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Exact invariants and adiabatic invariants for nonholonomic systems in non-Chetaev's type based on El-Nabulsi dynamical models

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El-Nabulsi动力学模型下非Chetaev型非完整系统的精确不变量与绝热不变量*

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研究El-Nabulsi动力学模型下非Chetaev型非完整系统精确不变量与绝热不变量问题. 首先, 导出El-Nabulsi-d'Alembert-Lagrange原理并建立系统的运动微分方程. 其次, 建立El-Nabulsi模型下未受扰动的非Chetaev 型非完整系统的Noether对称性与Noether对称性导致的精确不变量之间的关系; 再次, 引入力学系统的绝热不变量概念, 研究受小扰动作用下非Chetaev型非完整系统Noether对称性的摄动导致绝热不变量问题, 给出了绝热不变量存在的条件及其形式. 作为特例, 本文讨论了El-Nabulsi模型下Chetaev型非完整系统的精确不变量与绝热不变量问题. 最后分别给出非Chetaev型和Chetaev型两种约束下的算例以说明结果的应用.

关键词: 对称性摄动, 绝热不变量, 非Chetaev型非完整约束, El-Nabulsi动力学模型

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1 引 言

非完整约束可分为Chetaev型和非Chetaev型约束. Chetaev型约束是指所受非完整约束对虚位移的限制满足Chetaev条件, 而非Chetaev型约束是指所受非完整约束对虚位移的限制不满足Chetaev条件. 当满足一定条件时, 非Chetaev型非完整系统退化为Chetaev型非完整系统. 1957年, Novoselov给出了非Chetaev型非完整约束的例子, 之后建立了非Chetaev型非完整力学系统的运动微分方程[1].

1917年, Bugers[2]针对一类特殊的Hamilton系统首先提出了绝热不变量问题, 之后,Djuki[3], Bulanov[4]和Notte[5]等在这方面做了许多工作. 绝热不变量[6,7]也称缓渐不变量或浸渐不变量, 它是指当参数缓慢变化时几乎不变的量. 事实上, 参数的缓慢变化等同于小扰动的作用, 小扰动作用下对称性的改变及其不变量与力学系统的可积性密切相关. 约束力学系统对称性的摄动与绝热不变量研究取得了一系列重要成果[8−15]. El-Nabulsi动力学模型[16,17]是基于分数阶积分定义的非保守系统动力学模型, 其新颖之处在于分数阶时间积分仅依赖于一个实参数, 所得到的Euler-Lagrange方程出现相应于耗散力的广义分数阶外力但不出现分数阶导数. 基于El-Nabulsi动力学模型研究物理学系统的动力学行为已经取得了一些重要结果[18−27]. 本文将进一步研究El-Nabulsi模型下非Chetaev型非完整约束系统的精确不变量与绝热不变量问题.

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2 El-Nabulsi模型下非完整系统的运动微分方程

假设力学系统的位形由n个广义坐标$q_k(k=1,\ 2,\ \cdots,\ n)$来确定, 系统的Lagrange函数为$L=L(\tau,\ \boldsymbol{q},\ \dot{\boldsymbol{q}})$, 则基于Riemann-Liouville分数阶积分的El-Nabulsi变分问题为[25]: 求积分泛函

$$S=\frac{1}{\Gamma(\alpha)}\int_a^b L(\tau,q_k(\tau),\dot{q}_k(\tau))(t-\tau)^{\alpha-1}\,\mathrm{d}\tau,\quad(1)$$

在固定边界条件

$$q_k(a)=q_{k,a},\quad q_k(b)=q_{k,b},\ (k=1,2,\cdots,n)\quad(2)$$

下的极值问题, 其中$\dot{q}_k=\mathrm{d}q_k/\mathrm{d}\tau$, Γ是Euler Gamma函数$0<\alpha\leqslant 1$, τ是固有时间, t是观察者时间, $\tau\neq t$, 函数L是其变量的C^2类函数.

泛函(1)称为El-Nabulsi-Hamilton作用量. 当$\alpha=1$时, 上述变分问题成为力学系统的经典变分问题.

根据变分学理论, 泛函(1)在$q_k=q_k(\tau)$上取得极值的必要条件是其变分等于零, 即$\delta S=0$, 因此有

$$\delta S=\frac{1}{\Gamma(\alpha)}\int_a^b\left(\frac{\partial L}{\partial q_k}\delta q_k-\frac{\partial L}{\partial \dot{q}_k}\partial\dot{q}_k\right)\times(t-\tau)^{\alpha-1}\,\mathrm{d}\tau=0.\quad(3)$$

经过分部积分并利用边界条件(2),(3)式可化为

$$\delta S=\frac{1}{\Gamma(\alpha)}\int_a^b\left[\left(\frac{\partial L}{\partial q_k}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_k}\right)(t-\tau)^{\alpha-1}-(1-\alpha)\frac{\partial L}{\partial \dot{q}_k}(t-\tau)^{\alpha-2}\right]\delta q_k\,\mathrm{d}\tau=0.\quad(4)$$

由积分区间$[a,b]$的任意性, 我们有

$$\left[\left(\frac{\partial L}{\partial q_k}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_k}\right)(t-\tau)^{\alpha-1}-(1-\alpha)\frac{\partial L}{\partial \dot{q}_k}(t-\tau)^{\alpha-2}\right]\delta q_k=0.\quad(5)$$

(5)式称为El-Nabulsi-d'Alembert-Lagrange原理.

假设系统的运动受有g个彼此独立的理想非Chetaev型非完整约束

$$F_\beta(\tau,\boldsymbol{q},\dot{\boldsymbol{q}})=0,\ (\beta=1,2,\cdots,g),\quad(6)$$

约束(6)对虚位移上的限制条件为

$$f_{\beta k}(\tau,\boldsymbol{q},\dot{\boldsymbol{q}})\,\delta q_k=0,\quad(\beta=1,2,\cdots,g;k=1,2,\cdots,n).\quad(7)$$

一般说来, $f_{\beta k}$与$\dfrac{\partial F_\beta}{\partial\dot{q}_k}$无关. 特别地, 当$f_{\beta k}=\dfrac{\partial F_\beta}{\partial\dot{q}_k}$时, 非Chetaev型非完整约束退化为Chetaev型非完整约束.

由El-Nabulsi-d'Alembert-Lagrange原理(5)和虚位移方程(7), 利用通常的Lagrange乘子法, 可以得到

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial\dot{q}_k}-\frac{\partial L}{\partial q_k}=-\frac{1-\alpha}{t-\tau}\frac{\partial L}{\partial\dot{q}_k}+\lambda_\beta f_{\beta k},\quad(k=1,2,\cdots,n),\quad(8)$$

其中λ_β为约束乘子. 方程(8)为El-Nabulsi模型下非Chetaev型非完整系统的运动微分方程.

3 Noether对称性与精确不变量

引进时间和广义坐标的无限小变换

$$\bar{\tau}=\tau+\Delta\tau,\bar{q}_k(\bar{\tau})=q_k(\tau)+\Delta q_k,\quad(k=1,2,\cdots,n),\quad(9)$$

或其展开式

$$\bar{\tau}=\tau+\varepsilon\xi_0^0(\tau,\boldsymbol{q},\dot{\boldsymbol{q}}),\quad \bar{q}_k(\bar{\tau})=q_k(\tau)+\varepsilon\xi_k^0(\tau,\boldsymbol{q},\dot{\boldsymbol{q}}),\quad(k=1,2,\cdots,n),\quad(10)$$

其中ε为无限小参数, ξ_0^0,ξ_k^0为无限小变换的生成元或生成函数.

El-Nabulsi-Noether等式给出[25]

$$\frac{\partial L}{\partial\tau}\xi_0^0+\frac{\partial L}{\partial q_k}\xi_k^0+\frac{\partial L}{\partial\dot{q}_k}\left(\dot{\xi}_k^0-\dot{q}_k\dot{\xi}_0^0\right)+L\left(\dot{\xi}_0^0+\frac{1-\alpha}{t-\tau}\xi_0^0\right)=-\dot{G}^0(t-\tau)^{1-\alpha}.\quad(11)$$

由于

$$\delta q_k=\Delta q_k-\dot{q}_k\Delta\tau=\varepsilon\left(\xi_k^0-\dot{q}_k\xi_0^0\right),\quad(12)$$

将(12)式代入(7)式, 得到

$$f_{\beta k}\left(\xi_k^0-\dot{q}_k\xi_0^0\right)=0,\ (\beta=1,2,\cdots,g).\quad(13)$$

称(13)式为非Chetaev型非完整约束对无限小生成元的限制条件.

定理1　对于El-Nabulsi模型下未受扰动的非Chetaev型非完整系统(6), (8), 如果无限小变换的生成元ξ_0^0,ξ_k^0满足El-Nabulsi-Noether等式(11)

和限制条件(13), 则系统的Noether对称性直接导致Noether守恒量

$$I_0 = \left[L\xi_0^0 + \frac{\partial L}{\partial \dot{q}_k}\left(\xi_k^0 - \dot{q}_k\xi_0^0\right)\right] \times (t-\tau)^{\alpha-1} + G^0 = \text{const.} \tag{14}$$

守恒量(14)是系统的精确不变量, 它揭示了El-Nabulsi模型下未受扰动的非Chetaev型非完整系统(6), (8)的Noether对称性与不变量之间的关系.

4 Noether对称性的摄动与绝热不变量

定义1[6] 若 $I_z(\tau, \boldsymbol{q}, \dot{q}, v)$ 是力学系统的一个含有小参数 v 的最高次幂为 z 的物理量, 其对时间 τ 的一阶导数正比于 v^{z+1}, 则称 I_z 为力学系统的 z 阶绝热不变量.

假设El-Nabulsi模型下非Chetaev型非完整系统(6), (8)受到小扰动 vQ_k 作用, 则系统原有的对称性与守恒量将会相应地发生改变. 系统受扰动后的运动正轨满足如下运动微分方程为

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = -\frac{1-\alpha}{t-\tau}\frac{\partial L}{\partial \dot{q}_k} + \lambda_\beta f_{\beta k} + vQ_k,$$
$$(k = 1, 2, \cdots, n). \tag{15}$$

假设扰动后的无限小变换生成元 ξ_0, ξ_k 是在系统未受扰动时的无限小变换生成元基础上发生的摄动, 同时规范函数 G 也相应地发生了摄动, 即

$$\xi_0 = \xi_0^0 + v\xi_0^1 + v^2\xi_0^2 + \cdots,$$
$$\xi_k = \xi_k^0 + v\xi_k^1 + v^2\xi_k^2 + \cdots,$$
$$G = G^0 + vG^1 + v^2G^2 + \cdots \tag{16}$$

则对称性的摄动与绝热不变量之间的关系有如下定理.

定理2 El-Nabulsi模型下非Chetaev型非完整系统(6), (8)在受到小扰动 vQ_k 的作用下, 如果存在规范函数 $G^j(\tau, \boldsymbol{q}, \dot{\boldsymbol{q}})$, 使生成元 $\xi_0^j(\tau, \boldsymbol{q}, \dot{\boldsymbol{q}}), \xi_k^j(\tau, \boldsymbol{q}, \dot{\boldsymbol{q}})$ 满足结构方程

$$\frac{\partial L}{\partial \tau}\xi_0^j + \frac{\partial L}{\partial q_k}\xi_k^j + \frac{\partial L}{\partial \dot{q}_k}\left(\dot{\xi}_k^j - \dot{q}_k\dot{\xi}_0^j\right) + L\left(\dot{\xi}_0^j + \frac{1-\alpha}{t-\tau}\xi_0^j\right) + Q_k\left(\xi_k^{j-1} - \dot{q}_k\xi_0^{j-1}\right)$$
$$= -\dot{G}^j(t-\tau)^{1-\alpha}, \quad (j = 0, 1, 2, \cdots), \tag{17}$$

以及限制条件

$$f_{\beta k}\left(\xi_k^j - \dot{q}_k\xi_0^j\right) = 0,$$
$$(\beta = 1, \cdots, g; j = 0, 1, 2, \cdots), \tag{18}$$

其中 $j = 0$ 时, 约定 $\xi_0^{-1} = \xi_k^{-1} = 0$, 则

$$I_z = \sum_{j=0}^{z} v^j\left\{\left[L\xi_0^j + \frac{\partial L}{\partial \dot{q}_k}(\xi_k^j - \dot{q}_k\xi_0^j)\right] \times (t-\tau)^{\alpha-1} + G^j\right\} \tag{19}$$

是该系统的一个 z 阶绝热不变量.

证明

$$\begin{aligned}
&\frac{\mathrm{d}I_z}{\mathrm{d}\tau} \\
&= \sum_{j=0}^{z} v\left\{\left[\frac{\mathrm{d}L}{\mathrm{d}\tau}\xi_0^j + \frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}_k}(\xi_k^j - \dot{q}_k\xi_0^j)\right.\right. \\
&\quad \left. + L\dot{\xi}_0^j + \frac{\partial L}{\partial \dot{q}_k}(\dot{\xi}_k^j - \dot{q}_k\dot{\xi}_0^j - \ddot{q}_k\xi_0^j)\right](t-\tau)^{\alpha-1} \\
&\quad \left. + \left[L\xi_0^j + \frac{\partial L}{\partial \dot{q}_k}(\xi_k^j - \dot{q}_k\xi_0^j)\right](1-\alpha)(t-\tau)^{\alpha-2} + \dot{G}^j\right\} \\
&= \sum_{j=0}^{z} v^j\left[\left(\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} + \frac{1-\alpha}{t-\tau}\frac{\partial L}{\partial \dot{q}_k} - \lambda_\beta f_{\beta k}\right)\right. \\
&\quad \times (\xi_k^j - \dot{q}_k\xi_0^j) + \frac{\partial L}{\partial \tau}\xi_0^j + \frac{\partial L}{\partial q_k}\xi_k^j + \frac{\partial L}{\partial \dot{q}_k}(\dot{\xi}_k^j - \dot{q}_k\dot{\xi}_0^j) \\
&\quad + L\left(\dot{\xi}_0^j + \frac{1-\alpha}{t-\tau}\xi_0^j\right) + \lambda_\beta f_{\beta k}(\xi_k^j - \dot{q}_k\xi_0^j) \\
&\quad \left. + \dot{G}^j(t-\tau)^{1-\alpha}\right](t-\tau)^{\alpha-1} \\
&= \sum_{j=0}^{z} v^j\left[vQ_k(\xi_k^j - \dot{q}_k\xi_0^j) - Q_k(\xi_k^{j-1} - \dot{q}_k\xi_0^{j-1})\right] \\
&\quad \times (t-\tau)^{\alpha-1} \\
&= v^{z+1}Q_k(\xi_k^z - \dot{q}_k\xi_0^z)(t-\tau)^{\alpha-1}.
\end{aligned} \tag{20}$$

故 I_z 是El-Nabulsi模型下非Chetaev型非完整系统(6), (8)的一个 z 阶绝热不变量.

下面讨论一种特殊情况: El-Nabulsi模型下的Chetaev型非完整系统.

如果取 $f_{\beta k} = \dfrac{\partial F_\beta}{\partial \dot{q}_k}$, 则约束(6)为Chetaev型非完整约束, 虚位移方程(7)成为

$$\frac{\partial F_\beta}{\partial \dot{q}_k}\delta q_k = 0, \quad (\beta = 1, 2, \cdots, g). \tag{21}$$

于是, 系统未受扰动时的运动微分方程(8)和限制

条件 (13) 分别成为

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}_k}-\frac{\partial L}{\partial q_k}=-\frac{1-\alpha}{t-\tau}\frac{\partial L}{\partial \dot{q}_k}+\lambda_\beta\frac{\partial F_\beta}{\partial \dot{q}_k},\quad (k=1,2,\cdots,n), \tag{22}$$

$$\frac{\partial F_\beta}{\partial \dot{q}_k}\left(\xi_k^0-\dot{q}_k\xi_0^0\right)=0,(\beta=1,2,\cdots,g). \tag{23}$$

系统受到小扰动 vQ_k 作用后, 其运动微分方程 (15) 和限制条件 (18) 分别成为

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\partial L}{\partial \dot{q}_k}-\frac{\partial L}{\partial q_k}=-\frac{1-\alpha}{t-\tau}\frac{\partial L}{\partial \dot{q}_k}+\lambda_\beta\frac{\partial F_\beta}{\partial \dot{q}_k}+vQ_k,\quad (k=1,2,\cdots,n) \tag{24}$$

$$\frac{\partial F_\beta}{\partial \dot{q}_k}\left(\xi_k^j-\dot{q}_k\xi_0^j\right)=0,\quad (\beta=1,2,\cdots,g;j=0,1,2\cdots). \tag{25}$$

因此, 定理 1 和定理 2 分别成为如下定理 3 和定理 4.

定理 3 对于 El-Nabulsi 模型下未受扰动的 Chetaev 型非完整系统 (6), (22), 如果无限小变换的生成元 ξ_0^0, ξ_k^0 满足 El-Nabulsi-Noether 等式 (11) 和限制条件 (23), 则系统的 Noether 对称性直接导致精确不变量 (14) 式.

定理 4 El-Nabulsi 模型下 Chetaev 型非完整系统 (6), (22) 在受到小扰动 vQ_k 的作用下, 如果存在规范函数 $G^j(\tau,\boldsymbol{q},\dot{\boldsymbol{q}})$, 使生成元 $\xi_0^j(\tau,\boldsymbol{q},\dot{\boldsymbol{q}})$, $\xi_k^j(\tau,\boldsymbol{q},\dot{\boldsymbol{q}})$ 满足结构方程 (17) 和限制条件 (25), 则系统存在一个形如 (19) 式的 z 阶绝热不变量.

当 $\alpha=1$ 时, 上述结果退化为经典非完整系统的 Noether 对称性与精确不变量和对称性摄动与绝热不变量问题.

5 算 例

例 1 已知力学系统的 Lagrange 函数为

$$L=\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right), \tag{26}$$

所受的非完整约束为

$$F=\dot{q}_2-\tau\dot{q}_1=0, \tag{27}$$

假设系统是非 Chetaev 型的, 且虚位移方程有形式

$$\delta q_1-\delta q_2=0. \tag{28}$$

试研究 El-Nabulsi 模型下该非 Chetaev 型非完整约束系统 Noether 对称性的摄动与绝热不变量问题.

El-Nabulsi-Noether 等式 (11) 给出

$$\dot{q}_1\left(\dot{\xi}_1^0-\dot{q}_1\dot{\xi}_0^0\right)+\dot{q}_2\left(\dot{\xi}_2^0-\dot{q}_2\dot{\xi}_0^0\right)+\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right)\times\left(\dot{\xi}_0^0+\frac{1-\alpha}{t-\tau}\xi_0^0\right)=-\dot{G}^0\left(t-\tau\right)^{1-\alpha}, \tag{29}$$

由虚位移方程 (28) 式, 限制条件 (13) 式给出

$$\xi_1^0-\dot{q}_1\xi_0^0-\left(\xi_2^0-\dot{q}_2\xi_0^0\right)=0, \tag{30}$$

联立方程 (29) 和 (30), 有如下解

$$\xi_0^0=0,\xi_1^0=1,\xi_2^0=1,G^0=0. \tag{31}$$

生成元 (31) 相应于 El-Nabulsi 模型下所论非完整系统的 Noether 对称性. 由定理 1, 系统存在如下精确不变量:

$$I_0=\left(\dot{q}_1+\dot{q}_2\right)\left(t-\tau\right)^{\alpha-1}. \tag{32}$$

假设系统所受的小扰动为

$$vQ_1=v\dot{q}_1^2,\ vQ_2=v\dot{q}_1\dot{q}_2, \tag{33}$$

条件 (17) 给出

$$\dot{q}_1\left(\dot{\xi}_1^1-\dot{q}_1\dot{\xi}_0^1\right)+\dot{q}_2\left(\dot{\xi}_2^1-\dot{q}_2\dot{\xi}_0^1\right)+\frac{1}{2}\left(\dot{q}_1^2+\dot{q}_2^2\right)\times\left(\dot{\xi}_0^1+\frac{1-\alpha}{t-\tau}\xi_0^1\right)+Q_1\left(\xi_1^0-\dot{q}_1\xi_0^0\right)+Q_2\left(\xi_2^0-\dot{q}_2\xi_0^0\right)=-\dot{G}^1\left(t-\tau\right)^{1-\alpha}. \tag{34}$$

由虚位移方程 (28) 式, 限制条件 (18) 给出

$$\xi_1^1-\dot{q}_1\xi_0^1-\left(\xi_2^1-\dot{q}_2\xi_0^1\right)=0. \tag{35}$$

联立方程 (34) 和 (35), 有如下解:

$$\xi_0^1=0,\xi_1^1=-q_1,\xi_2^1=-q_1,G^1=0. \tag{36}$$

由定理 2 知, 系统存在如下一阶绝热不变量:

$$I_1=\left(\dot{q}_1+\dot{q}_2\right)\left(t-\tau\right)^{\alpha-1}-vq_1\left(\dot{q}_1+\dot{q}_2\right)\left(t-\tau\right)^{\alpha-1}. \tag{37}$$

进一步可求得系统的更高阶绝热不变量.

例 2 Appell-Hamel 模型 [28]:系统的 Lagrange 函数为

$$L=\frac{1}{2}m\left(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2\right)-mgq_3, \tag{38}$$

所受的非完整系约束

$$F=\dot{q}_1^2+\dot{q}_2^2-\dot{q}_3^2=0, \tag{39}$$

研究 El-Nabulsi 模型下 Chetaev 型非完整约束系统的 Noether 对称性的摄动与绝热不变量.

El-Nabulsi-Noether 等式 (11) 给出

$$-mg\xi_3^0+m\dot{q}_1\left(\dot{\xi}_1^0-\dot{q}_1\dot{\xi}_0^0\right)$$

$$+ m\dot{q}_2\left(\dot{\xi}_2^0 - \dot{q}_2\dot{\xi}_0^0\right) + m\dot{q}_3\left(\dot{\xi}_3^0 - \dot{q}_3\dot{\xi}_0^0\right) + \left[\frac{1}{2}m\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - mgq_3\right] \times \left(\dot{\xi}_0^0 + \frac{1-\alpha}{t-\tau}\xi_0^0\right) = -\dot{G}^0 (t-\tau)^{1-\alpha}. \quad (40)$$

条件 (23) 给出

$$\dot{q}_1\left(\xi_1^0 - \dot{q}_1\xi_0^0\right) + \dot{q}_2\left(\xi_2^0 - \dot{q}_2\xi_0^0\right) - \dot{q}_3\left(\xi_3^0 - \dot{q}_3\xi_0^0\right) = 0, \quad (41)$$

联立方程 (40) 和方程 (41), 系统有如下解

$$\xi_0^0 = (t-\tau)^{1-\alpha}, \quad \xi_1^0 = 0, \quad \xi_2^0 = 0, \quad \xi_3^0 = 0, \quad G^0 = 2m(\alpha-1)\int \dot{q}_3^2 (t-\tau)^{-1}\,\mathrm{d}\tau. \quad (42)$$

由定理 3, 系统有如下精确不变量

$$I_0 = -\frac{1}{2}m\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - mgq_3 + 2m(\alpha-1)\int \dot{q}_3^2 (t-\tau)^{-1}\mathrm{d}\tau. \quad (43)$$

当 $\alpha = 1$ 时, 守恒量 (43) 给出经典守恒量 [29]

$$I_0 = -\frac{1}{2}m\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - mgq_3. \quad (44)$$

假设系统受到小扰动

$$vQ_1 = 0, vQ_2 = v\left(m\ddot{q}_3 - mg\right)(t-\tau)^{\alpha-1}, \quad vQ_3 = vm\ddot{q}_2 (t-\tau)^{\alpha-1} \quad (45)$$

的作用, 条件 (17) 给出

$$- mg\xi_3^1 + m\dot{q}_1(\dot{\xi}_1^1 - \dot{q}_1\dot{\xi}_0^1) + m\dot{q}_2(\dot{\xi}_2^1 - \dot{q}_2\dot{\xi}_0^1) + m\dot{q}_3(\dot{\xi}_3^1 - \dot{q}_3\dot{\xi}_0^1) + \left[\frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) - mgq_3\right] \times \left(\dot{\xi}_0^1 + \frac{1-\alpha}{t-\tau}\xi_0^1\right) + Q_1(\xi_1^0 - \dot{q}_1\xi_0^0) + Q_2(\xi_2^0 - \dot{q}_2\xi_0^0) + Q_3(\xi_3^0 - \dot{q}_3\xi_0^0) = -\dot{G}^1(t-\tau)^{1-\alpha}. \quad (46)$$

由条件 (25) 有

$$\dot{q}_1\left(\xi_1^1 - \dot{q}_1\xi_0^1\right) + \dot{q}_2\left(\xi_2^1 - \dot{q}_2\xi_0^1\right) - \dot{q}_3\left(\xi_3^1 - \dot{q}_3\xi_0^1\right) = 0. \quad (47)$$

联立方程 (46) 和方程 (47), 得解

$$\xi_0^1 = 0, \xi_1^1 = 0, \xi_2^1 = \dot{q}_3, \xi_3^1 = \dot{q}_2, G^1 = 0. \quad (48)$$

由定理 4 可知, 系统有如下一阶绝热不变量

$$I_1 = -\frac{1}{2}m\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - mgq_3 + 2m(\alpha-1)\int \dot{q}_3^2 (t-\tau)^{-1}\mathrm{d}\tau + 2vm\dot{q}_2\dot{q}_3 (t-\tau)^{\alpha-1}. \quad (49)$$

当 $\alpha = 1$ 时, (49) 式给出经典一阶绝热不变量

$$I_1 = -\frac{1}{2}m\left(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2\right) - mgq_3 + 2vm\dot{q}_2\dot{q}_3. \quad (50)$$

进一步可求得系统的更高阶绝热不变量.

6 结 论

文章提出并研究了 El-Nabulsi 模型下非 Chetaev 型非完整系统的精确不变量与绝热不变量问题. 得到了该模型下非 Chetaev 型非完整系统 Noether 对称性导致的 Noether 型绝热不变量. 文章结果具有一般性, El-Nabulsi 模型下 Chetaev 型非完整约束系统是其特例. 当 $\alpha = 1$ 时, 本文结果退化为经典非完整系统 Noether 对称性的摄动与绝热不变量问题. 本文方法还可进一步推广应用于研究 El-Nabulsi 模型下 Lie 对称性和 Mei 对称性的摄动与绝热不变量问题等.

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Exact invariants and adiabatic invariants for nonholonomic systems in non-Chetaev's type based on El-Nabulsi dynamical models*

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Abstract

In this paper, the problem of exact invariants and adiabatic invariants for nonholonomic system in non-Chetaev's type based on the El-Nabulsi dynamical model is studied. First, the El-Nabulsi-d'Alembert-Lagrange principle is deduced and the differential equations of motion of the system are established. Then, the relation between the Noether symmetry and the exact invariant that is led directly by the symmetry for undisturbed nonholonomic system in non-Chetaev's type is given. Furthermore, by introducing the concept of high-order adiabatic invariant of a mechanical system, the conditions that the perturbation of symmetry leads to the adiabatic invariant and its formulation are studied for the nonholonomic system in non-Chetaev's type under the action of small disturbance. As a special case, the problem of the exact invariants and the adiabatic invariants for the nonholonomic system in Chetaev's type in El-Nabulsi model is discussed. At the end of the paper, two examples for the nonholonomic systems in non-Chetaev's type constraints and also the Chetaev's type constraints are given respectively to show the application of the methods and the results of this paper.

Keywords: perturbation of symmetry, adiabatic invariant, nonholonomic system in non-Chetaev's type, El-Nabulsi dynamical model

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