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Thermodynamic properties of Fermi gas under generalized uncertainty principle

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引用信息 Citation: *Acta Physica Sinica*, 64, 080502 (2015) DOI: 10.7498/aps.64.080502

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广义不确定性原理下费米气体低温热力学性质*

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(2014年10月13日收到; 2014年12月1日收到修改稿)

在考虑到广义不确定性原理时, 统计物理中的态密度必须做出修正, 这导致对传统统计物理的所有结果都有不同程度的修正. 在高能、高温条件下, 此修正是颠覆传统观念的, 在低温条件下, 也有一定的修正. 研究了低温条件下考虑到广义不确定性原理时, 理想费米气体和具有弱相互作用费米气体的热力学性质, 分别给出理想费米气体和弱相互作用费米气体的化学势、内能和定容热容的解析表达式, 并以铜电子气体为例进行了具体数值计算, 将计算结果与不考虑广义不确定性原理时的费米气体的热力学性质进行了比较, 探讨了广义不确定性原理对系统热力学性质的影响. 考虑到广义不确定性原理后费米气体的化学势、费米能和基态能增大, 热容减少, 内能随温度的增加先增大, 到某一温度(对于铜电子气体, $T/T_{F0} \sim 0.3$)时, 增值为零, 温度再增加内能减少. 这些修正的具体数值主要由粒子数密度决定, 粒子数密度越大, 修正越大.

关键词: 广义不确定性原理, 费米气体, 热力学性质

PACS: 05.30.-d, 51.30.+i

DOI: 10.7498/aps.64.080502

1 引言

在引力的量子理论中, 如弦论、双狭义相对论和黑洞物理, 存在可观察的最小长度的限制^[1-6], 不确定性原理被修正为广义不确定性原理(generalized uncertainty principle, GUP)^[7-10], 这导致统计物理中的量子态密度被修正^[11], 进而统计物理中已研究过的一些问题的热力学性质都有不同程度的修正, 如黑体辐射^[11]、黑洞熵^[12]、变形空间的高温热容^[13]、任意维空间的谐振子^[14]、理想气体^[15](经典理想气体、极端相对论理想气体、理想费米子和玻色子气体)等问题.

在高能、极高温条件下, GUP下的统计物理对传统统计物理具有颠覆性的冲击^[13,15]. 如采用GUP下的统计物理计算的热容随温度趋于无穷大而趋于零^[13], 但传统统计物理计算的热容在通常的高温情况下是能均分定理的结果, 即其与温度无关, 是常量; 对于经典理想气体, GUP下的统计物

理的结果是: 随温度趋于无穷大, 系统的激发态的粒子数会趋于一定的值, 即此时若再增加系统的粒子数, 它们将在基态“凝聚”(温度趋于无穷大时的凝聚), 在极限温度时系统能量不再增加, 出现了能量上限^[15]. 传统统计物理对于经典理想气体的推理结果没有“高温凝聚”与“能量上限”.

在低温条件下, GUP下的统计物理虽然不具有极高温情景时对传统观念的颠覆性修正, 但仍然有一些修正.

超冷费米子系统实验的重大进展^[16-20], 使低温费米子系统的理论研究成为热点^[21-24].

本文主要研究考虑到GUP时, 理想费米气体和弱相互作用费米系统在低温条件下的热力学性质. 具体内容为: 1) 解析计算了考虑GUP时低温条件下理想费米气体的化学势、内能和热容, 并以铜电子气体为例数值计算了这3个热力学量与温度的关系, 讨论了与不考虑GUP时的差异; 2) 解析计算了考虑GUP时低温条件下具有弱相互作用费米气体的化学势、内能和热容, 以铜电子气体为例进

* 国家自然科学基金(批准号: 61167002)和宁夏自然科学基金(批准号: NZ14055)资助的课题.

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行数值计算与差异的讨论.

2 GUP 下低温理想费米气体的性质

2.1 GUP 和态密度的修正

由文献 [7—10] 知位置的不确定性 Δx 和动量的不确定性 Δp 满足 GUP, 最简单的 GUP 为

$$\Delta x \Delta p \geq \frac{\hbar}{2} [1 + A(\Delta p)^2], \quad (1)$$

式中, A 是正比于普朗克长度的正数; $\hbar = h/(2\pi)$, h 是普朗克常数. 由 (1) 式可得位置存在一非零的最小长度的限制, $\Delta x_{\min} = \hbar\sqrt{A}$. 此最小长度对诸多物理问题产生影响 [1—15, 25—30].

在三维空间, 通常的统计物理得到的单粒子能量 ε 附近单位能量间隔内个体量子态数 (态密度) 为

$$g(\varepsilon) = 2\pi g_0 V (2m)^{3/2} \varepsilon^{1/2} / h^3, \quad (2)$$

式中, V 为体积, g_0 是费米子内部运动基态简并度, m 为费米子质量. 考虑到 GUP 对态密度有一修正因子 $(1 + Ap^2)^{-3}$, 则态密度为 [11]

$$g(\varepsilon) = 2\pi g_0 V (2m)^{3/2} \frac{\varepsilon^{1/2}}{h^3 (1 + Ap^2)^3}, \quad (3)$$

(3) 式中的动量 p 与能量 ε 间的关系为 $\varepsilon = p^2/(2m)$.

为便于计算系统的热力学性质, 文献 [15] 将 (3) 式对动量 p 展开成幂级数:

$$g(\varepsilon) = 2\pi g_0 V (2m)^{3/2} \frac{\varepsilon^{1/2}}{h^3} (1 - 3Ap^2 + 6A^2p^4 - 10A^3p^6 + \dots), \quad (4a)$$

式中第一项为不考虑 GUP 时的态密度. 我们更倾向于将 (3) 式按带有 Peano 型余项的 Maclaurin 公式对动量 p 展开:

$$g(\varepsilon) = 2\pi g_0 V (2m)^{3/2} \frac{\varepsilon^{1/2}}{h^3} \{1 - 3Ap^2 + 6A^2p^4 - 10A^3p^6 + \dots + o[(Ap^2)^j]\}, \quad (4b)$$

式中 $o[(Ap^2)^j]$ 为 Peano 型余项, $j = 1, 2, 3, \dots$.

2.2 GUP 下理想费米气体的热力学性质

理想费米系统的粒子数为

$$N = \int \frac{g(\varepsilon) d\varepsilon}{z_{G0}^{-1} e^{\varepsilon/(k_B T)} + 1}, \quad (5)$$

式中, $g(\varepsilon)$ 是 GUP 下的系统态密度, z_{G0} 是 GUP 下理想费米系统的逸度, k_B 是玻尔兹曼常数, T 是

绝对温度. 将 (4b) 式代入 (5) 式, 有 [15]

$$\begin{aligned} \frac{N}{V} = n &= \frac{2\pi g_0}{h^3} (k_B T/\eta)^{3/2} \Gamma(3/2) f_{3/2}(z_{G0}) \\ &\times \left\{ 1 - 3(Ak_B T/\eta) \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \\ &+ 6(Ak_B T/\eta)^2 \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(5/2) f_{3/2}(z_{G0})} + \dots \\ &\left. + o[(Ak_B T/\eta)^j] \right\}, \quad (6) \end{aligned}$$

式中,

$$\begin{aligned} f_l(x) &= \frac{1}{\Gamma(l)} \int_0^\infty \frac{t^{l-1} dt}{x^{-1} e^t + 1} \\ &\xrightarrow{x \leq 1} \sum_{i=1}^\infty (-1)^{i-1} \frac{x^i}{i^l}, \quad (7) \end{aligned}$$

$f_l(x)$ 是费米积分, $\Gamma(l)$ 是 Gamma 函数, $\eta = 1/(2m)$. 费米积分满足:

$$x \frac{df_l(x)}{dx} = f_{l-1}(x). \quad (8)$$

也可得压强的表达式 [15]:

$$\begin{aligned} \frac{P}{k_B T} &= \frac{2\pi g_0}{h^3} (k_B T/\eta)^{3/2} \Gamma(3/2) f_{5/2}(z_{G0}) \\ &\times \left\{ 1 - 3(Ak_B T/\eta) \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ &+ 6(Ak_B T/\eta)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(5/2) f_{5/2}(z_{G0})} + \dots \\ &\left. + o[(Ak_B T/\eta)^j] \right\}. \quad (9) \end{aligned}$$

同时根据

$$\frac{U}{V} = T^2 \left[\frac{\partial}{\partial T} \left(\frac{P}{T} \right) \right]_z$$

得到内能的表达式:

$$\begin{aligned} U &= \frac{3V k_B T}{2} \frac{2\pi g_0}{h^3} (k_B T/\eta)^{3/2} \Gamma(3/2) f_{5/2}(z_{G0}) \\ &\times \left\{ 1 - 5(Ak_B T/\eta) \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ &+ 14(Ak_B T/\eta)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(5/2) f_{5/2}(z_{G0})} + \dots \\ &\left. + o[(Ak_B T/\eta)^j] \right\}. \quad (10) \end{aligned}$$

注意到 k_B 和 m 的数量级, 在 A 不大时, 即 $\lim_{j \rightarrow \infty} \frac{o[(Ak_B T/\eta)^j]}{(Ak_B T/\eta)^j} \rightarrow 0$ 时, 知 (6), (9), (10) 式是快速收敛的. 为了把内能 U 表示成 T, V, N 的函数, 把 (6) 式代入 (10) 式, 有

$$\begin{aligned}
 U &= \frac{3}{2} N k_B T \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} \left[\frac{1 - 5(Ak_B T/\eta) \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} + 14(Ak_B T/\eta)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(5/2) f_{5/2}(z_{G0})} + \dots}{1 - 3(Ak_B T/\eta) \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} + 6(Ak_B T/\eta)^2 \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(5/2) f_{3/2}(z_{G0})} + \dots} \right] \\
 &\approx \frac{3}{2} N k_B T \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} \left[1 + (Ak_B T/\eta) \left(\frac{9f_{5/2}(z_{G0})}{2f_{3/2}(z_{G0})} - \frac{15f_{7/2}(z_{G0})}{2f_{5/2}(z_{G0})} \right) \right. \\
 &\quad \left. + (Ak_B T/\eta)^2 \left(\frac{35f_{9/2}(z_{G0})}{f_{5/2}(z_{G0})} - \frac{195f_{7/2}(z_{G0})}{4f_{3/2}(z_{G0})} + \frac{81f_{5/2}^2(z_{G0})}{4f_{3/2}^2(z_{G0})} \right) + \dots \right]. \quad (11)
 \end{aligned}$$

(11) 式对温度求导, 可得定容热容:

$$\begin{aligned}
 C_V &= \left(\frac{\partial U}{\partial T} \right)_{V,N} \approx \frac{3}{2} N k_B \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} \left[1 + \left(\frac{9f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} - \frac{15f_{7/2}(z_{G0})}{f_{5/2}(z_{G0})} \right) \frac{Ak_B T}{\eta} \right. \\
 &\quad \left. + \left(\frac{105f_{9/2}(z_{G0})}{f_{5/2}(z_{G0})} - \frac{585f_{7/2}(z_{G0})}{4f_{3/2}(z_{G0})} + \frac{243f_{5/2}^2(z_{G0})}{4f_{3/2}^2(z_{G0})} \right) \left(\frac{Ak_B T}{\eta} \right)^2 \right] \\
 &\quad + \frac{3}{2} N k_B T \left[1 - \frac{f_{5/2}(z_{G0}) f_{1/2}(z_{G0})}{f_{3/2}^2(z_{G0})} \right. \\
 &\quad \left. + \left(\frac{3f_{5/2}(z_{G0})}{2f_{3/2}(z_{G0})} + \frac{15f_{7/2}(z_{G0}) f_{1/2}(z_{G0})}{2f_{3/2}^2(z_{G0})} - \frac{9f_{5/2}^2(z_{G0}) f_{1/2}(z_{G0})}{f_{3/2}^3(z_{G0})} \right) \frac{Ak_B T}{\eta} \right. \\
 &\quad \left. + \left(-\frac{55f_{7/2}(z_{G0})}{4f_{3/2}(z_{G0})} + \frac{12f_{5/2}^2(z_{G0})}{f_{3/2}^2(z_{G0})} - \frac{35f_{9/2}(z_{G0}) f_{1/2}(z_{G0})}{f_{3/2}^2(z_{G0})} \right) \right. \\
 &\quad \left. + \frac{195f_{7/2}(z_{G0}) f_{5/2}(z_{G0}) f_{1/2}(z_{G0})}{2f_{3/2}^3(z_{G0})} - \frac{243f_{5/2}^3(z_{G0}) f_{1/2}(z_{G0})}{4f_{3/2}^4(z_{G0})} \right) \left(\frac{Ak_B T}{\eta} \right)^2 \left] \frac{1}{z_{G0}} \frac{dz_{G0}}{dT}.
 \end{aligned}$$

(6) 式在 N, V 不变的条件下对 T 求导数, 可得

$$\begin{aligned}
 \frac{1}{z_{G0}} \frac{dz_{G0}}{dT} &= -\frac{3f_{3/2}(z_{G0})}{2Tf_{1/2}(z_{G0})} + \left(\frac{45f_{5/2}(z_{G0})}{4f_{1/2}(z_{G0})} - \frac{27f_{3/2}^2(z_{G0})}{4f_{1/2}^2(z_{G0})} \right) \frac{Ak_B}{\eta} \\
 &\quad + \left(\frac{675f_{5/2}(z_{G0}) f_{3/2}(z_{G0})}{8f_{1/2}^2(z_{G0})} - \frac{215f_{7/2}(z_{G0})}{4f_{1/2}(z_{G0})} - \frac{243f_{3/2}^3(z_{G0})}{8f_{1/2}^3(z_{G0})} \right) \left(\frac{Ak_B}{\eta} \right)^2 T, \\
 C_V &= \frac{3}{2} N k_B \left[\frac{5f_{5/2}(z_{G0})}{2f_{3/2}(z_{G0})} - \frac{3f_{3/2}(z_{G0})}{2f_{1/2}(z_{G0})} \right. \\
 &\quad \left. + \left(\frac{63f_{5/2}(z_{G0})}{4f_{1/2}(z_{G0})} - \frac{105f_{7/2}(z_{G0})}{4f_{3/2}(z_{G0})} - \frac{27f_{3/2}^2(z_{G0})}{4f_{1/2}^2(z_{G0})} + \frac{45f_{5/2}^2(z_{G0})}{4f_{3/2}^2(z_{G0})} \right) \frac{Ak_B T}{\eta} \right. \\
 &\quad \left. + \left(-\frac{335f_{7/2}(z_{G0})}{4f_{1/2}(z_{G0})} + \frac{837f_{5/2}(z_{G0}) f_{3/2}(z_{G0})}{8f_{1/2}^2(z_{G0})} - \frac{243f_{3/2}^3(z_{G0})}{8f_{1/2}^3(z_{G0})} - \frac{99f_{5/2}^2(z_{G0})}{4f_{3/2}(z_{G0}) f_{1/2}(z_{G0})} \right) \right. \\
 &\quad \left. + \frac{315f_{9/2}(z_{G0})}{2f_{3/2}(z_{G0})} - \frac{1235f_{7/2}(z_{G0}) f_{5/2}(z_{G0})}{8f_{3/2}^2(z_{G0})} + \frac{405f_{5/2}^3(z_{G0})}{8f_{3/2}^3(z_{G0})} \right) \left(\frac{Ak_B T}{\eta} \right)^2 \left]. \quad (12)
 \end{aligned}$$

当系统的温度很低时, z_{G0} 可能变得很大, 这时可应用 Sommerfeld 引理将费米积分对大宗量 $\ln z_{G0}$ 做渐近展开, 其中 Sommerfeld 引理为^[31]

$$f_l(z_{G0}) = \frac{(\ln z_{G0})^l}{\Gamma(l+1)} \left[1 + l(l-1) \frac{\pi^2}{6(\ln z_{G0})^2} + l(l-1)(l-2)(l-3) \frac{7\pi^4}{360(\ln z_{G0})^4} + \dots \right]. \quad (13)$$

将 (13) 式代入 (6) 式, 并注意到不考虑 GUP, 低温条件下理想费米气体的化学势为

$$\mu_0 \approx E_{F0} \left[1 - \frac{\pi^2}{12} \left(\frac{T}{T_{F0}} \right)^2 \right], \quad (14)$$

式中, $E_{F0} = (\hbar^2/2m)(3\pi^2n)^{2/3}$ 为不考虑 GUP 理想费米系统的费米能, $T_{F0} = E_{F0}/k_B$ 为费米温度. 在 A/η 为小量时, 可得考虑 GUP 低温条件下理想费米气体的化学势:

$$\mu_{G0} \approx E_{F0} \left\{ 1 + \frac{6}{5} (A/\eta) E_{F0} + \frac{\pi^2}{2} \left[(A/\eta) E_{F0} - \frac{1}{6} \right] \left(\frac{T}{T_{F0}} \right)^2 + \dots \right\}. \quad (15)$$

将 (13) 式代入 (11) 式并注意到 (15) 式, 得到低温极限下系统的内能 (只取到温度 T/T_{F0} 和 A/η 的平方项):

$$U = \frac{3}{5} N E_{F0} \left\{ 1 + \frac{6}{7} (A/\eta) E_{F0} - \frac{1468}{1575} (A/\eta)^2 E_{F0}^2 + \pi^2 \left[\frac{5}{12} - \frac{67}{70} (A/\eta) E_{F0} - \frac{1156}{1575} (A/\eta)^2 E_{F0}^2 \right] \left(\frac{T}{T_{F0}} \right)^2 \right\}. \quad (16)$$

当温度 $T = 0$ K 时, 得到基态能

$$U_0 = \frac{3}{5} N E_{F0} \left[1 + \frac{6}{7} (A/\eta) E_{F0} - \frac{1468}{1575} (A/\eta)^2 E_{F0}^2 \right]. \quad (17)$$

(16) 式对温度求导可得热容

$$C_V = \frac{6}{5} N k \pi^2 \left[\frac{5}{12} - \frac{67}{70} (A/\eta) E_{F0} - \frac{1156}{1575} (A/\eta)^2 E_{F0}^2 \right] \left(\frac{T}{T_{F0}} \right). \quad (18)$$

当 $A \rightarrow 0$ 时, (15)—(18) 式回到传统的化学势、内能和热容的低温表达式^[31]:

$$\begin{aligned} \mu_0 &= E_{F0} \left[1 - \frac{\pi^2}{12} \left(\frac{T}{T_{F0}} \right)^2 \right]; \\ U &= \frac{3}{5} N E_{F0} \left[1 + \frac{5\pi^2}{12} \left(\frac{T}{T_{F0}} \right)^2 \right]; \\ U_0 &= \frac{3}{5} N E_{F0}; \quad C_V = \frac{\pi^2}{2} N k \frac{T}{T_{F0}}. \end{aligned} \quad (19)$$

$T = 0$ K 时, 系统的总粒子数

$$N = \int_0^{p_F} \frac{4\pi V p^2 g_0}{h^3 (1 + A p^2)^3} dp.$$

积分得到在 GUP 下费米能 E_F 满足的关系:

$$N = \frac{\pi V g_0}{h^3 A} \left[\frac{A (E_F/\eta)^{3/2} - (E_F/\eta)^{1/2}}{2(1 + A E_F/\eta)^2} \right.$$

$$\left. + \frac{1}{2\sqrt{A}} \arctan \sqrt{A} (E_F/\eta)^{1/2} \right]. \quad (20)$$

其中费米动量与费米能满足 $p_F = (2mE_F)^{1/2} = (E_F/\eta)^{1/2}$, 再注意到不考虑 GUP 时的费米动量 $p_{F0} = h \left(\frac{3N}{4\pi V g_0} \right)^{1/3}$, 由 (20) 式按 E_{F0} 展开, 可得 GUP 下的费米能:

$$E_F \approx E_{F0} \left[1 + \frac{6}{5} (A/\eta) E_{F0} + \frac{3}{35} (A/\eta)^2 E_{F0}^2 + \dots \right]. \quad (21)$$

GUP 下的费米动量 p_F 为^[15]

$$p_F \approx p_{F0} \left(1 + \frac{3}{5} A p_{F0}^2 - \frac{24}{175} A^2 p_{F0}^4 + \dots \right). \quad (22)$$

以铜电子气体为例, 铜的密度为 $8.9 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$, 原子量为 63, 如果一个铜原子贡献一个自由电子, 则 $n = (8.9/63) \times N_A = 8.5 \times 10^{28} \text{ m}^{-3}$, 电子的质量为 $9.1 \times 10^{-31} \text{ kg}$, 铜电子气体的费米能 $E_{F0} = 1.1 \times 10^{-18} \text{ J}$. GUP 中 A 是一个修正量, 在本文的计算中将 A 作为小量处理, 文献[24—26]给出了最小长度的范围 $\Delta x_{\min} \approx 10^{-16} \text{—} 10^{-17} \text{ m}$, 按 $\Delta x_{\min} = 10^{-16} \text{ m}$, 并由 $\Delta x_{\min} = \hbar\sqrt{A}$ 知 A 可取到 $\approx 10^{36}$. 注意到前述内能、热容、费米能和化学势等的级数表达式, 对于电子体系, A 取 10^{36} 仍是小量, 可保证上述级数形式的热力学量快速收敛. 将 (15)—(18) 式分别与 (19) 式中的对应项相差, 可分别得到考虑 GUP 时, 对化学势、内能和热容的修正:

$$\begin{aligned} \Delta\mu &= \mu_{G0} - \mu_0 \\ &= E_{F0}^2 (A/\eta) \left[\frac{6}{5} + \frac{\pi^2}{2} \left(\frac{T}{T_{F0}} \right)^2 \right]; \\ \Delta U &= \frac{3}{5} N E_{F0}^2 (A/\eta) \left\{ \frac{6}{7} - \frac{1468}{1575} (A/\eta) E_{F0} - \pi^2 \left[\frac{67}{70} + \frac{1156}{1575} (A/\eta) E_{F0} \right] \left(\frac{T}{T_{F0}} \right)^2 \right\}; \\ \Delta C_V &= -\frac{6}{5} N k \pi^2 (A/\eta) E_{F0} \\ &\quad \times \left[\frac{67}{70} + \frac{1156}{1575} (A/\eta) E_{F0} \right] \left(\frac{T}{T_{F0}} \right). \end{aligned}$$

图 1 给出了考虑 GUP 与不考虑 GUP 两种方法铜电子理想费米气体内能随温度的变化, 两者相差的数值较小, 图 1 中的内插图为内能差异随温度的变化.

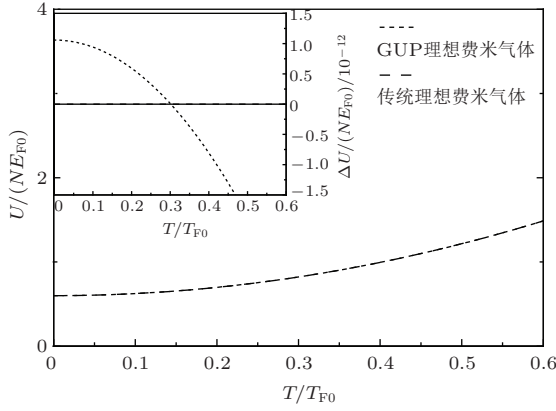


图1 铜电子理想费米气体内能及内能差异 (GUP对传统方法结果的修正) 随温度的变化. 长虚线为考虑GUP的结果, 短虚线为不考虑GUP的结果 ($A = 10^{36}$)

Fig. 1. Changes of the internal energy and its difference (amendment to the traditional method results by GUP) of ideal Fermi gas of copper electron with temperature. The long dash is the result of GUP, and short dash is the result without GUP ($A = 10^{36}$).

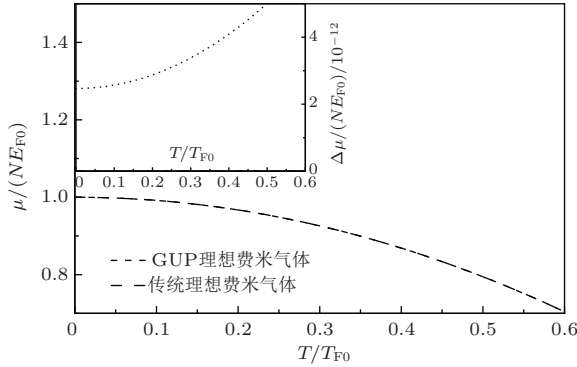


图3 铜电子理想费米气体化学势及化学势差异 (GUP对传统方法结果的修正) 随温度的变化. 长虚线为考虑GUP的结果, 短虚线为不考虑GUP的结果 ($A = 10^{36}$)

Fig. 3. Changes of the chemical potential and its difference (amendment to the traditional method results by GUP) of ideal Fermi gas of copper electron with temperature. The long dash is the result of GUP, and short dash is the result without GUP ($A = 10^{36}$).

图2和图3分别给出了铜电子气体热容和化学势在考虑GUP与不考虑GUP两种情形下随温度的变化, 内插图分别为差值随温度的变化.

3 GUP下弱相互作用费米气体的热力学性质

3.1 GUP下理想费米气体的自由能

注意到巨热力学势为

$$\Phi(\mu, T, V) = -PV, \quad (23)$$

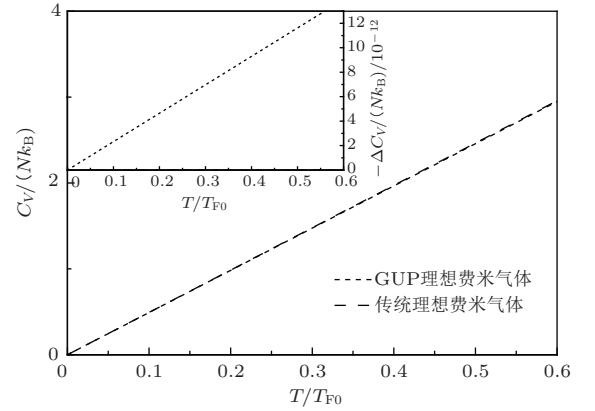


图2 铜电子理想费米气体热容及热容差异 (GUP对传统方法结果的修正) 随温度的变化. 长虚线为考虑GUP的结果, 短虚线为不考虑GUP的结果 ($A = 10^{36}$)

Fig. 2. Changes of the heat capacity and its difference (amendment to the traditional method results by GUP) of ideal Fermi gas of copper electron with temperature. The long dash is the result of GUP, and short dash is the result without GUP ($A = 10^{36}$).

式中 μ 是化学势, 它与逸度 z 的关系为 $z = e^{\mu/(k_B T)}$. 将(23)式代入(8)式, 得到GUP下理想费米气体巨热力学势:

$$\begin{aligned} \Phi_{\text{GUP}}(\mu_{G0}, T, V) &= -k_B T \frac{2\pi V g_0}{h^3} (k_B T / \eta)^{3/2} \Gamma(3/2) f_{5/2}(z_{G0}) \\ &\times \left[1 - 3A (k_B T / \eta) \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ &\left. + 6A^2 (k_B T / \eta)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} + \dots \right]. \end{aligned} \quad (24)$$

由自由能与巨热力学势间的关系 $F = \Phi + \mu N$ 及(6)式, 可得GUP下理想费米气体的自由能:

$$\begin{aligned} F_{\text{GUP}} &= -k_B T \frac{2\pi V g_0}{h^3} \left(\frac{k_B T}{\eta} \right)^{3/2} \\ &\times \Gamma(3/2) f_{5/2}(z_{G0}) \\ &\times \left[1 - 3 \frac{A k_B T}{\eta} \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ &\left. + 6 \left(\frac{A k_B T}{\eta} \right)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} + \dots \right] \\ &+ k_B T \ln z_{G0} \frac{2\pi V g_0}{h^3} \left(\frac{k_B T}{\eta} \right)^{3/2} \\ &\times \Gamma(3/2) f_{3/2}(z_{G0}) \\ &\times \left[1 - 3 \frac{A k_B T}{\eta} \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \end{aligned}$$

$$+ 6 \left(\frac{Ak_B T}{\eta} \right)^2 \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} + \dots \Big]. \quad (25)$$

(25) 式中逸度 z_{G0} 或化学势由下式或 (6) 式确定:

$$n = \frac{g_0}{\lambda^3} f_{3/2}(z_{G0}) \left[1 - 3A \frac{(k_B T/\eta)}{\Gamma(3/2) f_{3/2}(z_{G0})} \frac{3 f_{5/2}(z_{G0})}{2 f_{3/2}(z_{G0})} \right. \\ \left. + 6A^2 \frac{(k_B T/\eta)^2}{4 f_{3/2}(z_{G0})} \frac{15 f_{7/2}(z_{G0})}{f_{3/2}(z_{G0})} + \dots \right],$$

上式中 $\lambda = \sqrt{2\pi\hbar^2/(mk_B T)}$ 为热波长. 把 F_{GUP} 表示成 N, T, V 的函数, 可得

$$F_{GUP} \\ = -Nk_B T f_{5/2}(z_{G0}) \\ \times \left[1 - 5 \frac{Ak_B T}{\eta} \frac{\Gamma(5/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ \left. + 14 \left(\frac{Ak_B T}{\eta} \right)^2 \frac{\Gamma(7/2) f_{9/2}(z_{G0})}{\Gamma(3/2) f_{5/2}(z_{G0})} + \dots \right] \\ \times \left\{ f_{3/2}(z_{G0}) \left[1 - 3 \frac{Ak_B T}{\eta} \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \right. \\ \left. \left. + 6 \left(\frac{Ak_B T}{\eta} \right)^2 \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} + \dots \right] \right\}^{-1} \\ + Nk_B T \ln z_{G0}. \quad (26)$$

将 (26) 式按小量 $\frac{Ak_B T}{\eta}$ 级数展开, 得 GUP 下理想费米气体自由能的近似表达式:

$$F_{GUP} = Nk_B T \ln z_{G0} - Nk_B T \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} \\ \times \left\{ 1 + 3 \frac{Ak_B T}{\eta} \left(\frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right) \right. \\ \left. - \frac{5 \Gamma(5/2) f_{7/2}(z_{G0})}{3 \Gamma(3/2) f_{5/2}(z_{G0})} \right) \\ + 6 \left(\frac{Ak_B T}{\eta} \right)^2 \left[\frac{7 \Gamma(7/2) f_{9/2}(z_{G0})}{3 \Gamma(3/2) f_{5/2}(z_{G0})} \right. \\ \left. - \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} + \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \\ \left. \times \left(\frac{3 \Gamma(5/2) f_{5/2}(z_{G0})}{2 \Gamma(3/2) f_{3/2}(z_{G0})} \right) \right. \\ \left. - \frac{5 \Gamma(5/2) f_{7/2}(z_{G0})}{2 \Gamma(3/2) f_{5/2}(z_{G0})} \right) \Big] + \dots \Big\}. \quad (27)$$

3.2 基于赝势法 GUP 下弱相互作用费米气体的配分函数和自由能

考虑一限定在体积 V 中的 N 个具有弱相互作用自旋为 $1/2$ 的费米子组成的系统. 通过赝势法可

导出系统的能谱为

$$E = \sum_p (n_p^+ + n_p^-) \frac{p^2}{2m} + \frac{4\pi\hbar^2 a}{mV} N^+ N^-, \quad (28)$$

式中, $n_p^+ (n_p^-)$ 表示动量为 p 自旋向上(向下)的粒子数, $N^+ (N^-)$ 表示自旋向上(向下)的总粒子数, a 为粒子间二体相互作用的 s 波散射长度, 满足弱相互作用条件 $|a|/\lambda \ll 1$, $|a|n^{1/3} \ll 1$. 根据 (28) 式可得系统的正则配分函数

$$Q = \sum_{(n_p^+, n_p^-)} \exp \left\{ -\beta \left[\sum_p (n_p^+ + n_p^-) \frac{p^2}{2m} \right. \right. \\ \left. \left. + \frac{4\pi\hbar^2 a}{mV} N^+ N^- \right] \right\} \\ = \sum_{N^+=0}^N \exp \left\{ -\beta \left[F_0(N^+) + F_0(N - N^+) \right. \right. \\ \left. \left. + \frac{4\pi\hbar^2 a}{mV} N^+ (N - N^+) \right] \right\}, \quad (29)$$

式中, $\beta = 1/(k_B T)$, $\sum_{\{n_p^+, n_p^-\}}$ 为对所有满足

$$N = \sum_p (n_p^+ + n_p^-)$$

的分布 $\{n_p^+, n_p^-\}$ 求和. 仿照黄克逊、杨振宁等处理非理想玻色气体的方法, 我们引进

$$F_0(\xi) = -(1/\beta) \ln \sum_{\{\xi_p\}} \exp \left(-\beta \sum_p \xi_p \frac{p^2}{2m} \right)$$

表示限定在体积 V 中的 ξ 个“无自旋的”、无相互作用费米子所组成的一个“虚构系统”的自由能^[31,32], 其中 $\sum_{\{\xi_p\}}$ 为对所有满足 $\sum_p \xi_p = \xi$ 的分布 $\{\xi_p\}$ 求和. 由 (29) 式得系统的自由能:

$$F(\xi) = -(1/\beta) \ln \sum_{N^+=0}^N \exp \left\{ -\beta \left[F_0(N^+) \right. \right. \\ \left. \left. + F_0(N - N^+) \right. \right. \\ \left. \left. + \frac{4\pi\hbar^2 a}{mV} N^+ (N - N^+) \right] \right\}. \quad (30)$$

对于含有大量粒子的宏观系统, (30) 式中求和项的对数可近似等于求和项中最大项的对数, 可表示为^[22,31,32]

$$F = 2F_0 \left(\frac{N}{2} \right) + \frac{\pi a \hbar^2 N^2}{mV}. \quad (31)$$

而对 N 个无自旋、无相互作用的费米子组成的“虚构系统”, 把 (27) 式代入 (31) 式, 则在 GUP 下弱相互作用费米气体的自由能为

$$F_{GUP} = Nk_B T \ln z_{G0} - Nk_B T \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})}$$

$$\begin{aligned}
 & \times \left\{ 1 + 3 \frac{Ak_B T}{\eta} \left(\frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \right. \\
 & \left. \left. - \frac{5 \Gamma(5/2) f_{7/2}(z_{G0})}{3 \Gamma(3/2) f_{5/2}(z_{G0})} \right) \right. \\
 & + 6 \left(\frac{Ak_B T}{\eta} \right)^2 \left[\frac{7 \Gamma(7/2) f_{9/2}(z_{G0})}{3 \Gamma(3/2) f_{5/2}(z_{G0})} \right. \\
 & \left. - \frac{\Gamma(7/2) f_{7/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} + \frac{\Gamma(5/2) f_{5/2}(z_{G0})}{\Gamma(3/2) f_{3/2}(z_{G0})} \right. \\
 & \times \left(\frac{3 \Gamma(5/2) f_{5/2}(z_{G0})}{2 \Gamma(3/2) f_{3/2}(z_{G0})} \right. \\
 & \left. \left. - \frac{5 \Gamma(5/2) f_{7/2}(z_{G0})}{2 \Gamma(3/2) f_{5/2}(z_{G0})} \right) \right] + \dots \left. \right\} \\
 & + \frac{N \pi a n \hbar^2}{m}. \quad (32)
 \end{aligned}$$

3.3 基于赝势法 GUP 下弱相互作用费米气体的热力学函数

考虑 GUP 和赝势法后, 弱相互作用费米气体的化学势可由下式确定:

$$\begin{aligned}
 \mu_G &= (\partial F / \partial N)_{V, T} \\
 &= k_B T (\ln z_{G0} + a n \lambda^2). \quad (33)
 \end{aligned}$$

对于体积 V 和粒子数 N 不变的正则系统, 粒子数密度 n 不变, 有 $n_G = n_{G0} = n_0 = n$.

内能

$$\begin{aligned}
 U &= \left[\frac{\partial (\beta F_{GUP})}{\partial \beta} \right]_{N, V} \\
 &= \frac{3}{2} N k_B T \left\{ \frac{f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} \left[1 + \frac{3 Ak_B T}{2 \eta} \right. \right. \\
 & \times \left(\frac{3 f_{5/2}(z_{G0})}{f_{3/2}(z_{G0})} - \frac{5 f_{7/2}(z_{G0})}{f_{5/2}(z_{G0})} \right) \\
 & + \frac{3}{2} \left(\frac{Ak_B T}{\eta} \right)^2 \left(\frac{35 f_{9/2}(z_{G0})}{f_{5/2}(z_{G0})} - \frac{75 f_{7/2}(z_{G0})}{2 f_{3/2}(z_{G0})} \right. \\
 & \left. \left. + \frac{27 f_{5/2}^2(z_{G0})}{2 f_{3/2}^2(z_{G0})} \right) + \dots \right] + \frac{1}{3} a \lambda^2 n \left. \right\}; \quad (34)
 \end{aligned}$$

定容热容

$$\begin{aligned}
 C_V &= \left(\frac{\partial U}{\partial T} \right)_{V, N} \\
 &= \frac{3}{2} N k_B \left[\frac{5 f_{5/2}(z_{G0})}{2 f_{3/2}(z_{G0})} - \frac{3 f_{3/2}(z_{G0})}{2 f_{1/2}(z_{G0})} \right. \\
 & + \left(\frac{63 f_{5/2}(z_{G0})}{4 f_{1/2}(z_{G0})} - \frac{105 f_{7/2}(z_{G0})}{4 f_{3/2}(z_{G0})} \right. \\
 & \left. \left. - \frac{27 f_{3/2}^2(z_{G0})}{4 f_{1/2}^2(z_{G0})} + \frac{45 f_{5/2}^2(z_{G0})}{4 f_{3/2}^2(z_{G0})} \right) \frac{Ak_B T}{\eta} \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \left(- \frac{110 f_{7/2}(z_{G0})}{f_{1/2}(z_{G0})} \right. \\
 & + \frac{837 f_{5/2}(z_{G0}) f_{3/2}(z_{G0})}{8 f_{1/2}^2(z_{G0})} - \frac{243 f_{3/2}^3(z_{G0})}{8 f_{1/2}^3(z_{G0})} \\
 & - \frac{351 f_{5/2}^2(z_{G0})}{8 f_{3/2}(z_{G0}) f_{1/2}(z_{G0})} + \frac{945 f_{9/2}(z_{G0})}{4 f_{3/2}(z_{G0})} \\
 & \left. - \frac{1595 f_{7/2}(z_{G0}) f_{5/2}(z_{G0})}{8 f_{3/2}^2(z_{G0})} \right. \\
 & \left. + \frac{405 f_{5/2}^3(z_{G0})}{8 f_{3/2}^3(z_{G0})} \right) \left(\frac{Ak_B T}{\eta} \right)^2 \left. \right]. \quad (35)
 \end{aligned}$$

当系统的温度很低时, 同样应用 Sommerfeld 引理将费米积分对大宗量 $\ln z$ 做渐近展开, 将 (13) 式代入 (33), (34) 式, 可得化学势和内能, 保留到 T/T_{F0} 的二次项.

化学势

$$\begin{aligned}
 \mu_G &= \mu_{G0} + \frac{2 \pi \hbar^2 a n}{m} \\
 &= E_{F0} \left[1 + \frac{6 A}{5 \eta} E_{F0} \right. \\
 & + \frac{\pi^2}{2} \left(-\frac{1}{6} + \frac{A}{\eta} E_{F0} \right) \left(\frac{T}{T_{F0}} \right)^2 \\
 & \left. + \frac{8a}{3 \pi^{1/2} \lambda_{F0}} \right], \quad (36)
 \end{aligned}$$

式中 $\lambda_{F0} = \sqrt{2 \pi \hbar^2 / (m k_B T_{F0})}$.

内能

$$\begin{aligned}
 U &= \frac{3}{5} N E_{F0} \left\{ 1 + \frac{6}{7} (A/\eta) E_{F0} \right. \\
 & - \frac{1468}{1575} (A/\eta)^2 E_{F0}^2 \\
 & + \pi^2 \left[\frac{5}{12} - \frac{67}{70} (A/\eta) E_{F0} \right. \\
 & \left. \left. - \frac{1156}{1575} (A/\eta)^2 E_{F0}^2 \right] \left(\frac{T}{T_{F0}} \right)^2 \right\} \\
 & + \frac{N k_B T}{2} a \lambda^2 n \\
 & = \frac{3}{5} N E_{F0} \left\{ 1 + \frac{6}{7} (A/\eta) E_{F0} \right. \\
 & - \frac{1468}{1575} (A/\eta)^2 E_{F0}^2 \\
 & + \pi^2 \left[\frac{5}{12} - \frac{67}{70} (A/\eta) E_{F0} \right. \\
 & \left. \left. - \frac{1156}{1575} (A/\eta)^2 E_{F0}^2 \right] \left(\frac{T}{T_{F0}} \right)^2 \right. \\
 & \left. + \frac{20}{9 \pi^{1/2}} \frac{a}{\lambda_{F0}} \right\}. \quad (37)
 \end{aligned}$$

对 (37) 式求导, 可得热容

$$C_V = \frac{6}{5} N k \pi^2 \left[\frac{5}{12} - \frac{67}{70} (A/\eta) E_{F0} - \frac{1156}{1575} (A/\eta)^2 E_{F0}^2 \right] \left(\frac{T}{T_{F0}} \right). \quad (38)$$

当温度 $T = 0$ K 时, 费米能

$$E_F \approx E_{F0} \left(1 + \frac{6}{5} \frac{A}{\eta} E_{F0} + \frac{8a}{3\pi^{1/2} \lambda_{F0}} \right), \quad (39)$$

基态能

$$U_0 = \frac{3}{5} N E_{F0} \left[1 + \frac{6}{7} (A/\eta) E_{F0} - \frac{1467}{1575} (A/\eta)^2 E_{F0}^2 + \frac{20}{9\pi^{1/2}} \frac{a}{\lambda_{F0}} \right]. \quad (40)$$

由于本文中所使用的赝势法是以正则系统为基础的, 其特点就是粒子数 N 和体积 V 不变, 即粒子数密度 n 不变, 因而由赝势法计算弱相互作用费米系统的内能只是将理想费米系统的内能整体向上或向下平移 $\frac{4NE_{F0}}{3\pi^{1/2}} \frac{a}{\lambda_{F0}}$, 且此项与温度无关, 故计入 GUP 后, 对弱相互作用铜电子气体内能、热容和化学势修正的结果与图 1、图 2 和图 3 中曲线所显示的一样, 不再赘述. 当考虑粒子数 N 和体积 V 变化, 即粒子数密度 n 变化时, 就需要用其他统计分布, 如完全开放系统的统计分布^[33](N - E - V 分布), 其结果将有些变化.

4 结果与讨论

4.1 结果

在考虑到 GUP (1) 式后, 统计物理中的量子态密度被修正为 (3) 式. 基于修正后的态密度, 我们主要做了以下工作.

1) 在文献 [15] 给出理想费米气体以费米积分表示内能 (见 (11) 式) 的基础上, 计算出用费米积分表示的热容 (见 (12) 式), 以及低温极限条件下理想费米气体的内能、化学势、基态能和热容的解析表达式 (见 (15)–(18) 式), 并以铜电子气体为例做了数值计算. 图 1 显示了低温条件下内能随温度增加的数值关系, 图 1 中的内插图显示了考虑 GUP 之后内能与不考虑 GUP 的内能差值随温度的变化. 图 2 和图 3 分别显示了考虑 GUP 与不考虑 GUP 的热容和化学势随温度的变化, 内插图分别是它们差值随温度的变化. 3 个热力学量相对差异都很小, 当 T/T_{F0} 在 0–0.3 之间时, 相对差异在 10^{-12} 这一数量级. 这是非常小的数量级, 故对于铜电子气

体这类费米子系统, 在低温时, 不必考虑 GUP 所造成的修正. 但注意到关于费米子的有关科学实验和理论研究并不限于铜电子气体, 再注意到考虑到 GUP 后, 化学势、内能和热容等热力学量的修正中最大的一项的数量级由 AE_{F0}/η 决定, 此项并不总是小于 10^{-12} 这一数量级. 仍以电子费米子系统为例, A/η 约为 10^6 J⁻¹, 前面我们根据铜的质量密度, 取一个铜原子贡献一个电子, 估算出铜电子气体的粒子数密度 $n = 8.5 \times 10^{28}$ m⁻³, 注意到费米能 E_{F0} 正比于粒子数密度 $n^{3/2}$, 得铜电子气体费米能 E_{F0} 约为 10^{-18} J, 则 $AE_{F0}/\eta \sim 10^{-12}$; 当 $n \sim 10^{46}$ m⁻³ 时, 估算可得 $AE_{F0}/\eta \sim 1$, 此时 GUP 的修正就很大. 文献 [34] 认为: 电子的限度约为 10^{-22} m (也有观点认为电子是一点粒子, 不占有任何空间 [35], 这样电子密度可任意大), 那么一个电子的空间体积约为 10^{-66} m³, 电子的最大粒子数密度就为 10^{66} m⁻³, 设一个电子占有 10^9 个电子的空间体积, 即电子平均间距为 10^3 个电子的限度, 电子气为“稀薄”气体, 此时电子密度为 10^{57} m⁻³, 远远大于 10^{46} m⁻³. 即理论上, 在低温条件下, GUP 的修正会对费米子系统的性质造成影响, 影响大小视具体费米子系统 AE_{F0}/η 的数量级而定. 本文的理论计算也对相关费米系统的科学实验具有指导意义.

考虑 GUP 后: 内能随温度的升高先增加, 到 $T/T_{F0} \sim 0.3$ 时内能增值为零, 之后内能开始减少, 并且差值的绝对值越来越大; 热容减少, 热容差值的绝对值随温度的升高而增加; 化学势增大, 化学势差值随温度的升高而增加.

2) 以基于正则分布的赝势法, 对具有弱相互作用的费米系统进行了讨论. 计算了弱相互作用费米系统的化学势、内能和热容, 给出了解析表达式 (见 (33)–(35) 式); 并在低温极限条件下给出了化学势、内能、基态能和热容的解析表达式 (见 (36)–(39) 式); 最后以铜电子气体为例给出了数值计算的结果. 注意到在以正则分布为基础的赝势法中, 因粒子数密度不变, 弱相互作用约化为“赝势”, 其结果是内能、化学势等热力学量增加了与温度无关 (但与粒子数密度相关) 的一项, 因而内能、化学势和热容的差值随温度的数值关系与图 1、图 2 和图 3 所示的理想费米气体情况相同.

3) 所有热力学量的解析计算结果, 在 $A \rightarrow 0$ 时

都回到了修正前的已有文献中的结果; 在反映弱相互作用的散射长度 $a \rightarrow 0$ 时, 回到了理想费米气体的结果.

4.2 讨 论

GUP 的修正造成理想费米气体的基态能 (见 (17) 式) 和费米能 (见 (21) 式) 增大, 这是因为费米子在遵守泡利不相容原理的同时, 增加了“最小长度”的限制, 导致量子态密度变小, 在绝对温度 $T = 0$ K 时, 低能态填满费米粒子之后, 余下的费米粒子只得向高能态充填, 致使基态能和费米能增加.

从温度 $T = 0$ K 开始, 理想费米气体的内能 (基态能) 的增加值随温度逐渐减少 (图 3), 直至增加值变为负值, 即减少. $T = 0$ K 附近增大的原因与基态能、费米能增大的原因一样, 随着温度的升高, 粒子能量增加, 将去充填高能态, 因而内能增加, 但随着高能态粒子增多的同时, 将要充填的高能态的态密度远比低能态减少的快得多 (比较的是不考虑 GUP 时的态密度, 见 (3) 式), 且能量越高态密度及量子态数减少的越多, 即高能态费米粒子可占据的量子态数越来越少. 这样在温度相同或温差相同的前提下, 考虑到 GUP 时比不考虑 GUP 时的高能态粒子可占据的量子态少得多, 导致在高能态落脚的概率减小, 最终跃迁到高能态的粒子减少, 内能增加减缓, 此消彼长, 直至开始出现负增长, 即图 1 中的内插图两曲线 (一条是水平直线) 出现交点, 增值由正变负. 在温差相同的条件下, 跃迁到高能态的粒子减少, 导致热容也减少, 见图 2.

5 结 论

考虑 GUP 后, 量子态密度要做相应的修正. 在极端高温情况下, GUP 对传统统计物理已形成的观念具有颠覆性的冲击^[4,6]; 在低温条件下, 这一因素对理想费米气体和具有弱相互作用的费米气体的热力学性质有一些修正, 主要表现为 AE_{F0}/η . 通常情况下, 修正的数量级较小, 对于铜电子气体, 当温度 T/T_{F0} 在 0—0.3 之间时, 修正在 10^{-12} 这一数量级, 因而此时不必考虑 GUP 的修正. 但当粒子数密度增大时, 修正项会增加, 甚至会成为主导因素. 考虑到 GUP 后费米气体的化学势、费米能和基

态能增大, 热容减少, 内能随温度的增加先增大, 到某一温度 (铜电子气体为 $T/T_{F0} \sim 0.3$) 时, 增值为零, 温度再增加内能开始减少.

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Thermodynamic properties of Fermi gas under generalized uncertainty principle*

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(Received 13 October 2014; revised manuscript received 1 December 2014)

Abstract

When taking into account the generalized uncertainty principle in statistical physics, the density of states must make a correction, which causes all the results of traditional statistical physics to have different degrees of correction. In high-energy or high-temperature conditions, this amendment can subvert the traditional concept and there are also some certain amendments at low temperatures. In this paper we study the thermodynamic properties of the ideal and weakly interacting Fermi gas in low temperature conditions when the generalized uncertainty principle is taken into account. Firstly, analytical expressions of chemical potential, internal energy and heat capacity at constant volume of ideal or weakly interacting Fermi gas are given. Then the properties of copper electron gas are computed as an example, showing that when the generalized uncertainty principle is taken into account the chemical potential, Fermi energy and the ground state energy increase with the increase of temperature, while the heat capacity decreases. When the temperature is lower than 0.3 times T_{F0} , the internal energy increases with the increase of temperature, but becomes decreased when temperature is high than 0.3 times T_{F0} . These amendments are mostly dependent on particle density, which becomes bigger and bigger with particle density increasing.

Keywords: generalized uncertainty principle, Fermi gas, thermodynamic property

PACS: 05.30.-d, 51.30.+i

DOI: 10.7498/aps.64.080502

* Project supported by the National Natural Science Foundation of China (Grant No. 61167002) and the Natural Science Foundation of Ningxia, China (Grant No. NZ14055).

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