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二项-负二项组合光场态的光子统计性质及其在量子扩散通道中的生成*

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在组合二项-负二项分布的基础上, 提出了二项-负二项组合光场态, 这种态能在 Fock 态历经量子扩散通道的过程中实现. 导出了此光场的二阶相干度公式, $g^{(2)}(t) = 2 - \dfrac{m^2+m}{(m+\kappa t)^2}$, 发现随着时间的推移光场从非经典 Fock 态变为经典态, 光子数 m 经扩散通道后变成了 $m+\kappa t$, κ 是扩散常数, 相应的光子统计从亚泊松分布历经泊松分布再变成混沌光; 初始 Fock 态的光子数越多, 则扩散所需的时间越长.

关键词: 二项-负二项组合光场态, 二阶相干度, 亚泊松分布, 泊松分布

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1 引 言

量子光学的一个重要课题是发现光场的新量子态并研究其非经典性质, 以便对光的本性有新的了解. 例如, 相干态与压缩态光场和热光场有本质的区别, 除此以外, 人们还发现了光场的二项式态光场, 其密度算符为[1]

$$\rho_{\mathrm{B}} = \sum_{m=0}^{n} \binom{n}{m} r^m (1-r)^{n-m} |m\rangle\langle m|, \tag{1}$$

这里 $|m\rangle = \dfrac{a^{\dagger m}}{\sqrt{m!}}|0\rangle$ 是光子数态. 由二项式定理

$$\sum_{m=0}^{n} \binom{n}{m} r^m s^{n-m} = (r+s)^n \tag{2}$$

知 $\mathrm{tr}\rho_{\mathrm{B}} = 1$. 当一个粒子数态经历一个振幅衰减通道, 就会演化为二项式态[2], 光场的奇-偶二项式态在文献 [3] 中提出. 此后不久, 人们又发现了光场的负二项式态, 其密度算符为[4]

$$\rho_{\mathrm{A}} = \sum_{m=0}^{n} \binom{n+m}{m} r^{n+1} (1-r)^m |m\rangle\langle m|, \tag{3}$$

由负二项式定理

$$\sum_{m=0}^{\infty} \binom{n+m}{m} (-x)^m = (1+x)^{-n-1} \tag{4}$$

知

$$\mathrm{tr}\rho_{\mathrm{A}} = r^{n+1} \sum_{m=0}^{n} \binom{n+m}{m} (1-r)^m = 1. \tag{5}$$

当一个粒子数态经过双模压缩算符作用后就会呈现负二项分布. 处在负二项式态的光子数

$$\begin{aligned}\mathrm{tr}\left(\rho_{\mathrm{A}} a^\dagger a\right) &= r^{n+1} \sum_{m=0}^{n} \binom{n+m}{m} (1-r)^m m \\ &= r^{n+1} (1-r) \frac{\partial}{\partial (1-r)} r^{-(n+1)} \\ &= (n+1)(1-r)/r. \end{aligned}\tag{6}$$

光场的奇-偶负二项式态在文献 [5] 中提出.

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本文将指出让粒子数态经过量子扩散通道可以产生一个新的光场: 光场的二项式-负二项式态, 这是一个具有组合二项式和负二项式分布意义的态, 我们着重分析它的非经典性质. 本文安排如下: 在第二节简略介绍二项式和负二项式组合分布函数, 并在此基础上构造二项和负二项组合光场态; 在第三节为了说明当光的粒子数态 $|m\rangle\langle m|$ 通过一个扩散通道后会生成这样一个态, 先介绍量子扩散通道的Kraus算符解; 在第四和第五节分别计算此光场的平均光子数和二阶相干度, 光子数 m 经扩散通道后变成了 $m+\kappa t$, κ 是扩散常数, 此光场的二阶相干度公式 $g^{(2)}(t)=2-\dfrac{m^2+m}{(m+\kappa t)^2}$, 即相应的光子统计从亚泊松分布历经泊松分布再变成混沌光.

2 二项式和负二项式组合分布

引入如下的二项-负二项联合分布(或称为概率质量分布函数):

$$f(x,y,m)=\binom{m}{l}x^l\binom{m-l+j}{j}y^j. \tag{7}$$

根据二项式定理和负二项式定理得到

$$\begin{aligned}&\sum_{l=0}^{m}\sum_{j=0}^{\infty}\binom{m}{l}\binom{m-l+j}{j}x^ly^j\\&=\sum_{l=0}^{m}\binom{m}{l}x^l(1-y)^{-(m-l)-1}\\&=\left(x+\frac{1}{1-y}\right)^m\frac{1}{1-y}.\end{aligned} \tag{8}$$

在 $\sum_{l=0}^{m}\sum_{j=0}^{\infty}\binom{m}{l}\binom{m-l+j}{j}x^ly^j$ 中, 当 $x=0$, l 只能取为零, 这时此式约化为 $\sum_{j=0}^{\infty}\binom{m+j}{j}y^j$, 即负二项分布. 在此基础上引入二项和负二项组合光场态, 其密度算符为

$$\begin{aligned}\rho=C_m\sum_{l=0}^{m}\sum_{j=0}^{\infty}\binom{m}{l}\binom{m-l+j}{j}x^ly^j\\\times|m-l+j\rangle\langle m-l+j|,\end{aligned} \tag{9}$$

这里 C_m 待定,

$$|m-l+j\rangle=\frac{a^{\dagger m-l+j}}{(m-l+j)!}|0\rangle, \tag{10}$$

$a^\dagger$ 是光子产生算符, $|0\rangle$ 是真空态, $|0\rangle$ 由 a 湮没, $\left[a,a^\dagger\right]=1$. 由于要求 $\mathrm{tr}\rho=1$,

$$\mathrm{tr}\rho=C_m\sum_{l=0}^{m}\binom{m}{l}x^l\left(\frac{1}{1-y}\right)^{m-l+1}=1, \tag{11}$$

故 C_m 要满足

$$C_m=(1-y)\left(x+\frac{1}{1-y}\right)^{-m}. \tag{12}$$

3 量子扩散通道的Kraus算符解的简要回顾

众所周知, 系统和外界环境作用引起的量子退相干导致系统的初态密度矩阵 $\rho(0)$ 演化到 t 时刻系统的密度矩阵 $\rho(t)$, 两者之间通过以下方程关联 [6]:

$$\rho(t)=\sum_{m,n=0}^{\infty}M_{m,n}\rho_0M_{m,n}^\dagger, \tag{13}$$

此形式称为密度矩阵的无限算符和表示, $M_{m,n}$ 称之为Kraus算符. 描述扩散形式的主方程为 [7]

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}=-\kappa(a^\dagger a\rho-a^\dagger\rho a-a\rho a^\dagger+\rho aa^\dagger). \tag{14}$$

这里 κ 是扩散常数.

为了揭示获得方程(2) Kraus算符的方法, 我们引入双模纠缠态表象 [8]

$$\begin{aligned}|\eta\rangle=\exp\bigg(&-\frac{1}{2}|\eta|^2+\eta a^\dagger-\eta^*\tilde{a}^\dagger\\&+a^\dagger\tilde{a}^\dagger\bigg)|0\tilde{0}\rangle,\end{aligned} \tag{15}$$

这里 $\tilde{a}^\dagger$ 是独立于实模 $a^\dagger$ 的一个虚模, $|\tilde{0}\rangle$ 由 $\tilde{a}$ 湮没, $\left[\tilde{a},\tilde{a}^\dagger\right]=1$. 态 $|\eta=0\rangle\equiv|I\rangle$ 具有如下特性:

$$\begin{aligned}&a|I\rangle=\tilde{a}^\dagger|I\rangle,a^\dagger|I\rangle=\tilde{a}|I\rangle,\\&(a^\dagger a)^n|I\rangle=(\tilde{a}^\dagger\tilde{a})^n|I\rangle.\end{aligned} \tag{16}$$

将方程(14)两边作用于 $|I\rangle$ 上并标记 $|\rho\rangle=\rho|I\rangle$, 通过(16)式可知 $|\rho(t)\rangle$ 服从时间演化方程

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}|\rho(t)\rangle=&-\kappa(a^\dagger a\rho-a^\dagger\rho a-a\rho a^\dagger+\rho aa^\dagger)|I\rangle\\=&-\kappa(a^\dagger-\tilde{a})(a-\tilde{a}^\dagger)|\rho(t)\rangle.\end{aligned} \tag{17}$$

其形式解为

$$|\rho(t)\rangle=\exp[-\kappa t(a^\dagger-\tilde{a})(a-\tilde{a}^\dagger)]|\rho_0\rangle, \tag{18}$$

注意到本征方程

$$(a-\tilde{a}^\dagger)|\eta\rangle=\eta|\eta\rangle,\ (a^\dagger-\tilde{a})|\eta\rangle=\eta^*|\eta\rangle \tag{19}$$

以及

$$\langle\eta|(a^\dagger-\tilde{a})=\eta^*\langle\eta|,\ \langle\eta|(a-\tilde{a}^\dagger)=\eta\langle\eta|, \tag{20}$$

$|\rho(t)\rangle$ 与 $\langle\eta|$ 的内积函数可被表示为

$$\begin{aligned}\langle\eta|\rho\rangle &= \langle\eta|\exp[-\kappa t(a^\dagger-\tilde{a})(a-\tilde{a}^\dagger)]|\rho_0\rangle\\ &= \mathrm{e}^{-\kappa t|\eta|^2}\langle\eta|\rho_0\rangle.\end{aligned} \tag{21}$$

再利用热纠缠态表象的完备性关系

$$\int\frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta|=1, \tag{22}$$

可得

$$\begin{aligned}|\rho(t)\rangle &= \int\frac{\mathrm{d}^2\eta}{\pi}\mathrm{e}^{-\kappa t|\eta|^2}|\eta\rangle\langle\eta|\rho_0\rangle\\ &= \int\frac{\mathrm{d}^2\eta}{\pi}\colon\exp[-(1+\kappa t)|\eta|^2+\eta(a^\dagger-\tilde{a})\\ &\quad+\eta^*(a-\tilde{a}^\dagger)+a^\dagger\tilde{a}^\dagger+a\tilde{a}-a^\dagger a\\ &\quad-\tilde{a}^\dagger\tilde{a}]\colon|\rho_0\rangle\\ &= \frac{1}{1+\kappa t}\colon\exp\left[\frac{\kappa t}{1+\kappa t}(a^\dagger\tilde{a}^\dagger+a\tilde{a}\right.\\ &\quad\left.-a^\dagger a-\tilde{a}^\dagger\tilde{a})\right]\colon|\rho_0\rangle\\ &= \frac{1}{1+\kappa t}\mathrm{e}^{\frac{\kappa t}{1+\kappa t}a^\dagger\tilde{a}^\dagger}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a+\tilde{a}^\dagger\tilde{a}}\\ &\quad\times\mathrm{e}^{\frac{\kappa t}{1+\kappa t}a\tilde{a}}|\rho_0\rangle.\end{aligned} \tag{23}$$

再由 (16) 式, 有

$$\mathrm{e}^{\frac{\kappa t}{1+\kappa t}a\tilde{a}}|\rho_0\rangle=\sum_{l=0}^{\infty}\frac{1}{l!}\left(\frac{\kappa t}{1+\kappa t}a\right)^l\rho_0 a^{\dagger l}|I\rangle, \tag{24}$$

因此, (23) 式变为

$$\begin{aligned}|\rho(t)\rangle &= \mathrm{e}^{\frac{\kappa t}{1+\kappa t}a^\dagger\tilde{a}^\dagger}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a+1}\\ &\quad\times\sum_{n=0}^{\infty}\frac{1}{l!}\left(\frac{\kappa t}{1+\kappa t}a\right)^l\rho_0\\ &\quad\times a^{\dagger l}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}|I\rangle\\ &= \sum_{j,l=0}^{\infty}\frac{1}{j!l!}\frac{(\kappa t)^{j+l}}{(\kappa t+1)^{j+l+1}}\\ &\quad\times a^{\dagger j}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^l\rho_0\\ &\quad\times a^{\dagger l}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^j|I\rangle,\end{aligned} \tag{25}$$

所以密度算符 ρ 的 Kraus 算符和表示为

$$\begin{aligned}\rho(t) &= \sum_{j,l=0}^{\infty}\frac{1}{j!l!}\frac{(\kappa t)^{j+l}}{(\kappa t+1)^{j+l+1}}a^{\dagger j}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}\\ &\quad\times a^l\rho_0 a^{\dagger l}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^j\\ &\equiv\sum_{j,l=0}^{\infty}M_{j,l}\rho_0 M_{j,l}^\dagger.\end{aligned} \tag{26}$$

这里 Kraus 算符

$$\begin{aligned}M_{j,l} &= \sqrt{\frac{1}{j!l!}\frac{(\kappa t)^{j+l}}{(\kappa t+1)^{j+l+1}}}a^{\dagger j}\\ &\quad\times\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^l.\end{aligned} \tag{27}$$

4 粒子态密度矩阵 $\rho_0=|m\rangle\langle m|$ 在扩散通道中的演化

在 (26) 式中让

$$T_1=\frac{\kappa t}{1+\kappa t},\quad T_2=\frac{1}{1+\kappa t}, \tag{28}$$

鉴于

$$\begin{aligned}&\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^l=a^l(1+\kappa t)^l\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a},\\ &a^{\dagger l}\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}=\left(\frac{1}{1+\kappa t}\right)^{a^\dagger a}a^{\dagger l}(1+\kappa t)^l,\end{aligned} \tag{29}$$

因此有

$$\begin{aligned}\rho(t) &= T_2\sum_{j,l=0}^{\infty}\frac{1}{j!l!T_2^{2l}}T_1^{j+l}a^{\dagger j}a^l\,\mathrm{e}^{a^\dagger a\ln T_2}\\ &\quad\times\rho_0\,\mathrm{e}^{a^\dagger a\ln T_2}a^{\dagger l}a^j.\end{aligned} \tag{30}$$

设初态为粒子数态, 即 $\rho_0=|m\rangle\langle m|$, 代入 (30) 式, 利用

$$\begin{aligned}a|m\rangle &= \sqrt{m}|m-1\rangle,\\ a^\dagger|m\rangle &= \sqrt{m+1}|m+1\rangle\end{aligned} \tag{31}$$

计算

$$\begin{aligned}\rho_{|m\rangle}(t) &= T_2\sum_{j,l=0}^{\infty}\frac{1}{j!l!T_2^{2l}}T_1^{j+l}\,\mathrm{e}^{2m\ln T_2}\\ &\quad\times a^{\dagger j}a^l|m\rangle\langle m|a^{\dagger l}a^j\\ &= T_2\sum_{j,l=0}^{\infty}\frac{1}{j!l!}T_1^{j+l}T_2^{2(m-l)}\\ &\quad\times a^{\dagger j}\frac{m!}{(m-l)!}|m-l\rangle\langle m-l|a^j\\ &= T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\end{aligned}$$

$$\times\sum_{j=0}^{\infty}T_1^j\binom{m-l+j}{j}$$
$$\times|m-l+j\rangle\langle m-l+j|. \tag{32}$$

由于

$$(1-T_1)\left(\frac{T_1}{T_2^2}+\frac{1}{1-T_1}\right)^{-m}=T_2^{2m+1}, \tag{33}$$

再根据(11)和(12)式知 $\mathrm{tr}\rho_{|m\rangle}(t)=1$, 所以(32)式有资格成为量子光学理论中的一个二项-负二项联合分布态.

5 处于二项-负二项组合分布态的平均光子数

现在计算处于二项-负二项组合态的平均光子数

$$\begin{aligned}&\mathrm{tr}[\rho_{|m\rangle}(t)a^{\dagger}a]\\&=T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\sum_{j=0}^{\infty}T_1^j\binom{m-l+j}{j}\\&\quad\times(m-l+j).\end{aligned} \tag{34}$$

我们需分别考察其中的带 l 与带 j 的项的贡献. 带 l 的项的贡献为

$$\begin{aligned}&\sum_{l=0}^{\infty}\binom{m}{l}x^l\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j}l\\&=\sum_{l=0}^{\infty}\binom{m}{l}x^l(1-y)^{-(m-l)-1}l\\&=mx\left(x+\frac{1}{1-y}\right)^{m-1}\frac{1}{1-y},\end{aligned} \tag{35}$$

故

$$\begin{aligned}&T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\sum_{j=0}^{\infty}T_1^j\binom{m-l+j}{j}l\\&=T_2^{2m+1}m\frac{T_1}{T_2^3}\left(\frac{T_1}{T_2^2}+\frac{1}{1-T_1}\right)^{m-1}\\&=mT_1.\end{aligned} \tag{36}$$

又因带 j 的项的贡献是

$$\begin{aligned}&\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j}j\\&=y\frac{\partial}{\partial y}\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j}\\&=y\frac{\partial}{\partial y}(1-y)^{-(m-l)-1}\\&=(1-y)^{-(m-l)-2}(m+1-l),\end{aligned} \tag{37}$$

于是

$$\begin{aligned}&T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\sum_{j=0}^{\infty}T_1^j\binom{m-l+j}{j}j\\&=T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^lT_1(m+1-l)\\&\quad\times\left(\frac{1}{1-T_1}\right)^{m-l+2}.\end{aligned} \tag{38}$$

(38)式中的

$$\begin{aligned}&(m+1)T_1T_2^{2m+1}\\&\quad\times\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\left(\frac{1}{1-T_1}\right)^{m-l+2}\\&=(m+1)\frac{T_1}{T_2}\end{aligned} \tag{39}$$

以及

$$\begin{aligned}&T_1T_2^{2m+1}\sum_{l=0}^{\infty}l\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\left(\frac{1}{1-T_1}\right)^{m-l+2}\\&=m\frac{T_1^2}{T_2}.\end{aligned} \tag{40}$$

结合(36)和(38)—(40)式, 并用(28)式得到

$$\mathrm{tr}[\rho_{|m\rangle}(t)a^{\dagger}a]=m+\frac{T_1}{T_2}=m+\kappa t. \tag{41}$$

可见初态的光子数 m 经扩散通道后变成了 $m+\kappa t$, 当扩散常数 κ 较小, 光子数缓慢增加 κt, 故可以用于量子调控.

6 二项-负二项组合态的光子统计性质

为了计算二项-负二项组合态的二阶相干度, 考虑

$$\begin{aligned}&\mathrm{tr}[\rho_{|m\rangle}(t)a^{\dagger}aa^{\dagger}a]\\&=T_2^{2m+1}\sum_{l=0}^{\infty}\binom{m}{l}\left(\frac{T_1}{T_2^2}\right)^l\\&\quad\times\sum_{j=0}^{\infty}T_1^j\binom{m-l+j}{j}(m-l+j)^2,\end{aligned} \tag{42}$$

引入

$$\begin{aligned}R_1&\equiv\sum_{l=0}^{\infty}\binom{m}{l}x^l\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j}\\&=\left(x+\frac{1}{1-y}\right)^m\frac{1}{1-y},\end{aligned} \tag{43}$$

$$R_2 \equiv \sum_{l=0}^{\infty}\binom{m}{l}x^l\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j} \times (m-l+j), \tag{44}$$

$$R_3 \equiv \sum_{l=0}^{\infty}\binom{m}{l}x^l\sum_{j=0}^{\infty}y^j\binom{m-l+j}{j} \times (m-l+j)^2. \tag{45}$$

它们满足递推关系

$$R_2 = mR_1 - x\frac{\partial}{\partial x}R_1 + y\frac{\partial}{\partial y}R_1, \tag{46}$$

$$R_3 = mR_2 - x\frac{\partial}{\partial x}R_2 + y\frac{\partial}{\partial y}R_2. \tag{47}$$

从 (43) 式导出

$$\frac{\partial}{\partial x}R_1 = m\left(x+\frac{1}{1-y}\right)^{m-1}\frac{1}{1-y}, \tag{48}$$

$$\frac{\partial}{\partial y}R_1 = \left(x+\frac{1}{1-y}\right)^{m}\frac{1}{(1-y)^2} + m\left(x+\frac{1}{1-y}\right)^{m-1}\frac{1}{(1-y)^3}. \tag{49}$$

将它们代入 (46) 式得到

$$R_2 = \left(x+\frac{1}{1-y}\right)^{m-1}\frac{m+xy-xy^2+y}{(1-y)^3}. \tag{50}$$

用同样的方法计算 R_3 后, 得到

$$\begin{aligned}&\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger aa^\dagger a]\\ &= T_{2m+1}R_3 = T_{2m+1}\left(mR_2 - x\frac{\partial}{\partial x}R_2 + y\frac{\partial}{\partial y}R_2\right)\\ &= m^2+4mkt+2k^2t^2+kt.\end{aligned} \tag{51}$$

具体的过程见附录 A.

根据二阶相干度公式[9]

$$g^{(2)}(t) = \frac{\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger aa^\dagger a] - \mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger a]}{\left(\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger a]\right)^2}, \tag{52}$$

当 $g^{(2)}=1$, $\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger aa^\dagger a] - \left(\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger a]\right)^2 = \mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger a]$, 即光子数的方差等于 $\bar{n}$, 呈现如相干态那样的泊松分布性质. 处于初态 $\rho_0 = |m\rangle\langle m|$ 时

$$g^{(2)}(0) = 1-\frac{1}{m}.$$

把 (51) 和 (41) 式代入 (52) 式得到终态的二阶相干度

$$g^{(2)}(t) = 2-\frac{m^2+m}{(m+\kappa t)^2}. \tag{53}$$

显然,

1) 当 $\kappa t < \sqrt{m}(\sqrt{m+1}-\sqrt{m})$ 时,

$$g^{(2)} < 1, \tag{54}$$

此态仍呈现亚泊松分布和反聚束效应 (非经典态);

2) 当 $\kappa t = \sqrt{m}(\sqrt{m+1}-\sqrt{m})$ 时,

$$g^{(2)} = 1, \tag{55}$$

此态呈现泊松分布; 随着时间的增加,

3) 当 $\kappa t > \sqrt{m}(\sqrt{m+1}-\sqrt{m})$ 时,

$$g^{(2)} > 1, \tag{56}$$

特别地, 当 t 趋于无穷大, $g^{(2)} \to 2$, 就呈现混沌光的性质. 这说明, 随着时间的推移发生量子扩散, 光场从非经典态变为经典态. 当初始态的光子数越多, 则扩散所需的时间越长.

7 结 论

本文在理论上构造了新型的二项-负二项组合光场态, 并且提出了用扩散量子通道可以实现从泊松分布到二项-负二项组合分布过渡的方法, 从而给相应的实验验证提供了一个可能的方向. 此外, 我们通过理论计算分析了这种新型光场的光子统计性质, 包括平均光子数和二阶相干度: 前者的计算结果给这种光场提供了量子调控的可能性; 后者的计算结果则揭示了光场由非经典态向经典态过渡的过程. 本文的结论在理论意义上清晰可靠, 不足之处在于未能找到相关实验佐证. 此后我们会尝试与相关实验室合作探究在实验上构造此种光场的可能性, 以验证本文得出的相关结论.

附录 A $\mathrm{tr}[\rho_{|m\rangle}(t)a^\dagger aa^\dagger a]$ 表达式的推导

由 (44) 式得到

$$x\frac{\partial}{\partial x}R_2 = x\left[(m-1)\left(x+\frac{1}{1-y}\right)^{m-2}\frac{m+xy-xy^2+y}{(1-y)^3} + \left(x+\frac{1}{1-y}\right)^{m-1}\frac{y}{(1-y)^2}\right]$$

和

$$y\frac{\partial}{\partial y}R_2 = y\left[(m-1)\left(x+\frac{1}{1-y}\right)^{m-2}\frac{m+xy-xy^2+y}{(1-y)^5} + \left(x+\frac{1}{1-y}\right)^{m-1}\frac{3m+x+2y-xy^2+1}{(1-y)^4}\right].$$

代入 (47) 式得到

$$R_3 = m\left(x+\frac{1}{1-y}\right)^m\frac{1}{1-y} - xm\left(x+\frac{1}{1-y}\right)^{m-1}\frac{1}{1-y}$$

$$
\begin{aligned}
&+\left(x+\frac{1}{1-y}\right)^{m}\frac{y}{(1-y)^2}\\
&+m\left(x+\frac{1}{1-y}\right)^{m-1}\frac{y}{(1-y)^3}\\
=&\left(x+\frac{1}{1-y}\right)^{m-1}\frac{m+xy-xy^2+y}{(1-y)^3}.
\end{aligned}
$$

再代入 $x=\dfrac{T_1}{T_2^2}$, $y=T_1$, $T_1=\dfrac{\kappa t}{1+\kappa t}$ 和 $T_2=\dfrac{1}{1+\kappa t}$, 注意 $T_1+T_2=1$, $\dfrac{T_1}{T}=\kappa t$, 得到

$$
\begin{aligned}
&mR_2=\frac{m(m+\kappa t)}{T_2^{2m+1}},\\
&x\frac{\partial}{\partial x}R_2=T_1\left[(m-1)\frac{m+\kappa^2t^2-\kappa^2t^2T_1+T_1}{T_1^{2m+1}}+\frac{\kappa t}{T_1^{2m+1}}\right],\\
&y\frac{\partial}{\partial y}R_2=T_1\left[(m-1)\frac{m+\kappa^2t^2-\kappa^2t^2T_1+T_1}{T_1^{2m+1}}\right.\\
&\qquad\left.+\frac{3m+\kappa t(1+\kappa t)+2T_1-\kappa^2t^2T_1+1}{T_2^{2m+2}}\right].
\end{aligned}
$$

于是有

$$
\begin{aligned}
&\operatorname{tr}[\rho_{|m\rangle}(t)a^{\dagger}aa^{\dagger}a]\\
&=T_{2m+1}R_3=T_{2m+1}\left(mR_2-x\frac{\partial}{\partial x}R_2+y\frac{\partial}{\partial y}R_2\right)\\
&=m^2+4m\kappa t+2\kappa^2t^2+\kappa t.
\end{aligned}
$$

这就是 (51) 式.

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Statistical properties of binomial and negative-binomial combinational optical field state and its generation in quantum diffusion channel*

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Abstract

According to the combinational binomial-negative-binomial distribution, we propose a binomial-negative-binomial combinational optical field state, which can be generated in the process of a Fock state $|m\rangle\langle m|$ passing through a quantum-mechanical diffusion channel. We derive the second-order coherence degree formula, $g^{(2)}(t) = 2 - \dfrac{m^2+m}{(m+\kappa t)^2}$, which is the diffusion constant. We find that in the process of the Fock state undergoing quantum diffusion and becoming classical, the corresponding photon statistics evolves from sub-Poissonian distribution to Poisson distribution and finally goes to a chaotic state. We also find that the more photons in the initial Fock state, the longer time is needed for quantum decoherence.

Keywords: binomial-negative-binomial combinational optical field state, second-order coherence, Poisson distribution, sub-Poissonian distribution

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