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Combined analysis of tunable phase mask within spatial and frequency domain

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可调谐相位板空域频域联合分析*

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波前编码系统采用在传统光学系统中加入相位板来扩大光学系统的景深而避免传统景深延拓技术的不利影响. 由于相位板的参数不可调, 整个系统的景深延拓扩展率也不能动态可调. 采用两相位板组合的方法可以有效克服这一点. 本文首先从光线差的角度提出了两三次相位板组合下的光线像差分布以及点扩散函数尺寸的具体关系表达式, 直观体现了系统的光线结构, 指出了光线结构和点扩散函数尺寸受两三次相位板的面型和相对位移量的影响. 其次采用稳相法从空间域给出了系统点扩散函数表达式, 依据点扩散函数的振荡性质给出了有效带宽表达式, 提出了点扩散函数在像面的位置会随两相位板面型参数以及相对于光瞳中心的位移量而发生平移. 最后利用菲涅耳积分给出两三次相位板任意面型参数和相对位移组合下的准确光学传递函数. 在得到的调制传递函数中直观体现出了面型参数和相对位移量对调制传递函数和相位传递函数以及有效带宽的影响, 并说明了此系统相位传递函数的非线性性质. 通过空间域与频率域相结合的方法分析验证了传统的两三次相位板组合具有景深可调和带宽可调的性质, 为设计可调谐波前编码系统提供了理论依据.

关键词: 可调谐波前编码, 点扩散函数, 光学传递函数

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1 引 言

波前编码系统有效克服了传统的景深延拓技术, 避免了光学系统光通量降低和系统分辨能力减弱等的负面影响. 自三次相位板[1]提出以来, 研究者们提出了很多形式的相位板, 然而, 由于三次相位板的形式简单以及可以得到其解析解等性质, 其作为典型的相位板在空域[2]和频域[3]受到了广泛的研究. 在传统的波前编码系统中, 相位板物理参数一般是根据特定的光学系统进行优化设计, 一旦给定就不能再动态改变, 整个系统的景深延拓扩展率也不会再发生变化, 这种特殊性限制了它的使用. 近年来, 可调谐波前编码概念的出现打破了这种限制, 实现了离焦不变性可调. Ojeda-Castaneda 等[4]设计了由两块余弦型相位板组成的可分离相位板, 通过将两块相位板反方向移动相同的位移来实现离焦敏感性可调. Zhao 和 Wei[5]在分析了由两块余弦相位板组成的可分离相位板的基础上, 提出了更一般的情况, 可以实现相同或者相反方向的不同位移量, 从而实现了离焦不敏感性可调, 并指出此种系统可实现带宽可调, 可用于信息隐藏等. 文献[5]中指出可以采用两三次相位板的组合形式, 并在附录中直接给出了系统的调制传递函数 (magnitude transfer function, MTF) 解和有效带宽, 由于采用稳相法, 虽然得到了近似的MTF, 但无法在MTF中直观体现出系统的有效带宽, 并且忽略了离焦对带宽的影响. 本文在空间域采用光线差理论[6]和稳相法理论[7]给出此种系统的光线结构和点扩散函数 (point spread function, PSF) 表达式, 并在频域利用菲涅耳积分性质[8]给出准确的光学传递函数 (optical transfer function, OTF), 分析验证以传统的两三次相位板组合可实现离焦敏感性可调和系统带宽可调. 首先, 根据光线差理论

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给出了两三次相位板组合的光线结构以及点扩散函数的大小, 说明了点扩散函数的大小受到了两可分离相位板面型参数以及它们之间相对位移量的影响. 然后, 在空间域, 依据稳相法理论给出系统的点扩散函数的理论表达式, 并得出有效带宽表达式, 验证了离焦敏感性可调和带宽可调的性质, 指出点扩散函数在像平面的位置会随相位板面型参数及相对光瞳中心的位移量变化而发生平移. 最后, 在频率域, 给出了此系统在任意面型参数和相对位移量情况下的OTF的精确解, 得到了其有效带宽, 并且最终在MTF直观地反映出面型参数和相位位移量对MTF和有效带宽的影响, 并说明此系统仍然保持了传统三次相位板的相位传递函数(phase transfer function, PTF)的非线性变化会对后期图像处理造成伪像效应.

2 空间域分析

2.1 光线差分析

一对传统的三次相位板组合的一般形式可以表示为

$$Q(x)=\alpha_1(x+m_1)^3+\alpha_2(x+m_2)^3, \qquad (1)$$

其中, α_1, α_2 分别为两三次相位板的面型参数; m_1, m_2 代表两相位板相对光瞳中心的位移量, 且满足 $m_1, m_2\in[-1,1]$.

光线差采用几何光学的观点直观体现出光线分布. 光线差可由对波前的微分得到, 对于含有离焦量 δl 的上述波前编码光学系统来说其光线差为

$$\Delta X=3(n-1)f[\alpha_1(|x|+m_1)^2 + 3\alpha_2(|x|+m_2)^2]-\frac{\delta l}{f}x. \qquad (2)$$

若 $\alpha_1+\alpha_2>0$ 并依据极值理论, 可求得(2)式取最小值时 x 的值:

$$x_{\min}=-\frac{\alpha_1 m_1+\alpha_2 m_2}{\alpha_1+\alpha_2}+\frac{|\delta l|}{6(n-1)(\alpha_1+\alpha_2)f^2}. \qquad (3)$$

因而光线差的最小值、最大值以及点扩散函数的表达式为

$$\begin{cases}\Delta X_{\min}=3(n-1)f[\alpha_1(|x_{\min}|+m_1)^2 \\ \qquad +\alpha_2(|x_{\min}|+m_2)^2]-\dfrac{|\delta l|}{f}x_{\min}, \\ \Delta X_{\max}=3(n-1)f[\alpha_1(r+m_1)^2 \\ \qquad +\alpha_2(r+m_2)^2]-\dfrac{|\delta l|}{f}r, \\ D=\Delta X_{\max}-\Delta X_{\min}.\end{cases} \qquad (4)$$

$\delta l=8W_{20}(F/\#)^2$ 表示了离焦量 W_{20} 之间的转换关系; r 为半口径, $F/\#=2r/f$ 为光学系统 F 数. 以焦距 $f=10$ mm, 相位板折射率 $n=1.5$ 的系统为例, 选取不同的面型参数 α_1, α_2 以及相位板相对于光瞳中心的位移量 m_1, m_2 进行分析.

图1给出了在 $\alpha_1=0.0084$, $\alpha_2=0.0056$, $m_1=0.4$ 情况下改变 m_2 的值对系统光线结构, 光线差的上边界、下边界以及点扩散函数尺寸的影响. 图2给出了 $m_1=0.4$, $m_2=0.3$, $\alpha_1=0.0084$ 情况下改变 α_2 的值对系统光线结构, 光线差的上边界、下边界以及点扩散函数尺寸的影响.

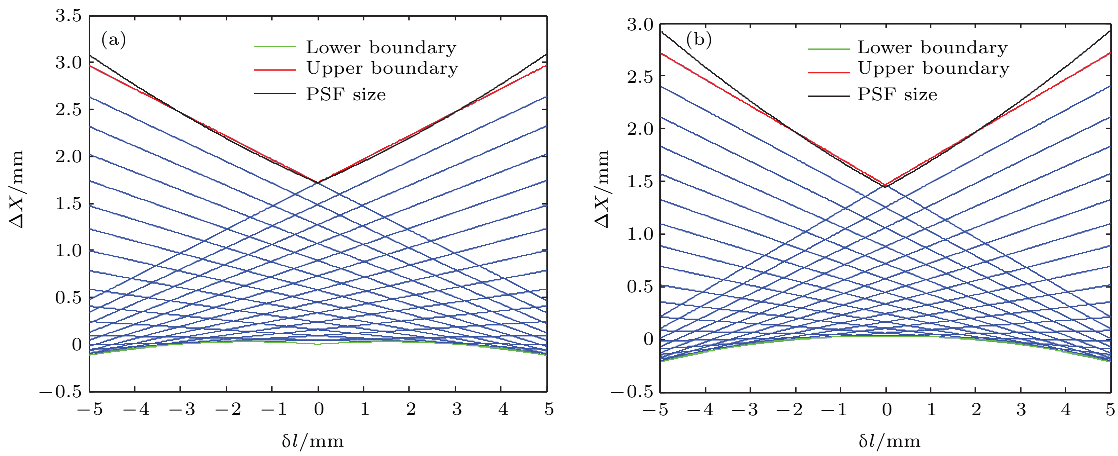

图1 (网刊彩色) 光线结构 (a) $m_2=0.3$; (b) $m_2=-0.3$

Fig. 1. (color online) Ray structure: (a) $m_2=0.3$; (b) $m_2=-0.3$.

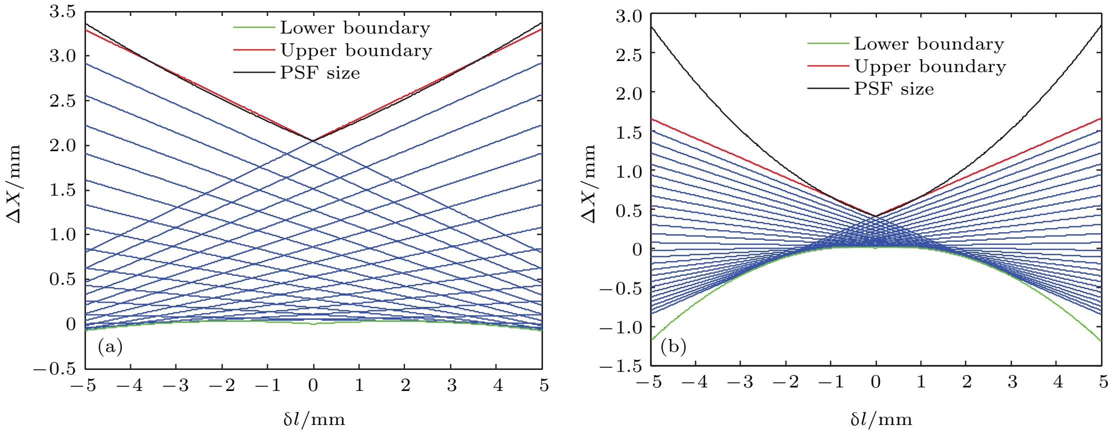

图2 (网刊彩色) 光线结构 (a) $\alpha_2 = 0.0084$; (b) $\alpha_2 = -0.0056$

Fig. 2. (color online) Ray structure: (a) $\alpha_2 = 0.0084$; (b) $\alpha_2 = -0.0056$.

由图1可以看出, 在面型参数一定的情况下, 位移量的方向影响点扩散函数的大小以及离焦敏感性. 由图2可以看出, 在位移量一定的情况下, 相位板的放置方向直接影响着点扩散函数的尺寸以及离焦的敏感性.

2.2 点扩散函数分析

非相干光学系统的点扩散函数是光瞳函数的傅里叶变换的模平方[9](文中以后提到的α_1, α_2的值是第一部分的值乘以波数), 假设$\alpha_1 + \alpha_2 > 0$.

$$h(u,\psi) = \frac{1}{2}\left|\int_{-1}^{1}\exp\Big\{\mathrm{j}\big[\alpha_1(x+m_1)^3 + \alpha_2(x+m_2)^3 + \psi x^2 - 2\pi u x\big]\Big\}\mathrm{d}x\right|^2. \quad (5)$$

采用稳相法从空域的角度给出系统的点扩散函数表达式.

令$\varphi(x) = ax^3 + bx^2 + cx + d$, 其中

$$a = \alpha_1 + \alpha_2, \quad (6a)$$

$$b = 3(\alpha_1 m_1 + \alpha_2 m_2) + \psi, \quad (6b)$$

$$c = 3(\alpha_1 m_1^2 + \alpha_2 m_2^2) - 2\pi u, \quad (6c)$$

$$d = \alpha_1 m_1^3 + \alpha_2 m_2^3. \quad (6d)$$

依据稳相法求取稳相点, 相比文献[5]中运用稳相法分析MTF, 此时(5)式存在两个鞍点:

$$x_{\rm s1} = \frac{-2b + \sqrt{4b^2 - 12ac}}{6a}, \quad (7a)$$

$$x_{\rm s2} = \frac{-2b - \sqrt{4b^2 - 12ac}}{6a}. \quad (7b)$$

两稳相点处的积分结果分别为

$$S(x_{\rm s1}) = \sqrt{\frac{2\pi}{|\varphi''(x_{\rm s1})|}}\exp(\mathrm{j}\,\mathrm{sgn}(\varphi''(x_{\rm s1})))\exp(\mathrm{j}\varphi(x_{\rm s1})), \quad (8a)$$

$$S(x_{\rm s2}) = \sqrt{\frac{2\pi}{|\varphi''(x_{\rm s2})|}}\exp(\mathrm{j}\,\mathrm{sgn}(\varphi''(x_{\rm s2})))\exp(\mathrm{j}\varphi(x_{\rm s2})). \quad (8b)$$

在$u_{\rm l} < u \leqslant u_{\rm m}$之间存在两稳相点, 在$u_{\rm m} < u \leqslant u_{\rm u}$仅存在一个稳相点, 其他区域值为零, 故点扩散函数有如下结果:

$$h(u,\psi) = \begin{cases} 0, & u \leqslant u_{\rm l}, \\ \dfrac{1}{2}\left|S(x_{\rm s1}) + S(x_{\rm s2})\right|^2, & u_{\rm l} \leqslant u \leqslant u_{\rm m}, \\ \dfrac{1}{2}\left|S(x_{\rm s1})\right|^2, & u_{\rm m} \leqslant u \leqslant u_{\rm u}, \\ 0, & u \geqslant u_{\rm u}. \end{cases} \quad (9)$$

其中,

$$u_{\rm l} = \frac{3(\alpha_1 m_1^2 + \alpha_2 m_2^2)}{2\pi} - \frac{b^2}{6\pi a}, \quad (10a)$$

$$u_{\rm m} = \frac{3(\alpha_1 m_1^2 + \alpha_2 m_2^2) - (-3a + 2b)}{2\pi}, \quad (10b)$$

$$u_{\rm u} = \frac{3(\alpha_1 m_1^2 + \alpha_2 m_2^2) - (-3a - 2b)}{2\pi}. \quad (10c)$$

依据上下界可得点扩散函数的尺寸为

$$u_{\rm u} - u_{\rm l} = \frac{(3a + 2b)^2}{6\pi a}, \quad (11a)$$

如果给定实际光学系统的衍射极限频率$f_{\rm cutoff}$, 则可得点扩散函数大小为

$$D = \frac{(3a + 2b)^2}{3\pi a f_{\rm cutoff}}. \quad (11b)$$

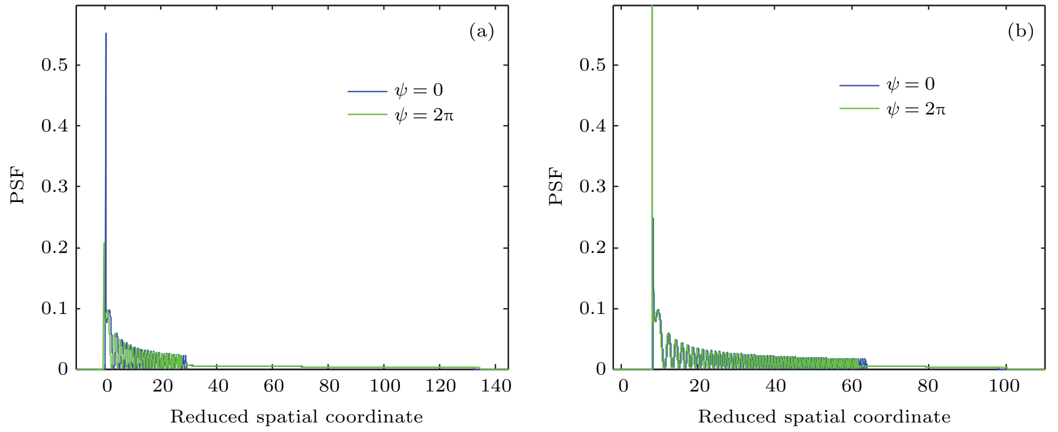

图 3 (网刊彩色) 面型参数一定, 不同离焦量下的一维点扩散函数 (a) $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = 0.3$; (b) $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = -0.3$

Fig. 3. (color online) One dimensional point spread function with defined figures in different defocus values: (a) $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = 0.3$; (b) $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = -0.3$.

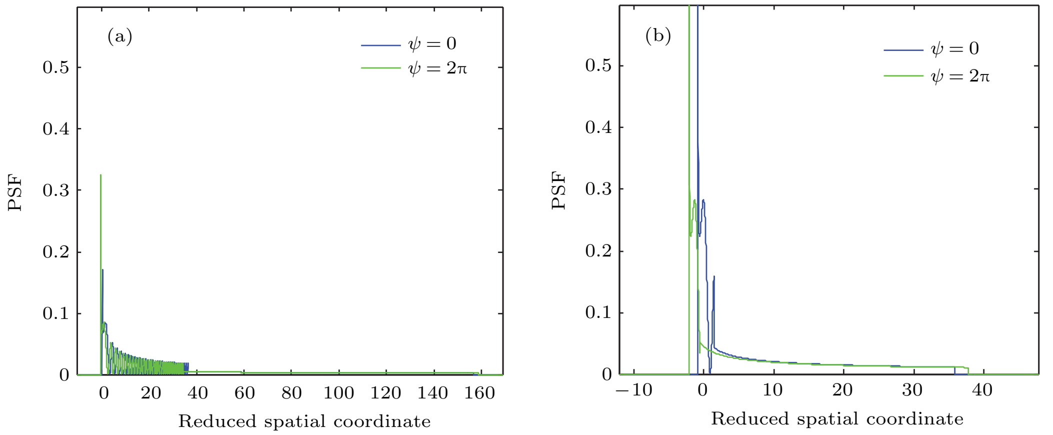

图 4 (网刊彩色) 位移量一定, 不同离焦量下的一维点扩散函数 (a) $m_1 = 0.4$, $m_2 = 0.3$, $\alpha_1 = 90$, $\alpha_2 = 90$; (b) $m_1 = 0.4$, $m_2 = 0.3$, $\alpha_1 = 90$, $\alpha_2 = -60$

Fig. 4. (color online) One dimensional point spread function with defined relative displacements in different defocus values: (a) $m_1 = 0.4$, $m_2 = 0.3$, $\alpha_1 = 90$, $\alpha_2 = 90$; (b) $m_1 = 0.4$, $m_2 = 0.3$, $\alpha_1 = 90$, $\alpha_2 = -60$.

根据 (9) 式, 采用前述参数给出系统正焦和离焦 2π 情况下的点扩散函数如图 3 和图 4 所示.

从图 3 和图 4 中可以看出, 点扩散函数在区间 $[u_l, u_m]$ 表现为振荡效应, 这是由于在此区间内存在两个鞍点, 点扩散函数受到了调制, 调制频率为

$$f(x) = \frac{\sqrt{4b^2 - 12ac}}{3a}. \tag{12}$$

最大调制频率为

$$f_{\max} = 2 - \left|\frac{2(\alpha_1 m_1 + \alpha_2 m_2)}{\alpha_1 + \alpha_2} + \frac{2\psi}{3(\alpha_1 + \alpha_2)}\right|. \tag{13}$$

(13) 式即为系统在离焦情况下的最大有效带宽, 在 $\alpha_1 + \alpha_2 \gg \psi$ 的情况下与文献 [5] 对传递函数采用稳相法分析得到的有效带宽一致.

从图 3 可以看出, 面型参数一定的情况下, 相位板相对于光瞳中心的位移量大小和方向直接影响点扩散函数的幅度值、大小和像面位置, 离焦影响着点扩散函数的幅度值. 从图 4 可以看出, 在位移量一定的情况下, 点扩散函数的振荡性质、幅度以及大小都会受到相位板的放置方向的影响, 进而影响着系统对离焦的敏感性. 总体而言相位板面型参数和相对光瞳中心位移量都会对点扩散函数的振荡性质和平移造成影响, 离焦会造成点扩散函数的幅度值发生变化, 但点扩散函数的形状不会发生变化.

3 频率域分析

采用稳相法近似虽然得出了有效带宽, 但无法在传递函数上体现出有效带宽以及无法得到 PTF, 并且上述方法都假设 $\alpha_1 + \alpha_2 > 0$. OTF 包含了 MTF 和 PTF, 可准确反映系统的性质, 对波前编码

系统后续图像处理尤为重要. 因此对系统OTF进行精确求解, 得到了任意一对三次相位板组合下的精确的OTF. 非相干OTF为光瞳函数的自相关[9].

$$OTF = \frac{1}{2}\int_{-1+\left|\frac{v}{2}\right|}^{1-\left|\frac{v}{2}\right|} \exp\left\{\mathrm{j}\left[Q\left(x+\frac{v}{2}\right) - Q\left(x-\frac{v}{2}\right) + 2v\psi x\right]\right\}\mathrm{d}x. \tag{14}$$

将(1)式代入(14)式, 则有

$$OTF = \frac{1}{2}\int_{-1+\left|\frac{v}{2}\right|}^{1-\left|\frac{v}{2}\right|} \exp\left\{\mathrm{j}\left[3\nu(\alpha_1+\alpha_2)x^2 + 6v\left(\alpha_1 m_1 + \alpha_2 m_2 + \frac{\psi}{3}\right)x + 3v(\alpha_1 m_1^2 + \alpha_2 m_2^2) + \frac{\alpha_1+\alpha_2}{4}v^3\right]\right\}\mathrm{d}x. \tag{15}$$

为了方便求解, 对(15)式做如下变量替换:

$$a = 3\nu(\alpha_1+\alpha_2), \tag{16a}$$

$$b = 6v\left(\alpha_1 m_1 + \alpha_2 m_2 + \frac{\psi}{3}\right), \tag{16b}$$

$$c = 3v(\alpha_1 m_1^2 + \alpha_2 m_2^2) + \frac{\alpha_1+\alpha_2}{4}v^3. \tag{16c}$$

依据菲涅耳积分, (14)式的精确积分结果如下:

$$OTF = \begin{cases} \exp\left\{\mathrm{i}\left[\frac{b(t_1+t_2)}{2}+c\right]\right\}(t_2-t_1)\mathrm{sinc}\left(\frac{b(t_2-t_1)}{2\pi}\right), & a=0, \\ \exp\left[\mathrm{i}\left(c-\frac{b^2}{4a}\right)\right]\sqrt{\frac{\pi}{2a}}(F(t_2')-F(t_1')), & a>0, \\ OTF^*(-a,-b,-c,t_1,t_2), & a<0, \end{cases} \tag{17}$$

其中,

$$t_1' = \sqrt{\frac{2}{\pi}}\left(t_1\sqrt{a} + \frac{b}{2\sqrt{a}}\right),$$

$$t_2' = \sqrt{\frac{2}{\pi}}\left(t_2\sqrt{a} + \frac{b}{2\sqrt{a}}\right),$$

$$t_1 = -1 + \left|\frac{v}{2}\right|, \quad t_2 = 1 - \left|\frac{v}{2}\right|,$$

$$F(x) = \int_0^x \exp\left(\frac{\mathrm{i}\pi t^2}{2}\right)\mathrm{d}t$$

为菲涅耳积分.

依据$t_1' = 0$, 可得系统的最大带宽为

$$\begin{aligned} v_{\max} &= 2 - |b/a| \\ &= 2 - \left|\frac{2(\alpha_1 m_1 + \alpha_2 m_2)}{\alpha_1+\alpha_2} + \frac{2\psi}{3(\alpha_1+\alpha_2)}\right|. \end{aligned} \tag{18}$$

(17)式给出了系统在任意面型参数和任意相对光瞳中心位移量情况下的OTF, 可以准确反映此种波前编码系统的性质. 为了方便分析, 结合(17)式, 只给出在前述不同参数下系统的MTF图和PTF图, 并在图中同时给出了有效带宽, 结果如图5—图8所示.

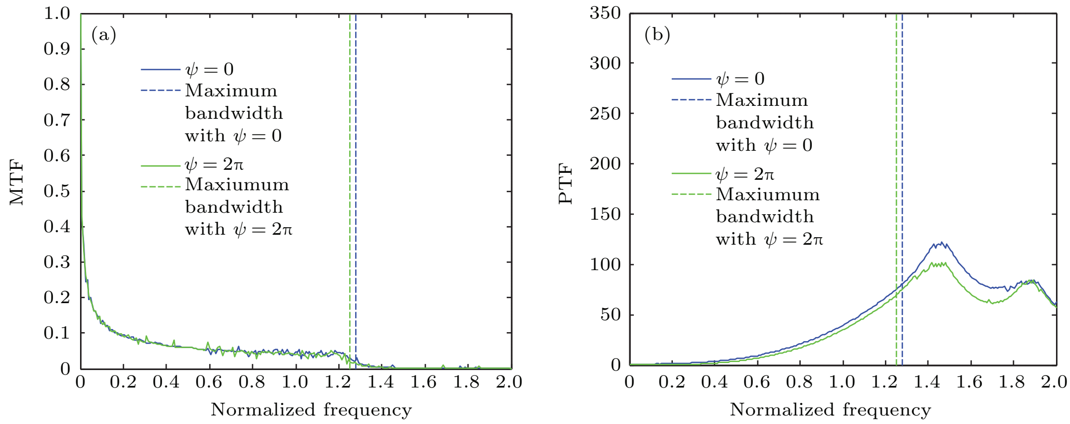

图5 (网刊彩色) $\alpha_1=90$, $\alpha_2=60$, $m_1=0.4$, $m_2=0.3$时的(a) MTF, (b) PTF

Fig. 5. (color online) When $\alpha_1=90$, $\alpha_2=60$, $m_1=0.4$, $m_2=0.3$: (a) MTF map and maximum bandwidth in defocus value $\psi=0$ and $\psi=2\pi$; (b) PTF map and maximum bandwidth in defocus value $\psi=0$ and $\psi=2\pi$.

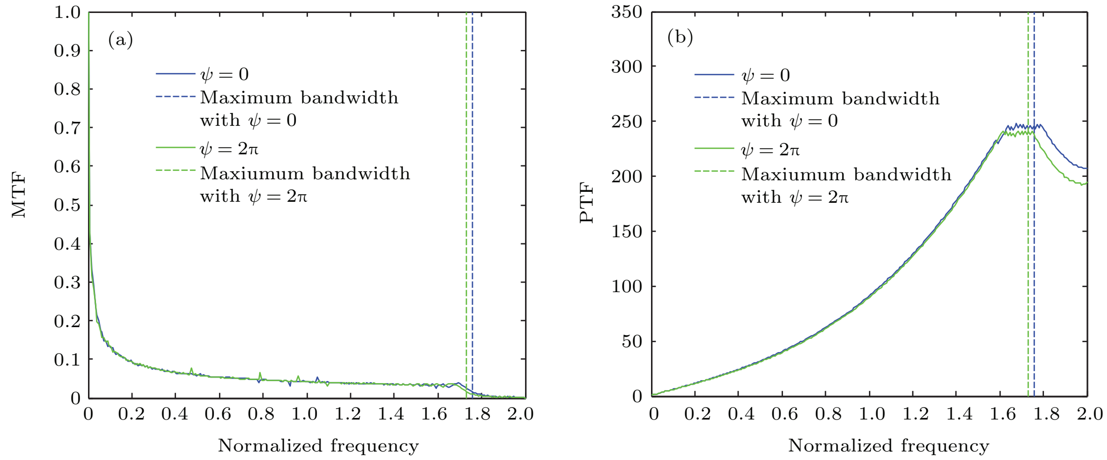

图 6 (网刊彩色) $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = -0.3$ 时的 (a) MTF; (b) PTF

Fig. 6. (color online) When $\alpha_1 = 90$, $\alpha_2 = 60$, $m_1 = 0.4$, $m_2 = -0.3$: (a) MTF map and maximum bandwidth in defocus value $\psi = 0$ and $\psi = 2\pi$; (b) PTF map and maximum bandwidth in defocus value $\psi = 0$ and $\psi = 2\pi$.

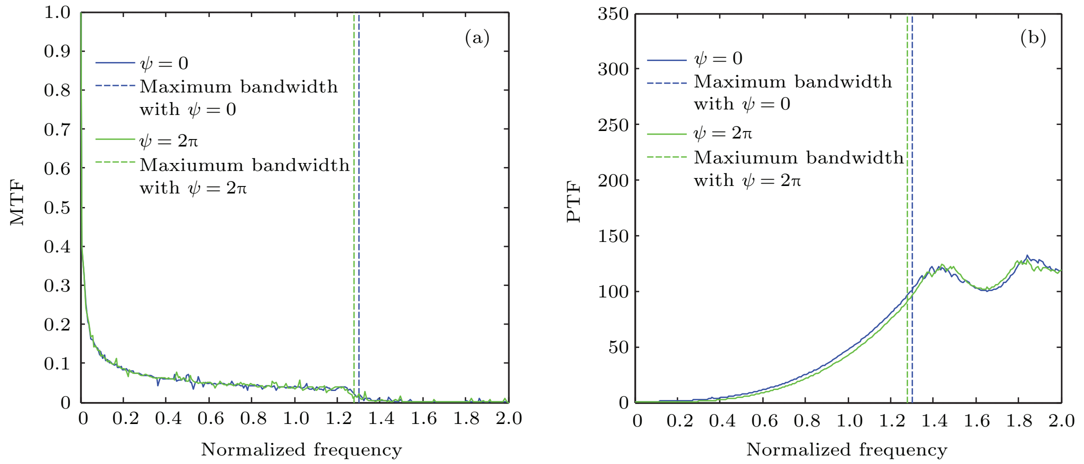

图 7 (网刊彩色) $\alpha_1 = 90$, $\alpha_2 = 90$, $m_1 = 0.4$, $m_2 = 0.3$ 时的 (a) MTF; (b) PTF

Fig. 7. (color online) When $\alpha_1 = 90$, $\alpha_2 = 90$, $m_1 = 0.4$, $m_2 = 0.3$: (a) MTF map and maximum bandwidth in defocus value $\psi = 0$ and $\psi = 2\pi$; (b) PTF map and maximum bandwidth in different defocus value $\psi = 0$ and $\psi = 2\pi$.

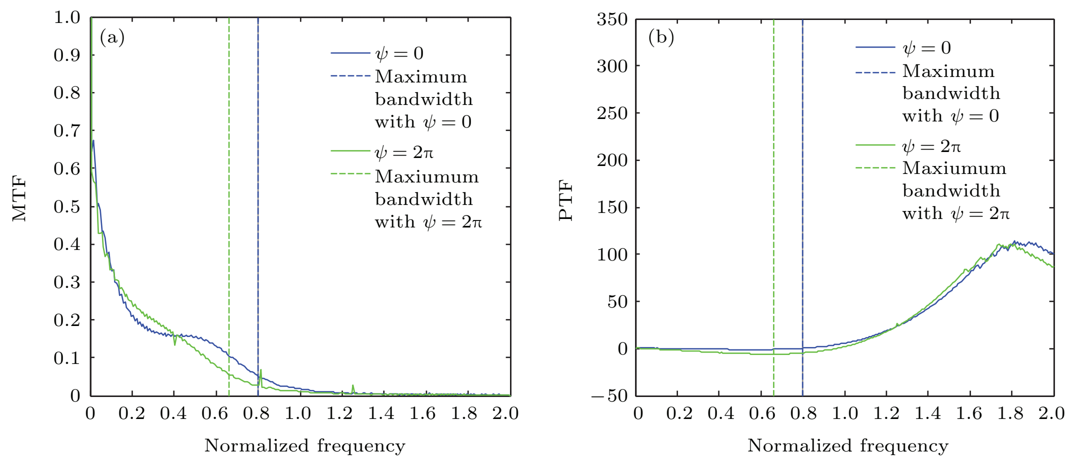

图 8 (网刊彩色) $\alpha_1 = 90$, $\alpha_2 = -60$, $m_1 = 0.4$, $m_2 = 0.3$ 时的 (a) MTF; (b) PTF

Fig. 8. (color online) When $\alpha_1 = 90$, $\alpha_2 = -60$, $m_1 = 0.4$, $m_2 = 0.3$: (a) MTF map and maximum bandwidth in defocus value $\psi = 0$ and $\psi = 2\pi$; (b) PTF map and maximum bandwidth in different defocus value $\psi = 0$ and $\psi = 2\pi$.

由图5—图7可以看出, MTF的精确解能直观反映出系统的带宽, 在带宽内MTF的值不为零, 超过系统带宽, MTF的值降为零. 虽然图中系统MTF具有离焦不敏感性, 但是不同离焦量的PTF呈非线性变化, 而编码图像复原采用同一OTF进行去卷积运算时, 不同离焦量PTF的差异就会使解码图像造成伪像效应. 由图8可以看出, 此时相位板反向放置时会减弱系统对离焦不敏感的性质, MTF随离焦量不同而发生变化, 此时的PTF在截止频率以内表现出线性性质. MTF图直观反映出了系统的有效带宽, 系统的最终性质受到了系统各个参数的调节, 不但具备传统三次相位板的景深延拓性质, 同时还可以实现系统性质的可调谐性.

4 结 论

本文采用光线差法对由传统的一对三次相位板组合的情况进行分析, 直观反映了系统光线结构以及点扩散函数尺寸受两相位板面型参数和相对光瞳中心位移量大小的影响. 在空间域, 采用稳相法对系统的一维点扩散函数进行了理论分析, 得出点扩散函数的幅度值会随离焦量发生变化, 但形状不会发生变化. 指出点扩散函数位置会随着相对位移量的大小而发生变化. 此外, 点扩散函数还与相位板的面型参数以及组合方式有关. 在频率域, 利用菲涅耳积分对系统的OTF进行了精确求解, 得到的MTF能直观反映出有效带宽, 这点弥补了稳相法求得的MTF不能直观反映有效带宽的不足, 系统的PTF仍表现出非线性变化, 这对后续图像处理是不利的. 带宽的可调性则通过相对于光瞳中心不同的位移量来实现. 通过一对不同位移量和面型参数的三次相位板组合可以实现离焦敏感性和带宽可调, 有效解决单一三次相位板不能动态可调的缺点.

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Combined analysis of tunable phase mask within spatial and frequency domain*

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Abstract

Wavefront coding technique is a powerful technique which overcomes the defects of traditional way to extend depth of field. By inserting a phase mask into the traditional incoherent imaging system, wavefront coding technique does not reduce the resolution and the light gathering power of the optical system but enlarges the depth of field of incoherent imaging system. Although several kinds of phase masks have been reported, cubic phase mask is still of a classical type which has been investigated widely both in spatial and frequency domain. Since the phase profiles of phase masks adopted in classical wavefront coding systems are predefined with specific optical systems, the extension of depth of field is not tunable. Tunable wavefront coding systems are introduced by using a pair of detachable phase masks, which is possible to control the depth of field and bandwidth of system by changing the position of each component with respect to the pupil center. Ojeda-Castañeda [Ojeda-Castañeda J, Rodríguez M, Naranjo R 2010 *Proceedings of Progress in Electronics Research Symposium*, Cambridge, July 5–8, 2010 p531] proposed to use a pair of cosine phase masks to make defocus sensitivity tunable. Zhao [Zhao H, Wei J X 2014 *Opt. Commun.* **326** 35] investigated an improved version of Ojeda-Castaneda's design in frequency domain and found that the proposed system realized tunable bandwidth. The present study, based on the work of Zhao, analyzes the tunable characteristics of a pair of simple modified detachable cubic phase masks in spatial domain and frequency domain. Firstly, the ray aberration theory is adopted to give mathematical analyses and ray aberration maps of the proposed tunable phase mask. Based on the mathematical derivations, the size of point spread function (PSF) of system can be changed not only by profile of each cubic mask but also by the each mask displacement relative to pupil center. Secondly, a mathematical PSF based on the stationary phase method is derived in spatial domain. Simulations indicate that the positions of PSF translate in the image plane with the displacements of phase mask profile and the position of each component with respect to the pupil center. By analyzing the oscillations of PSF, the effective bandwidth is obtained. Through the expression, we can conclude that the effective bandwidth can be changed by the position, mask profile of each component and defocus. Only when the addition of two mask profiles is large enough, can the effective bandwidth be simplified without adding the influence of defocus. In addition, though the approximate expression of magnitude transfer of function (MTF) has been given by adopting stationary phase method in the appendix of previous work, it cannot give an intuitive grasp of the effective bandwidth in MTF map. Unlike the MTF expression derived before, the exact optical transfer function (OTF) expression is derived by adopting Fresnel integral in frequency domain. Exact MTF and phase transfer function (PTF) can be derived from OTF. Based on the exact MTF expression, simulations give an intuitive effective bandwidth in MTF map. Simulations also show the nonlinear property of PTF. The effective bandwidth and MTF can be changed by different phase mask profiles and positions, which indicate that the effective bandwidth and defocus sensitivity can be tuned. Analyses are conducted both in spatial domain and in frequency domain to verify the tunable property of the proposed phase mask, which provides theoretical foundation for tunable wavefront coding system design.

Keywords: tunable wavefront coding, point spread function, optical transfer function

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