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四瓣高斯光束的Gyrator变换性质和矩形空心光束的产生

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基于 Gyrator 变换, 推导了四瓣高斯光束场分布的解析表达式, 研究了四瓣高斯光束通过 Gyrator 变换后的光强分布和相位分布. 结果表明: 在 Gyrator 变换过程中, 四瓣高斯光束能够转换为具有光涡旋的矩形空心光束, 在获得矩形空心光束时其四顶角处光束强度最强, 而四条边上的光束强度分布几乎是均匀的. 对影响矩形空心光束强度和相位分布的光束参数和变换角进行了详细的分析, 发现光束阶数不同, 产生不同类型的空心光束; Gyrator 变换的变换角则影响空心光束能量分布; 空心光束亮环的大小由四瓣高斯光束的束腰宽度决定, 束腰宽度越大, 矩形空心光束的宽度越小.

关键词: 四瓣高斯光束, Gyrator 变换, 矩形空心光束,

PACS: 42.60.Jf, 42.55.–f, 42.15.Eq, 42.25.–p **DOI:** 10.7498/aps.65.124204

1 引 言

Gyrator 变换是傅里叶变换、分数傅里叶变换的推广形式, 且 Gyrator 变换实现了相空间中空间频率域和空间域中的任意旋转, 从数学看都属于线性正则积分变换, 线性积分正则变换可以描述一阶光学系统. 而 Rodrigo 等[1,2] 将 Gyrator 变换运用于光学领域, 并设计出了实现相应变换的光学系统, 利用 Gyrator 变换实现了厄米高斯激光束到拉盖尔高斯激光束的模式转换. 同时, Gyrator 变换被广泛地应用于数字与光学图像信息加密处理、滤波和去噪等方面[3,4].

空心光涡束是一种在传播方向上中心光强或轴向光强为零的光束, 此类光束的中心存在相位奇异点, 由于在生命科学与纳米技术、二元光学、原子光学以及微观粒子的激光控制和导向等方面的应用而受到人们的广泛关注. 近年来, 空心光束的产生以及空心光束类型等问题引发了国内外学者的密切关注, 成为重要的研究课题[5–9]. 常用椭圆对称的模型来描述空心光束, 因为椭圆空心光束可以看成是圆形空心光束与矩形空心光束之间的桥梁, 已提出多种理论及实验研究方案产生圆形和椭圆形空心光束, 如 Sun 等[10] 提出了利用双曲正弦高斯函数表示椭圆空心光束的模型; Zhao 等[11] 设计了组合棱镜光学系统实现圆形空心光束与椭圆形空心光束之间的转换. Cai 和 Zhang[12] 提出了描述矩形空心光束的理论模型并分析了矩形空心光束在近轴光学系统中的传播特性. 在实验中也得到了矩形空心光束, 如 He 等[13] 提出用模式转换的方法得到矩形空心光束; Shi 等[14] 用波晶片产生了可调矩形空心光束.

四瓣高斯光束是一种特殊的高斯光束类型, 基于四瓣高斯光束的传播性质, 其在微光学、光通信与分束技术等方面有着广泛的应用[15]. 本文基于 Gyrator 变换理论研究了四瓣高斯光束经过 Gyrator 变换后的场强和相位分布. 结果表明, 在适当的条件下, 四瓣高斯光束经 Gyrator 变换后可获得矩形空心光束. 本文提出了一种产生矩形空心光束的方案, 丰富了空心光束的种类, 推广了 Gyrator 变换的应用.

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2 四瓣高斯光束的Gyrator变换公式

设在入射平面的四瓣高斯光束的场分布$E(x,y)$为

$$\begin{aligned}&E(x,y,0)\\&=x^{2n}y^{2m}\exp\left(-\frac{x^2+y^2}{\omega^2}-\frac{\mathrm{i}kx^2}{2R_x}-\frac{\mathrm{i}ky^2}{2R_y}\right),\end{aligned}\tag{1}$$

其中, n, m为光束的阶数; ω为高斯光束的束腰宽度; k为波矢; R_x, R_y分别为四瓣高斯光束在x, y方向上的曲率半径.

Gyrator变换对二维函数$E\,(x,y)$进行如下的积分变换:

$$\begin{aligned}&U(x,y)\\&=\frac{1}{|\sin\alpha|}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E(x_i,y_i)\\&\quad\times\exp\left[2\pi\mathrm{i}\frac{(xy+x_iy_i)\cos\alpha-(x_iy+xy_i)}{\sin\alpha}\right]\\&\quad\times\mathrm{d}x_i\mathrm{d}y_i,\end{aligned}\tag{2}$$

其中α为Gyrator变换的变换角. 变换角α与Gyrator变换系统中广义透镜的旋转角度有关. 容易得到, $\alpha=0$为恒等变换, 当$\alpha=\pi/2$表示坐标转动了$\pi/2$的傅里叶变换或者逆变换. 同时Gyrator变换对于变换角α具有周期性和可加性. Rodrigo等已经详细讨论了Gyrator变换的数学性质, 设计了三个相互之间距离确定的广义透镜组成的特殊光学系统, 成功实现了Gyrator变换的大幅角度变换.

将(1)式代入变换(2)式中, 并利用积分公式和厄米多项式展开式:

$$\int_{-\infty}^{\infty}t^n\exp\left[-(x-t)^2\right]\mathrm{d}t=(2\mathrm{i})^{-n}\,\mathrm{H}_n\,(\mathrm{i}x)\,,\tag{3}$$

$$\mathrm{H}_n\,(\xi)=\sum_{m=0}^{[m/2]}(-1)^m\frac{n!}{m!(n-2m)!}(2\xi)^{n-2m}.\tag{4}$$

整理得到

$$\begin{aligned}U\,(x,y)=&\frac{1}{|\sin\alpha|}\times\left(\sqrt{\frac{R_y\omega^2}{R_y+\mathrm{i}Z_{\mathrm{R}}}}\right)^{2m+1}\exp\left[2\pi\mathrm{i}xy\cot\alpha-\frac{R_y\omega^2\pi^2x^2}{(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha}\right.\\&\left.+\frac{R_x\omega^2\left[R_y\omega^2\pi^2x\cos\alpha-\pi\mathrm{i}y\sin\alpha\,(R_y+\mathrm{i}Z_{\mathrm{R}})\right]^2}{(R_x+\mathrm{i}Z_{\mathrm{R}})\,(R_y+\mathrm{i}Z_{\mathrm{R}})^2\sin^4\alpha+R_xR_y\omega^4\pi^2\,(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha\cos^2\alpha}\right]\\&\times\sum_{a=0}^{m}\sum_{b=0}^{2m-2a}\sum_{d=0}^{\frac{2n+b}{2}}(2\mathrm{i})^{-2m-2n-b}(-1)^{a+b+d}x^{2m-2a-b}\cos^b\alpha\frac{2m!(2n+b)!}{a!b!d!(2m-2a-b)!(2n+b-2d)!}\\&\times\left[\frac{2\mathrm{i}R_y\omega^2\pi^2x\cos\alpha+2\pi y\,(R_y+\mathrm{i}Z_{\mathrm{R}})\sin\alpha}{(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha}\right]^{2n+b-2d}\times\left(\frac{4\pi^2R_y\omega^2}{(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha}\right)^{m-a}\\&\times\left[\sqrt{\frac{R_x\omega^2\,(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha}{(R_x+\mathrm{i}Z_{\mathrm{R}})\,(R_y+\mathrm{i}Z_{\mathrm{R}})\sin^2\alpha+R_xR_y\omega^4\pi^2\cos^2\alpha}}\right]^{4n+2b-2d+1},\end{aligned}\tag{5}$$

其中, Z_{R}为瑞利长度, $Z_{\mathrm{R}}=\pi\omega^2/\lambda$. 由$k=2\pi/\lambda$ (k为波矢, λ为波长)得到:

$$Z_{\mathrm{R}}=k\omega^2/2.\tag{6}$$

3 结果与讨论

利用数值计算方法, 由(5)式可得到Gyrator变换后四瓣高斯光束在变换平面上的光强分布, 并对光强进行归一化. 取$R_x=-R_y=CZ_{\mathrm{R}}$, C为常数, 且

$$R_x=-R_y=Ck\omega^2/2=K\omega^2,\tag{7}$$

其中, $K=Ck/2$为束参数. 由此可知, 对于四瓣高斯光束, 其Gyrator变换的光强与Gyrator变换的变换角α、基模高斯光束的束腰宽度ω、光束阶数n, m和束参数K有关.

图1给出了在Gyrator变换平面上四瓣高斯光束的归一化光强$I(x,y)$分布及对应场的相位分布. 当$n=m=3$, $\omega=0.9$, $K=30$, $\alpha=0.4133\pi$, 0.5867π时, 四瓣高斯光束的分布呈现明显的矩形空心结构, 矩形四角的光强最强, 矩形四条边上的光强几乎均匀分布. 相位分布图则表明, 在矩形中间存在光涡旋, 由于所选取的变换角α互补, 故图1中(a), (a′)和(b), (b′)的光强分布相同, 而相位分布则有$\pi/2$的旋转.

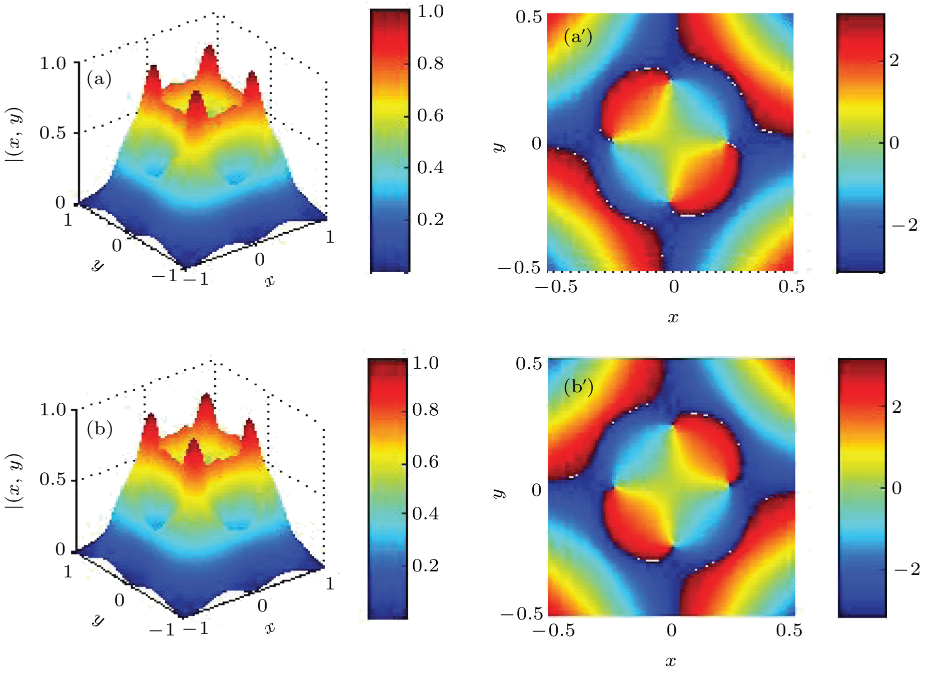

图 1 (网刊彩色) 在 Gyrator 变换面上四瓣高斯光束的归一化光强 $I(x,y)$ 分布 ((a), (b)) 及对应场的相位分布 ((a′), (b′)) ($n=m=3$, $\omega=0.9$, $K=30$) (a), (a′) $\alpha=0.4133\pi$; (b), (b′) $\alpha=0.5867\pi$

Fig. 1. (color online) Intensity distributions ((a), (b)) and the corresponding phase distribution ((a′), (b′)) of a Gyrator transform four-petal Gaussian beam for $n=m=3$, $\omega=0.9$, $K=30$: (a), (a′) $\alpha=0.4133\pi$; (b), (b′) $\alpha=0.5867\pi$.

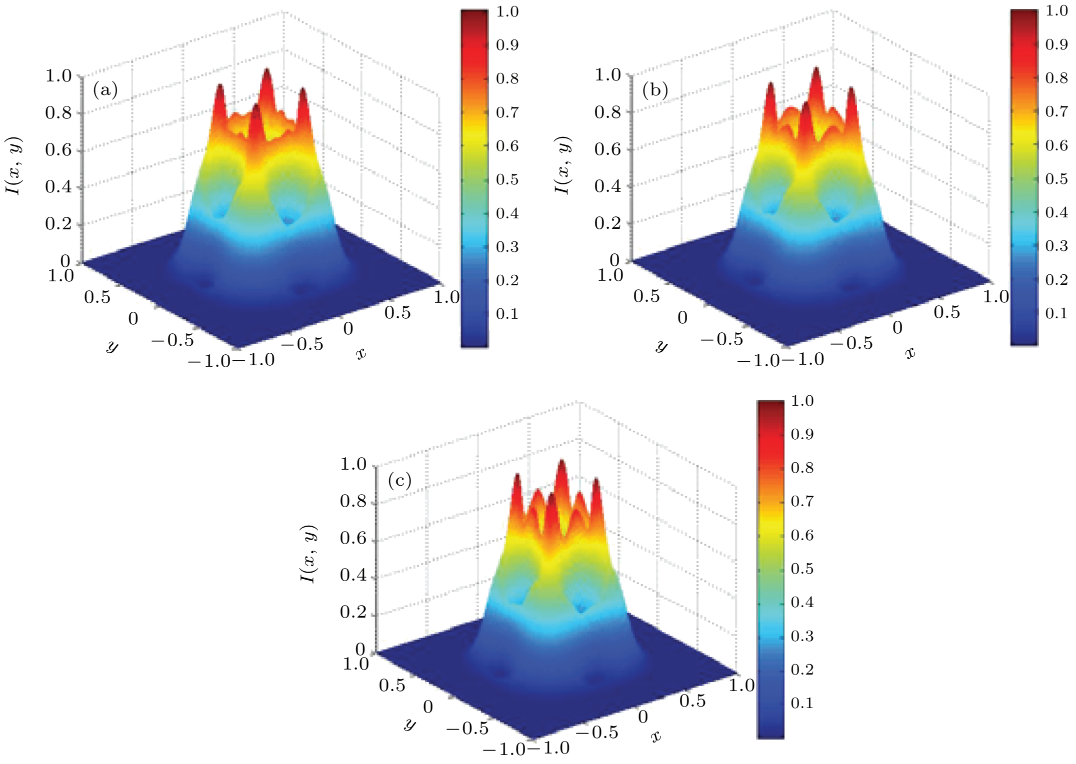

图 2 (网刊彩色) 变换角 α 不同时, 在 Gyrator 变换面上四瓣高斯光束的归一化光强 $I(x,y)$ 分布 ($n=m=3$, $\omega=1.6$, $K=30$) (a) $\alpha=0.4718\pi$; (b) $\alpha=0.4725\pi$; (c) $\alpha=0.4732\pi$

Fig. 2. (color online) Intensity distributions of a Gyrator transform four-petal Gaussian beam for $n=m=3$, $\omega=1.6$, $K=30$: (a) $\alpha=0.4718\pi$; (b) $\alpha=0.4725\pi$; (c) $\alpha=0.4732\pi$.

图2给出了变换角α不同时的光强分布，取$n = m = 3$, $\omega = 1.6$, $K = 30$，图2(a)变换角$\alpha = 0.4718\pi$得到的矩形空心光束的光强分布最为均匀，当变换角α在较小的范围内变化时，矩形空心光束的光强分布会失去均匀性，变换角α变化越大，光束光强均匀性损失越严重，图2(b) $\alpha = 0.4725\pi$ 和图2(c) $\alpha = 0.4732\pi$所示．为了获得光强均匀分布的矩形空心光束，在确定四瓣高斯光束的其他参数条件下，须仔细调节Gyrator变换的变换角α.

图3表示束腰宽度不同时的光强分布，$n = m = 3$, $K = 30$，图中所取的束腰宽度分别为(a) $\omega = 0.8$, (b) $\omega = 1.2$, (c) $\omega = 2.0$．随着四瓣高斯光束束腰宽度ω的增大，矩形空心光束宽度反而越小．因此，可以通过选择合适的四瓣高斯光束的束腰宽度来控制矩形空心光束的大小．束参数K不同时，在Gyrator变换面上四瓣高斯光束的归一化光强分布如图4所示，取$n = m = 3$, $\omega = 1.0$, $\alpha = 0.4286\pi$, (a) $K = 10$, (b) $K = 20$, (c) $K = 30$. 数值计算结果表明束参数K大于10时，总能得到矩形空心光束，且束参数K取较大值时，矩形空心光束的光强分布更为均匀.

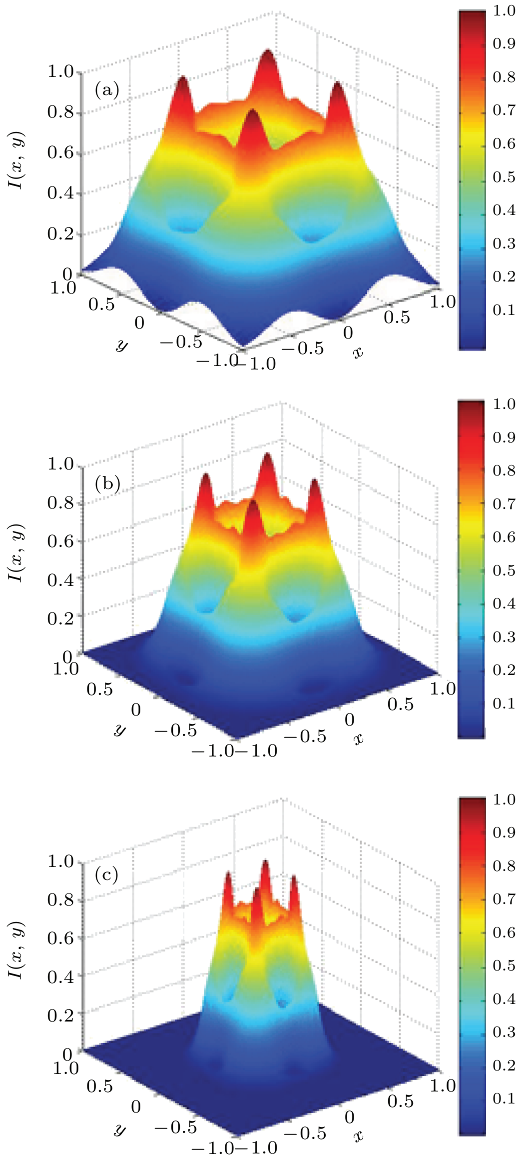

图3 (网刊彩色) 光束束腰宽度ω不同时，在Gyrator变换面上四瓣高斯光束的归一化光强$I(x,y)$分布 ($n = m = 3$, $K = 30$) (a) $\omega = 0.8$; (b) $\omega = 1.2$; (c) $\omega = 2.0$

Fig. 3. (color online) Intensity distributions of a Gyrator transform four-petal Gaussian beam for $n = m = 3$, $K = 30$: (a) $\omega = 0.8$; (b) $\omega = 1.2$; (c) $\omega = 2.0$.

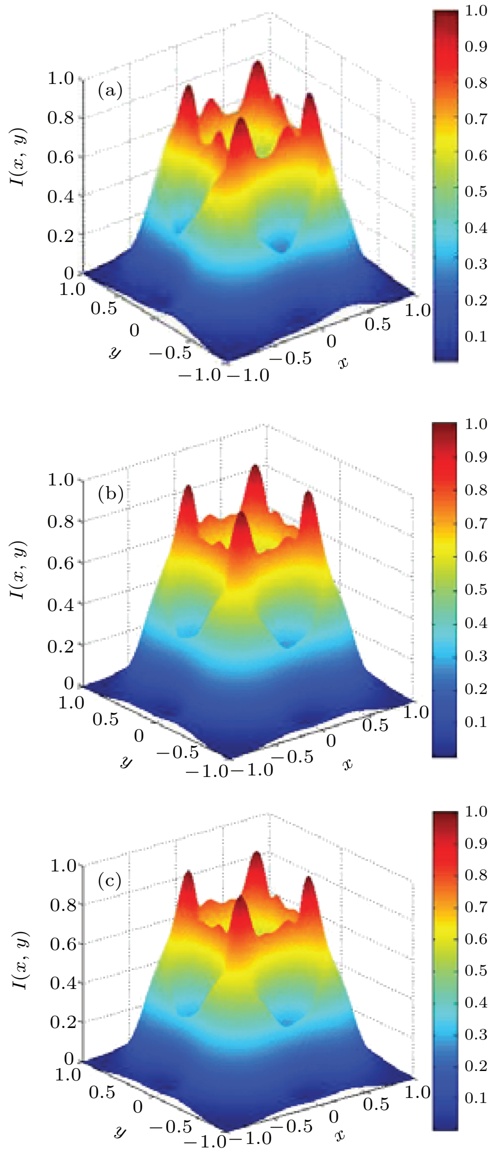

图4 (网刊彩色) 束参数K不同时，在Gyrator变换面上四瓣高斯光束的归一化光强$I(x,y)$分布 ($n = m = 3$, $\omega = 1.0$, $\alpha = 0.4286\pi$) (a) $K = 10$; (b) $K = 20$; (c) $K = 30$

Fig. 4. (color online) Intensity distributions of a Gyrator transform four-petal Gaussian beam for $n = m = 3$, $\omega = 1.0$, $\alpha = 0.4286\pi$: (a) $K = 10$; (b) $K = 20$; (c) $K = 30$.

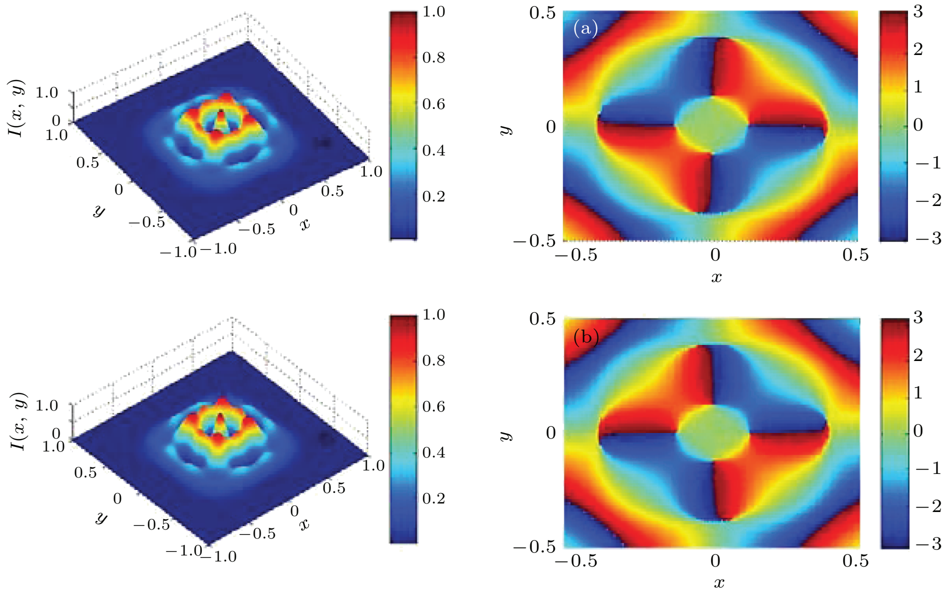

图5　(网刊彩色) 在Gyrator变换面上四瓣高斯光束的归一化光强$I(x, y)$分布(左)和相位分布(右) ($n = m = 2$, $\omega = 1.2$, $K = 30$)　(a) $\alpha = 0.4591\pi$; (b) $\alpha = 0.5409\pi$

Fig. 5. (color online) Intensity distributions (left) and the corresponding phase distribution (right) of a Gyrator transform four-petal Gaussian beam for $n = m = 2$, $\omega = 1.2$, $K = 30$: (a) $\alpha = 0.4591\pi$; (b) $\alpha = 0.5409\pi$.

另外, 如图5所示, 当$n = m = 2$, $\omega = 1.2$, $K = 30$, 图5(a)中$\alpha = 0.4591\pi$; 图5(b)中$\alpha = 0.5409\pi$, 得到了一种新的奇异空心光束, 奇异空心光束和传统空心光束最大的区别在于光束中心带有圆形的实心核, 理论上可以看成是像散高斯模型和像散圆环模型的叠加. 图5(a)和图5(b)所取的变换角α互补, 所以其光强分布相同, 而相位分布则完全相反.

4　结　论

本文由Gyrator变换公式出发, 推导出四瓣高斯光束的场分布解析表达式, 研究了四瓣高斯光束Gyrator变换后的光强分布和相位特性. 通过Gyrator变换, 四瓣高斯光束可以转换成中央具有涡旋的矩形空心结构光束, 矩形空心光束的大小由其束腰宽度ω决定, 束腰宽度ω越大, 矩形空心光束反而越小. 而变换角α则影响Gyrator变换后四瓣高斯光束光强的能量分布. 束参数K大于10时总能得到矩形空心光束. 光束阶数对得到的空心光束的形状影响较大, 当$n = m = 3$时获得矩形空心光束, 当$n = m = 2$得到奇异空心光束. 本文研究结果进一步丰富了Gyrator变换系统与四瓣高斯光束在光束整形方面的应用, 拓展了空心光束在微观粒子控制、信息处理、光镊技术等方面的应用.

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Gyrator transform of four-petal Gaussian beam and generation of rectangular hollow beam

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Abstract

Four-petal Gaussian beam is a special type of Gaussian beam, and its propagation properties are widely used in micro optics, optical communication and splitting technology. Recently, the generations and the properties of different types of hollow beams have become a hot research topic, such as research on hollow optical vortex beams. The Gyrator transform can be used to fulfill the mode conversion of laser beam. In this paper, based on the Gyrator transform, the analytical expression of four-petal Gaussian beam passing through such a transform system is derived, and the intensity distribution and the corresponding phase distribution associated with the transforming four-petal Gaussian beam are analyzed by numerical simulations. It is found that the four-petal Gaussian beam can be transformed into rectangular hollow beam by Gyrator transform, under the appropriate conditions of the beam order, the beam parameter, the transform angle of Gyrator transform, and the waist width. For the beam order $n = m = 3$, the transform angle of Gyrator transform $\alpha = 0.4133\pi$, the beam parameter $K = 30$, and the waist width $\omega = 0.9$, the rectangular hollow optical vortex beams can be obtained. Under such conditions, the maximum intensities appear in the four corners, and they are almost uniform on the four sides. The effects of the beam parameters, the transform angle, and the beam order on the distributions of intensity and phase of the rectangular hollow beam are analyzed in detail. The numerical results show that for the beam parameter $K > 10$, the rectangular hollow beam always is obtained, and for a lager beam parameter, the intensity distribution of the rectangular hollow beam is more uniform. Different beam order generates different type of hollow beam. For example, for $n = m = 2$, $\omega = 1.2$, $K = 30$, and $\alpha = 0.5409\pi$, a new strange circular hollow beam with solid circular nucleus can be obtained. The transform angle of Gyrator transform has a significant effect on the energy distribution of the hollow beam. When the transform angle changes in a small range, the uniformity of the intensity distribution of the rectangular hollow beam is lost. The bigger the transform angle change, the more serious the loss of uniformity of the hollow beam intensity is. The size of the hollow beam bright ring is determined by the waist width of the four-petal Gaussian beam: the larger the waist width, the smaller the bright ring is. The results further enriches the applications of Gyrator transform system and the four-petal Gaussian beam in the beam shaping.

Keywords: four-petal Gaussian beams, Gyrator transform, rectangular hollow beams

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