambda s}\, \mathrm{e}^{z^{*}a^{\dagger}\mathrm{e}^{\lambda/2}} |0\rangle\langle 0| \\
&\quad \times \mathrm{e}^{z a \mathrm{e}^{\lambda/2}} \langle\tilde{z}|\,\tilde{0}\rangle\langle\tilde{0}\,|\tilde{z}\rangle \\
&= \int \frac{\mathrm{d}^2 z}{\pi} \langle\tilde{z}|\, z^s z^{*s}\, \mathrm{e}^{\lambda s}\, \mathrm{e}^{z^{*}a^{\dagger}\mathrm{e}^{\lambda/2}} |0,\tilde{0}\rangle\langle 0,\tilde{0}| \\
&\quad \times \mathrm{e}^{z a \mathrm{e}^{\lambda/2}} |\tilde{z}\rangle \\
&= \int \frac{\mathrm{d}^2 z}{\pi} \langle\tilde{z}|\, \tilde{a}^{\dagger s}\, \mathrm{e}^{\lambda s}\, \mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\mathrm{e}^{\lambda/2}} |0,\tilde{0}\rangle\langle 0,\tilde{0}| \\
&\quad \times \mathrm{e}^{\tilde{a} a \mathrm{e}^{\lambda/2}} \tilde{a}^s |\tilde{z}\rangle \\
&= \mathrm{e}^{\lambda s}\tilde{\operatorname{tr}}\Big[\int \frac{\mathrm{d}^2 z}{\pi} \tilde{a}^{\dagger s}\, \mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\mathrm{e}^{\lambda/2}} |0,\tilde{0}\rangle\langle 0,\tilde{0}| \\
&\quad \times \mathrm{e}^{\tilde{a} a\, \mathrm{e}^{\lambda/2}} \tilde{a}^s |\tilde{z}\rangle\langle\tilde{z}|\Big] \\
&= (1-\gamma)^s\, \tilde{\operatorname{tr}}\Big[\tilde{a}^{\dagger s}\, \mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\mathrm{e}^{\lambda/2}} |0,\tilde{0}\rangle\langle 0,\tilde{0}| \\
&\quad \times \mathrm{e}^{\tilde{a} a\, \mathrm{e}^{\lambda/2}} \tilde{a}^s\Big].
\end{aligned} \tag{24}
$$

将上式代入 (19) 式, 并用 (20) 式, 得到

$$
\rho_s = \frac{\gamma}{s!n_{\mathrm{c}}^s} a^s\, \mathrm{e}^{\lambda a^{\dagger}a} a^{\dagger s}
$$

$$= \frac{\gamma^{s+1}}{s!}\tilde{\mathrm{tr}}\left[\tilde{a}^{\dagger s}\,\mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}}\,\mathrm{e}^{\lambda/2}\left|0,\tilde{0}\right\rangle\left\langle 0,\tilde{0}\right|\mathrm{e}^{\tilde{a}a}\,\mathrm{e}^{\lambda/2}\tilde{a}^{s}\right]. \tag{25}$$

对照(12)式可知相应于光场负二项式态的热真空态为

$$\left|\psi\left(\beta\right)\right\rangle_{s} = \sqrt{\frac{\gamma^{s+1}}{s!}}\tilde{a}^{\dagger s}\,\mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\sqrt{1-\gamma}}\left|0,\tilde{0}\right\rangle. \tag{26}$$

该热真空态是在混沌光场所对应的热真空态上的虚模激发, 这是对负二项式态的新看法. 注意相对于 $\left|\psi\left(\beta\right)\right\rangle_{s}$ 而言虚模激发等价于实模的湮没, 这与 $a^{s}e^{\lambda a^{\dagger}a}a^{\dagger s}$ 的表达式自洽; 进一步将(26)式改写为

$$\begin{aligned}&\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \sqrt{\gamma^{s+1}}\sum_{n=0}^{\infty}\frac{\left(\tilde{a}^{\dagger}a^{\dagger}\sqrt{1-\gamma}\right)^{n}}{n!}\left|0,\tilde{s}\right\rangle\\&= \sum_{n=0}^{\infty}\sqrt{\binom{n+s}{n}\gamma^{s+1}\left(1-\gamma\right)^{n}}\left|n,\tilde{s}+\tilde{n}\right\rangle.\end{aligned} \tag{27}$$

对照(16)式, 可见(27)式中的因子 $\sqrt{\binom{n+s}{n}\gamma^{s+1}\left(1-\gamma\right)^{n}}$ 恰好是负二项分布系数 $\binom{n+s}{n}\gamma^{s+1}\left(1-\gamma\right)^{n}$ 的开根号, $\left|\psi\left(\beta\right)\right\rangle_{s}$ 确实是一个负二项纯态. 我们得出结论: 相应于负二项光场

$$\rho_{s} = \sum_{n=0}^{\infty}\frac{(n+s)!}{n!s!}\gamma^{s+1}(1-\gamma)^{n}\left|n\right\rangle\left\langle n\right|$$

的热真空态为(27)式, 其中

$$\left|n,\tilde{s}+\tilde{n}\right\rangle = \tilde{a}^{\dagger s+n}a^{\dagger n}\left|0,\tilde{0}\right\rangle/\sqrt{n!\left(n+s\right)!}.$$

再由 $\mathrm{tr}\,\rho_{s}=1$, 可知

$$\begin{aligned}&\mathrm{tr}\,\tilde{\mathrm{tr}}\left|\psi\left(\beta\right)\right\rangle_{ss}\left\langle\psi\left(\beta\right)\right|\\&= \mathrm{Tr}\left|\psi\left(\beta\right)\right\rangle_{ss}\left\langle\psi\left(\beta\right)\right|\\&= {}_{s}\left\langle\psi\left(\beta\right)\right|\left.\psi\left(\beta\right)\right\rangle_{s}=1.\end{aligned} \tag{28}$$

(28)式也可以用(26)式直接证明, 即

$$\begin{aligned}&{}_{s}\left\langle\psi\left(\beta\right)\right|\left.\psi\left(\beta\right)\right\rangle_{s}\\&= \frac{\gamma^{s+1}}{s!}\left\langle\tilde{0},0\right|\tilde{a}^{s}\,\mathrm{e}^{\tilde{a}a\sqrt{1-\gamma}}\int\frac{d^{2}z_{1}d^{2}z_{2}}{\pi^{2}}\left|z_{1}z_{2}\right\rangle\\&\quad\left\langle z_{1}z_{2}\right|\tilde{a}^{\dagger s}\,\mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\sqrt{1-\gamma}}\left|0,\tilde{0}\right\rangle\\&= \frac{\gamma^{s+1}}{s!}\int\frac{\mathrm{d}^{2}z_{1}\mathrm{d}^{2}z_{2}}{\pi^{2}}\left|z_{2}\right|^{2s}\\&\quad\mathrm{e}^{z_{1}z_{2}\sqrt{1-\gamma}+z_{1}^{*}z_{2}^{*}\sqrt{1-\gamma}-\left|z_{2}\right|^{2}-\left|z_{1}\right|^{2}}\\&= \frac{\gamma^{s+1}}{s!}\int\frac{\mathrm{d}^{2}z_{2}}{\pi}\left|z_{2}\right|^{2s}\,\mathrm{e}^{-\gamma\left|z_{2}\right|^{2}}=1.\end{aligned} \tag{29}$$

由此证明纯态 $\left|\psi\left(\beta\right)\right\rangle_{s}$ 是归一化的.

4 用纯态 $\left|\psi\left(\beta\right)\right\rangle_{s}$ 的优点

由负二项式态对应的热真空(26)式给出

$$\begin{aligned}&a\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \sqrt{\frac{\gamma^{s+1}}{s!}}\sqrt{1-\gamma}\tilde{a}^{\dagger s+1}\,\mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\sqrt{1-\gamma}}\left|0,\tilde{0}\right\rangle\\&= \sqrt{1-\gamma}\sqrt{\frac{s+1}{\gamma}}\left|\psi\left(\beta\right)\right\rangle_{s+1},\end{aligned} \tag{30}$$

所以求 $a^{\dagger}a$ 的纯态平均立即可得光子数分布

$$\begin{aligned}&{}_{s}\left\langle\psi\left(\beta\right)\right|a^{\dagger}a\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \left(1-\gamma\right)\frac{s+1}{\gamma}{}_{s+1}\left\langle\psi\left(\beta\right)\right|\left.\psi\left(\beta\right)\right\rangle_{s+1}\\&= \left(1-\gamma\right)\frac{s+1}{\gamma}=\left(s+1\right)n_{\mathrm{c}}.\end{aligned} \tag{31}$$

又从

$$\begin{aligned}&a^{2}\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \sqrt{\frac{\gamma^{s+1}}{s!}}\left(1-\gamma\right)\tilde{a}^{\dagger s+2}\,\mathrm{e}^{\tilde{a}^{\dagger}a^{\dagger}\sqrt{1-\gamma}}\left|0,\tilde{0}\right\rangle\\&= \frac{\left(1-\gamma\right)}{\gamma}\sqrt{\left(s+1\right)\left(s+2\right)}\left|\psi\left(\beta\right)\right\rangle_{s+2}\end{aligned} \tag{32}$$

和

$$\begin{aligned}&{}_{s}\left\langle\psi\left(\beta\right)\right|a^{\dagger 2}a^{2}\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \frac{\left(1-\gamma\right)^{2}}{\gamma^{2}}\left(s+1\right)\left(s+2\right)\\&= \left(s+1\right)\left(s+2\right)n_{\mathrm{c}}^{2},\end{aligned} \tag{33}$$

可知处于负二项式态的光子数涨落为

$$\begin{aligned}&{}_{s}\left\langle\psi\left(\beta\right)\right|\left(a^{\dagger}a\right)^{2}\left|\psi\left(\beta\right)\right\rangle_{s}\\&\quad-\left[{}_{s}\left\langle\psi\left(\beta\right)\right|a^{\dagger}a\left|\psi\left(\beta\right)\right\rangle_{s}\right]^{2}\\&= \left(s+1\right)\left(n_{\mathrm{c}}+1\right)n_{\mathrm{c}}.\end{aligned} \tag{34}$$

负二项式态的二阶相干度为

$$\frac{{}_{s}\left\langle\psi\left(\beta\right)\right|a^{\dagger 2}a^{2}\left|\psi\left(\beta\right)\right\rangle_{s}}{\left[{}_{s}\left\langle\psi\left(\beta\right)\right|a^{\dagger}a\left|\psi\left(\beta\right)\right\rangle_{s}\right]^{2}}=\frac{s+2}{s+1}>1. \tag{35}$$

上述计算体现了用纯态求平均和求涨落的便利. 另一方面, 由 Wigner 算符的相干态表象[13–16]

$$\varDelta\left(\alpha\right)=\int\frac{\mathrm{d}^{2}z}{\pi^{2}}\left|\alpha+z\right\rangle\left\langle\alpha-z\right|\mathrm{e}^{\alpha z^{*}-\alpha^{*}z}, \tag{36}$$

和求纯态平均可立刻得负二项式态的 Wigner 函数

$$\begin{aligned}&{}_{s}\left\langle\psi\left(\beta\right)\right|\varDelta\left(\alpha\right)\left|\psi\left(\beta\right)\right\rangle_{s}\\&= \frac{\gamma^{s+1}}{s!}\left\langle 0,\tilde{0}\right|\mathrm{e}^{\tilde{a}a\sqrt{1-\gamma}}\tilde{a}^{s}\int\frac{\mathrm{d}^{2}z}{\pi^{2}}\left|\alpha+z\right\rangle\end{aligned}$$

$$\begin{aligned}&\langle\alpha-z|\,\mathrm{e}^{\alpha z^*-z\alpha^*}\\&\times\int\frac{\mathrm{d}^2z'}{\pi}\,|\tilde{z}'\rangle\,\langle\tilde{z}'|\,\tilde{a}^{\dagger s}\,\mathrm{e}^{\tilde{a}^\dagger a^\dagger\sqrt{1-\gamma}}\,|0,\tilde{0}\rangle\\=&\,\frac{\gamma^{s+1}}{s!}\int\frac{\mathrm{d}^2z}{\pi^2}\int\frac{\mathrm{d}^2z'}{\pi}\,|z'|^{2s}\\&\times:\,\mathrm{e}^{-|z'|^2-|\alpha|^2-|z|^2}\\&{}^{+\alpha z^*-z\alpha^*+\left[z'(\alpha+z)\right.}\\&{}^{\left.+z'^*(\alpha^*-z^*)\right]\sqrt{1-\gamma}}\,:\\=&\,\frac{\gamma^{s+1}}{\pi s!}\exp\left[-\frac{2\left(1-\mathrm{e}^{\lambda}\right)|\alpha|^2}{\mathrm{e}^{\lambda}+1}\right]\\&\times\sum_{l=0}^{s}\frac{s!s!\left(4\,|\alpha|^2\,\mathrm{e}^{\lambda}\right)^{s-l}}{l![(s-l)!]^2\left(\mathrm{e}^{\lambda}+1\right)^{2s-l+1}},\end{aligned}\tag{37}$$

其中 $\lambda=\ln(1-\gamma)=-\hbar\omega/kT$, 在最后一步我们用了积分公式

$$\begin{aligned}&\int\frac{\mathrm{d}^2z}{\pi}z^nz^{*m}\exp\left(\zeta\,|z|^2+\xi z+\eta z^*\right)\\=&\,\mathrm{e}^{-\xi\eta/\zeta}\sum_{l=0}^{\min(m,n)}\frac{m!n!\xi^{m-1}\eta^{n-1}}{l!\,(m-l)!\,(n-l)!\,(-\zeta)^{m+n-l+1}},\\&\mathrm{Re}\ \zeta<0.\end{aligned}\tag{38}$$

5 结　论

本文首次用有序算符内的积分方法找到了相应于负二项式光场的热真空态, 发现该热真空态是在混沌光场所对应的热真空态上的虚模激发, 并取负二项纯态的形式,

$$\sum_{n=0}^{\infty}\sqrt{\binom{n+s}{n}\gamma^{s+1}\left(1-\gamma\right)^n}\,|n,\tilde{s}+\tilde{n}\rangle\,.$$

由热真空态求纯态平均立刻可方便地得到负二项式光场的涨落和 Wigner 函数.

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Thermo-vacuum state in a negative binomial optical field and its application *

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Abstract

The core of optical field theory at finite temperature is how to introduce the thermo-vacuum state which is the basis of comprehensive investigation of electromagnetic field by virtue of quantum statistic method. Based on the spirit of thermo-field dynamics initiated by Takahashi and Umezawa, we first employ the integration method within the ordered product of operators to search for thermo-vacuum state for the optical negative binomial state (NBS) $\rho_s = \sum_{n=0}^{\infty}\binom{n+s}{n}\gamma^{s+1}(1-\gamma)^n |n\rangle\langle n|$.We find that the thermo-vacuum state we search for is just the fictitious-mode excitation in the thermo vacuum state for chaotic state or expressed as$\sum_{n=0}^{\infty}\sqrt{\binom{n+s}{n}\gamma^{s+1}(1-\gamma)^n}\,|n,\tilde{s}+\tilde{n}\rangle$,which takes the form of pure negative binomial state. The newly found thermo-vacuum state brings convenience for evaluating the Wigner function of NBS and the fluctuation of photon numbers in NBS.

Keywords: negative binomial optical field, thermo vacuum state, integration method within ordered product of operators, fictitious-mode excitation

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